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"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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## JOURNAL

OF THE

## INSTITUTE OF ACTUARIES.

*Opening Address by the President, FREDERICK SCHOOLING, Esq.*

[Delivered 25 November 1912.]

GENTLEMEN,—It falls to my lot as a new President, in accordance with the unbroken custom, to open the proceedings of this Session with an address. In the first place, I am desirous of thanking you for the indulgent consideration you have shown in electing me as your President for the time being, and I wish to assure you that it will be my constant endeavour to follow in the footsteps of my able predecessors as far as in me lies. With your assistance and with the advice and help of the Council it will be my pleasure and duty to try and maintain the prestige, the usefulness, and the dignity of the Institute.

The difficulty of finding suitable matter for an address grows greater for each succeeding President, although the range of subjects is undoubtedly a wide one. With considerable trepidation I have decided to adopt for the first part of my address the well-worn, but always interesting subject of the rate of interest on Stock Exchange investments—a subject which we actuaries have daily before us, and one which is, I think, of general interest at the present time. I am well aware that there must be a diversity of opinion on many points, and it is, perhaps, a good thing for me that a President's address cannot be discussed.

In the first place, it is well, I think, to investigate and see whether we can gather any lesson from past history, observing

what directions the curve produced by the rate of interest has taken during the last hundred years or so.

We find from Diagrams and Tables that have been prepared for various Societies and Congresses, and from a study of the reports of our old Assurance Companies, that there was a steady and general rise in the rate of interest throughout the financial world during the progress of Napoleonic wars. Then during the years 1815 to 1845 there followed a progressive decline in the rate of interest, after which, for twenty-five years or so, there was, on the whole, a steady rise in the curve. Then from 1872 till 1896 or 1897, we again have a period of progressive decline.

Finally, from 1897 until the present time—a period of fifteen years—there has been a marked advance in the rate of interest.

Now, writers of political economy tell us that the rate of interest is a function of the average rate of productiveness of capital. Professor Charlton T. Lewis, of the United States, pointed out in a discussion on this subject at the New York Congress that to this definition there should be added the fact that a human element comes into play in practice, “that the “actual rate of interest always depends upon the estimates formed “in men’s minds of what the future productiveness of capital “will be”, so that in optimistic times a high rate of interest is prevalent, the reverse being the case when pessimistic views hold the field.

I do not think that the truth of this view can be disputed. If we examine the past we find that the theory applies, generally, that when the spirit of enterprise is abroad, no matter how rich the world has become, the rate of interest will advance.

Take the period between the years 1845–1872, during which the rate of interest rose ; it was during this very period that great discoveries of gold were made in Australia and California—discoveries of enormous magnitude, so large that it is estimated that the annual supply of gold was thereby increased tenfold. We find, however, that the influence of the spirit of enterprise was more than sufficient to counteract the effect produced by the abundance of money. This was the period of railroad construction and the expectations that were formed as to the possibilities of the future of railroads were such as ultimately to bring about a financial crisis owing to scarcity of movable capital in the markets of the world.

It should be noted that fluctuations in the rate of interest are not sudden, but are gradual, moving slowly but surely in

one direction for periods extending over many years, the turn from one direction to another commencing some time after the cause of such variation has been at work. For example, the period of railway construction commenced years before 1845, but it was not until that date that the rise in the general rate of interest made itself felt. Any rise in the rate of interest is naturally fought against by holders of securities, who do not want to see their holding depreciated in value; dealers on the Exchanges of the world who have purchased enormous quantities of stocks to sell again, strain every nerve and use every endeavour to maintain prices rather than sell at a loss. Hence any rise in the rate of interest, or what is the same thing, any general fall in the value of securities, follows, tardily but surely, times of commercial activity and enterprise. In making these remarks, I am, of course, referring to rates of interest in the widest sense, for it must always be the case that particular investments—or even a class of investments—may be influenced by other considerations than those which are shaping the destinies of the financial world as a whole.

Coming now to present times, we shall, I think, find many reasons which account for the progressive increase in the rate of interest which has taken place since the year 1896.

In the first place, it will be found that during the last twenty years or so, the investing public, not only of this country, but of France, Germany, Belgium and Holland, have made the discovery that the field for investments is a world-wide one. Stockbrokers tell us that, twenty or thirty years ago, eighty per-cent of the securities purchased by their clients were British securities; now the proportion is reversed. Insurance Companies have a much larger proportion of their funds invested in foreign securities than they had twenty years ago. Lawyers and bankers advise foreign securities as investments to their clients without any hesitation in these modern days. The very fact that this state of things exists itself helps to maintain, if it does not create, the spirit of enterprise all over the world. When at the International Congress of Actuaries at Amsterdam a few months ago, I took advantage of the opportunity and discussed the question with representatives from all parts of the globe; they one and all told the same tale—that capital was being used for the development of the national resources of the country to which they belonged, but that still more was required. I had been told by a high authority before leaving for Amsterdam

that at least two hundred million pounds a year (not dollars) would be wanted for many years to come to develop and widen the railroads of the United States. I found that the representatives of that country looked upon the figure as a conservative estimate, because in addition to the new railway construction necessary to keep pace with the ever-growing population, the tracks of many existing railroads will have to be doubled; and in some States, owing to recent legislation, large capital outlay will be required in order to carry the tracks over or under the public roads, instead of along them.

If we turn to Canada we find the demand for capital for railroad construction greater in proportion than even in the United States; and the same want exists in a varying degree in all countries—all over the continents of Asia, Africa, and South America.

Another force that has played a very important part in the wonderful industrial activity of recent times is that of electricity. The installations of telephonic systems in every part of the civilized world has called for the use of enormous sums of money. This country has, perhaps, lagged behind in this development in comparison with others, but even here we recognize the revolution it has caused in business and social methods of communication. The capital required for telephonic installations is, however, but one side of the part played by the use of electricity in modern developments requiring great capital. It is no exaggeration to say that the introduction of the electric tramway has caused to be repeated in some degree the universal demand for capital that was experienced during the railroad construction period in early Victorian times. No town—I was almost saying no village—is now without its system of electric tramways. Largely owned by municipalities who have borrowed the money to build them, they undoubtedly supply a want, and are one of the means used to solve, or to attempt to solve, the great housing question which is so great a problem of modern life in the big cities of the world.

Then, again, we have the use of electricity as an illuminant. Electric-light companies have absorbed, during the last thirty years or so, an enormous amount of money without disturbing in any marked degree the capital invested in gas companies.

Another feature of modern times which, I think, is worthy of notice, is the craze for practically rebuilding towns and cities. Look at the main thoroughfares of London; we find them filled

with new and palatial buildings, all costing large sums of money, mostly borrowed in some form or another. As a rule, either debentures are issued, ground rents created, or mortgages granted to cover the whole or part of the cost. What is true of London is true of many another city, both within and without the United Kingdom. Berlin, for example, is a wonderful city of new buildings.

I might go on enumerating examples of industrial activity during recent years, but enough has been said to show that the rise in the rate of interest which has occurred is due to the wonderful expansion in industrial enterprises which began years before its effect was allowed to be felt on the Stock Exchanges of the world, and to the fact that the whole world is now looked upon as a field for safe investment.

No remarks on the rate of interest would be complete without some reference to the part that wars play in causing fluctuations. The immediate effect is, of course, to cause depreciation in the value of securities and a rise in the rate of interest, but the after-effect is probably the reverse. We find that after the Napoleonic wars there was a progressive decline in the rate of interest for a long period, and the same feature is observable after the close of the Franco-German war. This, I think, is not mainly because the war is over, but because a great war kills the spirit of enterprise, makes men act with extreme caution, and renders invention stagnant. Some people thought that the rate of interest would fall after the close of the Boer war, but the industrial activity was such as to more than counteract any tendency in this direction. The experience of recent years may be taken as showing that the Boer war played but an insignificant part, if any, in bringing about the present condition of the investment market. Neither do I think that recent legislation has contributed in any marked degree towards that condition.

It has been the fashion with some writers on financial matters to treat Consols as a thing apart from other investments, and to argue that a magic circle should in some mysterious manner be formed around the British Premier Security, whereby the price should be maintained in spite of a general rise in the rate of interest. It is difficult to see how any device can achieve this object unless it offers some real and valuable addition to the interest-yield, or, in other words, unless the Chancellor of the Exchequer makes a present to the investors in Consols at the expense of the taxpayer—a thing no Chancellor is likely to do.

Investors are not fools, and no offer of a shadow without substance would tempt them to invest in Government securities against their better judgment. As loyal Englishmen, we do believe that Consols are the finest security that can be purchased, but as investors we find that there are many other very safe investments which answer our purpose, because of the better interest-yield. It has been suggested that the heavy rate of Death Duties now required has been one cause of the fall in the price of Consols, because, owing to the large sums needed for this purpose, huge blocks of stock have been thrown on the market which otherwise would have been held. No doubt this is the case, and it may have hastened the rate of the fall in the price of Consols, but I doubt whether it has done more than this. Consols are the premier security, but so long as there is a world-wide market for excellent investments of unimpeachable safety, the price of Consols can only be expected to be such as to return an investor a slightly lower rate of interest than other Trustee Stocks.

It is sometimes claimed that United States Bonds yield a lower rate of interest than the Government debts of other countries, but on examination this is found to be so only because of the artificial market created by Law. As a matter of fact, it may be said that United States Bonds are not an investment security at all ; they are almost all held by banks for a special purpose. National banks are required to deposit with the Treasurer of the United States at least \$50,000 par value of United States registered bonds upon the issue of their Charter, unless their capital is less than \$150,000, in which case they must deposit United States registered bonds to one-fourth of their capital. Also, any excess in the circulation of notes over the amount of their Charter Bonds must be met by the deposit with the Treasurer of an equal amount of Government Registered Bonds. United States Bonds are also largely used by the National banks as the security for public monies received by them, the bonds being deposited with the Treasurer. Thus, to compare the rate of interest yielded by United States Bonds with that yielded by Consols is to compare like with the unlike—a thing an expert should avoid doing, whatever else he does.

Italian Rentes form another example of a Government security which is likely to be maintained at an artificial standard in years to come, for under their Insurance Laws a monopoly is created which compels Life Insurance Funds to be invested in Italian

Rentes—an indirect method of taxing policyholders repugnant to British ideas.

As to the probable rate of interest obtainable on Stock Exchange investments in the future, I have no intention of prophesying—the possibilities are so many and varied. The human element in the problem is so great as to render any forecast into the future, indeed, foolhardy. It may be that under stress of circumstances, repudiation by some Country, State, or Municipality will bring about a panic, and cause a rush back to trustee stocks and so-called gilt-edged securities. It is, however, no good indulging in conjecture as to what may or may not happen, for if anything does occur to disturb the present custom of world-wide investment it will probably be the unexpected.

Hitherto I have dealt with only one class of investment in which Life Assurance Companies have great sums at stake; there is, however, one other which certainly should be mentioned at the present time, not for the purpose of discussing the interest-yield, but because of threatened taxation. The class is that of investments dependent upon the value of land. I find that, according to the last Board of Trade Returns, the British Life Companies held the following amounts :

Land and House Property and Ground Rents £44,483,203

Mortgages on property in the United Kingdom 68,000,000  
In other words, British Life Companies are directly interested in property of the nature of realty in the United Kingdom to the extent of over £110,000,000 in capital value. They are indirectly interested in respect of other classes of investment whose security is largely dependent upon the value of land, such as Commercial and Industrial Debentures, Railway Ordinary and Debenture Stocks, and loans on, and investments in, Reversions and Life Interests.

It is thus very evident that any question affecting the manner of taxing land must be a matter of the first importance to all Life Assurance Companies, whatever be the nature of the suggested alteration, or whatever the form of the proposed new tax. The creation of a feeling of unrest in itself depreciates the value of property, one result of which will naturally be a smaller return from the new increment-duty tax than was anticipated.

Any entry upon the realms of controversial politics would, of course, be wrong and out of place in an address of this description, but questions involving taxation have, and always must have, an interest for actuaries individually, and for the Institute

of Actuaries as a body, so that when a proposition is seriously put forward which affects directly over one hundred million pounds of the invested funds of Life Assurance Companies, it is only natural that some remarks on the subject should appear in a Presidential Address.

The suggested taxation of land or site values is a somewhat obscure proposition, and it is difficult to get details as to the exact nature of the proposals. The advocates of the tax vary between the extreme followers of the late Mr. Henry George, who openly advocate the levy of a tax of twenty shillings in the pound in order to bring about the nationalization of the land, and those who aver that they have no designs upon agricultural land, but who agree in wishing to put additional taxation upon urban and semi-urban site values. It is worthy of note that the present agitation has been largely carried on in agricultural districts, has been pushed to the front in parliamentary contests in agricultural constituencies, and that enquiries have been made by a self-appointed inquisitorial committee as to the customs and methods of large agricultural landlords.

In a Report of the Land Values Group of Members of Parliament, Messrs. Chomley and Outhwaite estimate the total site-value of the United Kingdom to be about six thousand million pounds. Mr. Savill, in a paper recently contributed to the Surveyors' Institute, estimates the value to be just half this sum, viz., three thousand million pounds.

There is a prevailing idea that a system of land taxation would fall upon a few large land owners in the United Kingdom, but have little or no effect upon the general body of the public. This impression is quite fallacious, for not only are an enormous number of the population themselves landowners, but an immense amount of property in the nature of realty is held by public companies, Banks, Insurance Companies, &c., in all of which the public are interested, either as shareholders or policyholders.

With regard to the number of people with small incomes who are interested in land, it may be noted that while the gross income received from the ownership of land, houses, &c., in the United Kingdom in 1909-10, amounted to two hundred and seventy-two million pounds, over thirty-five and a half millions of this amount was received by persons whose total incomes from all sources was under £160 per annum; *i.e.*, over one-eighth of the total income received from land and houses in the United Kingdom was received by persons who did not pay Income Tax

(*see Inland Revenue Report*). The last date on which any official information is available showing the number of landowners in England and Wales is 1876. This Return shows that (exclusive of London) there were no less than 825,272 persons in England and Wales owning land of less than ten acres in area. The Return also shows that these holdings are, as might be expected, worth more per acre than the large estates.

We, here, although much concerned and interested in the general question, are vitally concerned in maintaining the position, that in any future legislation existing contracts made as investments must be considered sacred.

The importance of this is at once seen if we look at one of the mildest proposals that has been instanced as a new form of taxation. It has been suggested that an annual tax of 2*d.* in the £ on the capital value of land should be levied. As a rule, the suggestors excuse themselves for being so moderate by explaining that this 2*d.* is, of course, only a commencement.

Take the case of ground rents, where the rents have not been improved, and if capitalized, fairly represent the value of the land ; ground rents that have been purchased as an investment under existing laws by Insurance Companies, Trustees, or private individuals. Now, assuming that such a ground rent is worth twenty-five years' purchase, a tax of 2*d.* in the £ on the capital value would be equivalent to a deduction of 4*s.* 2*d.* in the £ from the ground rent payable, because the tax would have to be paid annually out of the income derived from the investment. In other words, a ground rent of £100 per annum would be reduced to £79 4*s.* per annum, and the market value of such a ground rent would be reduced from £2,500 to £1,980—a depreciation of over 20 per-cent, not taking into account the further depreciation which would inevitably take place owing to the attack on this class of security.

The gentlemen who go about the country talking of thousands of millions include in their totals all descriptions of property in whatever manner it may have been acquired, and assume that all will be taxed if their nostrums are swallowed.

But we have reason to believe in the fairness and equity of Governments, for on two occasions when the Death Duties have been increased, exceptions have been made in favour of purchasers of reversions for value, the duty payable at the death of the reversioner being at the rate leivable at the time the reversion was purchased, and more than this, we have the comforting assurance of the present Prime Minister who, when Chancellor of the Exchequer, said, on 10 July 1907, on the

occasion of the opening of the new Head Offices of the United Kingdom Temperance and General Provident Institution : " I believe that insurance companies, both here and in Scotland, " are largely interested as investors in ground values, or what are " called in Scotland 'feu duties' or 'ground annuals,' and securities " of that type; and I imagine that there is a certain amount of " apprehension among investors in this class of security as to the " possible effects on their investments of prospective or projected " legislation. I think that they need be under no apprehension " whatever on the subject. So far as I am acquainted with the " facts—and I suppose I ought to know—in any legislation that " is likely to be proposed in regard to matters of that kind they " may be certain that existing contracts will be rigidly respected " as sacred. There is no intention under any pretext of public " policy or otherwise to rip up obligations which have been incurred " in good faith and for value. Legislation must proceed with that " for its starting point and underlying assumption."

Another class of investment which would be seriously disturbed by any alteration in the method of taxation—such as the substitution of site values for assessable values—is that of Loans on Public Rates, Municipal and County Securities. When it is remembered that the total local debt in the United Kingdom, according to the latest report available (1908-9), is just under six hundred million pounds, it will be seen that any alteration in nature of the security upon which this huge sum has been borrowed must cause uncertainty and confusion, if it does nothing else. British Life Offices have about thirty-two million pounds invested in this class of security. The legal titles of the Municipalities, Counties, or Boards borrowing are investigated by the lawyers as the loans are contracted, for it is not an unheard of thing for a local body to imagine they have borrowing powers, or that they have power to borrow for a certain purpose when in reality their sanction is for another. The legal difficulties which would follow any disturbance of the present security given for local indebtedness can probably only be fully understood and revelled in by the legal mind. Companies might also find that a loan which had been looked upon as quite a gilt-edged security on account of the small debt of the borrowing body in comparison to the assessable value, and of the low rate per £ at which the rates were levied, had become one of quite a different description through the alteration in the method of taxation.

Although it may be that legislation affecting land values is not now as imminent as it at one time appeared to be, it is the duty of all of us to guard jealously the interests of policyholders by keeping a watchful eye on any proposed legislation.

My Address would probably be considered incomplete unless some reference were made to the National Insurance Act, 1911, more particularly to Part I, National Health Insurance. It is worthy of note how little is heard of Part II, Unemployment Insurance. In ordinary times the coming into operation of a new Act which compels insurance against unemployment in certain trades would have created much discussion and no little irritation, for there are many fine points to be decided, and it speaks well for the permanent officials at the head of the administration that things have worked so smoothly. No doubt the wide-spread and universal interest taken in Part I of the Act has to an extent monopolized the public ear, and the unemployment sections have not received the attentive criticism which they would have attracted had they stood by themselves as an independent Act of Parliament. Unemployment insurance is under control of the Board of Trade whose powers under the Act are very extensive; as in the Health Insurance Scheme there is no Government guarantee that any stated rate of benefit shall always be paid. Under Section 102, the Board of Trade have the power to revise the rates of contribution, the section being as follows :

“ If at any time after the expiration of seven years from the commencement of this Act it appears to the Board of Trade that the Unemployment Fund is insufficient or more than sufficient to discharge the liabilities imposed upon the Fund under this Part of this Act, or that the rates of contributions are excessive or deficient as respects any particular insured trade, or any particular branch of any such trade, the Board may, with the sanction of the Treasury, by special order made in manner hereinafter provided revise the rates of contribution of employers and workmen under this Part of this Act, and any such order may, if the Board think fit, prescribe different rates of contribution for different insured trades or branches thereof, and, where any such order is made, the rates prescribed by the order shall, as from such date as may be specified in the order, be substituted as respects trades or branches thereof to which it relates for the rates prescribed by this Act.

“ Provided that, where such a revision has been made, no

“further revision under this section shall be made before the expiration of seven years from the last revision, and that no order under this section shall increase the rates of contribution from employers or workmen by more than one penny per workman per week above the rates specified in the Eighth Schedule to this Act, or shall vary such rates unequally as between employers and workmen.”

The Seventh Schedule deals with the rates and periods of unemployment benefit, and gives the Board of Trade power to vary the rate of benefit between the limits of six and eight shillings per week. I mention these points because they involve actuarial questions. The data upon which the rates of contribution and benefit were decided were prepared by an eminent Fellow of this Institute, Mr. Ackland, whose able Report on the subject has been published. No doubt the Board of Trade will seek actuarial advice and assistance before deciding whether an unemployment fund is insufficient or more than sufficient to provide the benefits given.

It is, however, the National Health Insurance Part of the Act which engrosses the attention of the public, and is also of more interest to actuaries. Comparison has often been made between our new Act and the German laws dealing with the same subjects. We know that our Government made a careful study of German methods before drafting their Bill, but different laws, habits, and customs prevented imitation to any extent, although, no doubt, the experience of the twenty years which the Germans have passed through was of great value to our legislators. It is worthy of note that since the introduction of the sickness and invalidity laws in Germany there has been a constant stream of amending legislation, culminating in a great measure for the unification and reform of the whole system which was considered by the Committee of the Reichstag for two years, and passed rather more than a year ago by the German Imperial Parliament. It is to be hoped that we shall escape many of the dangers experienced in Germany, for there they have had not only difficulty with doctors, but communal and political difficulties and an immense amount of litigation owing to the enormous number of disputes which numbered hundreds of thousands per annum. In spite of all this, Germans tell me that, in their opinion, the Insurance Laws have, on the whole, been for the benefit of the public, and have undoubtedly helped towards maintaining the welfare, security, and contentment of the German working

classes. One disquieting fact is that during the early years of the operation of the German laws, the Old Age Pensions and the Invalidity Pensions were about the same in amount per annum. In the year 1907 the total amount paid for invalidity pensions was nearly seven times as great as that paid for old age pensions.

The question of malingering is a difficult one, and it is improbable that deliberate shamming has been common in Germany, but probable that the sickness insurance laws have had a tendency to make people's consciences unconsciously elastic, to accustom them to consider that what was not a sufficient cause to incapacitate from work before the passing of the Act, had now become a sufficient cause to do so. No doubt the change has been of gradual growth, almost imperceptible, but none the less sure. Action has recently been taken by the German Government in order to introduce a more rigid spirit of administration, and as a consequence there has been a considerable decrease in the number of new invalidity pensions granted per annum. It is admitted with regret that there has not been the development of character anticipated as a check upon abuses, and as a help to prevent possible miscarriage of justice.

The proportions of contributions paid respectively by the workman, the employer, and the Government under the National Health Insurance Act are well known to all here, and there is no need to repeat them, but it will be as well, as I have given some particulars of the German experience, if I here state the proportions of their insurance contributions. In their sickness insurance, two-thirds of the cost is paid by the workman, one-third by the employer, and the Government make no contribution. In the case of accident insurance, which is included within the scope of the German Insurance Act, the whole cost is paid by the employer. In the case of invalidity and old age pensions, one-half of the contribution is paid by the workman and one-half by the employer, and to this the Imperial Government adds a subsidy of M.50 per annum per pension granted under any one of the various provisions of the law. It should be stated that the Germans have one governing body, whose headquarters are in the Imperial Insurance Office at Berlin, to regulate accident insurance and old age pensions, as well as sickness and invalidity insurance, although different subsidiary agencies are employed, whereas we in the United Kingdom have our Workmen's Compensation Act, and our

Old Age Pensions Act separately administered by different governing bodies from that administering sickness and disablement insurance.

It has been mentioned that there is an immense amount of litigation in Germany, and it should therefore be stated that all Court proceedings are free. They begin in the local Arbitration Courts, and, if there is an appeal, end in Senate proceedings in the Imperial Insurance Office Buildings. These final Courts are in continuous session and consist of seven members, one of whom is a workman and one an employer of labour. Quite a common description of case is that in which there is a dispute as to whether a claimant for an invalidity pension has or has not paid the necessary 200 weeks' contribution, or whether the claimant has not come under some rule under which he or she would be exempted from paying the full 200.

Turning now to our own Act, we note that the actuarial investigations and calculations necessary were of a nature so difficult and complicated, that I am sure we are all proud of my two eminent predecessors who so ably dealt with so many problems and such seemingly overwhelming difficulties. The joint actuarial reports of Messrs. G. F. Hardy and Wyatt are productions which will for many years be a source of profitable study, not only to actuarial students but to many others outside the actuarial profession. In these reports the dangers and difficulties of the problem are courageously faced and pointed out. Without their aid it would have been difficult to insist that nothing more than the amount allowed in their calculations could be taken from the contributions for the purpose of settling with the doctors, and the thanks of the insured public are due, whether they know it or not, for this service.

As to whether the Manchester Unity experience will be a true guide for the future, and as to whether there will be a continuance of the conditions observed by their Lodges, time alone will prove, but it was the only standard available. It is admittedly a leap in the dark, at any rate as far as female risks are concerned ; the actuaries, in their report, pointed out that there were no complete data dealing with sickness rates among women. The question of the adequacy of the rates, however, must largely depend upon whether the approved societies can exercise the same amount of strictness in supervision as the Affiliated Orders have done in the past. In the fact that accidents are provided for under the Workmen's Compensation Act, and that, therefore, nothing

will be payable, or at any rate only the amount (if anything) by which compensation falls short of the allowance under the Health Insurance Act, the actuaries provided a margin ; and, again, a further margin arises in consequence of no modification having been made to allow for the sickness benefit beginning with the fourth day of sickness. Further, there was an addition of 10 per-cent, made to the Manchester Unity rates, and other means adopted, to strengthen the actuarial basis. During the passage of the Act, however, various concessions and increases of benefit were made which resulted in considerably decreasing these possible margins.

One of the main questions that had to be settled at the outset was the respective merits of the level premium plan and the assessment plan, as applied to State insurance. The plan adopted marks a great advance upon the Old Age Pensions scheme, inasmuch as provision is made for the accumulation of the necessary reserves within a period of eighteen and a-half years. We have shouldered our own burdens in this respect instead of leaving them for posterity to bear. The ingenious manner in which the question of the reserves required under the level premium plan has been solved is worthy of the greatest admiration.

On the question of the valuations the sound actuarial principle has been adopted of insisting that where a deficiency has been disclosed, it shall be promptly dealt with. Most of us who have had to make reports on Friendly Societies in the past know the difficulty of making members realize that the deficiency is real. Too often, when pointing out the necessity of taking the matter in hand, we have been met with the immediate rejoinder : “ There always has been a deficiency and there always will be one. ” “ Why should we trouble about it, when we have a large sum of ‘‘ money in hand ? ” It is eminently satisfactory, therefore, that the Act insists that where any deficiency is disclosed in the valuation of any approved society the immediate result will be a reduction of benefits or a call for extra levies.

The introduction of the National Insurance Act has necessitated the establishment of a New Government Department with an army of officials. The work of starting on its course so complex an Act is stupendous ; no one who has not been behind the scenes can have any idea of what it really amounts to. Not only has it entailed work upon civil servants, but the staffs of the approved societies have had to work day and night in order

to cope with the endless questions that arise in dealing with fourteen million people, each one of whom had to be instructed. It is doubtful whether sufficient progress could have taken place to enable the act to be put in force by the specified time if the services of great organizations had not been available for the purpose.

It has been the fashion in days gone by to represent civil servants as "eleven to three" gentlemen of the Tite Barnacle description. If Dickens or Thackeray had come across the Insurance Commissioners, their actuary, or secretaries, or any member of their staff, another type of man would have been portrayed, for these gentlemen have had no limit as to office hours—no eight hours a day for them; they have courteously devoted their great abilities to solving problems and mastering details which must have taxed their health and energies to the utmost. It is a great pleasure to me, and I hope it will not be considered presumption, to have this opportunity of placing upon record my opinion of the modern civil servant, that in the permanent officials we have a self-denying, able, hard-working set of men who form a most valuable national asset.

Since the close of our last Session the Seventh International Congress of Actuaries has been held. It was opened on 2 September by His Royal Highness Prince Henry of the Netherlands. Professor Müller was elected President of the Congress, and Dr. Van Schevichaven, Secretary. Dr. Humskerk, the Dutch Prime Minister, welcomed the Congress in the name of the Dutch Government in felicitous terms, in which he said that all Governments must in the present times take into account the uses to which actuarial science can be applied.

During the week interesting papers were discussed, including many written by members of our Institute; the subjects chosen were practical and up-to-date, but were fewer in number than at some previous Congresses. The first discussion was on the subject of "Reassurances in Life Assurance", and Dr. Paraira gave a summary of all the various papers written for the Congress on this subject. This method was followed throughout the proceedings, at the commencement of each separate subject, someone, usually a member of the Dutch Committee, gave a short and concise summary of the various arguments which the respective writers on that subject had used; then followed the discussion, and after that the writers of the papers had their say. One of the most interesting discussions was that on the

Public Administration of Old Age Pensions, the question as to whether such schemes should be contributory or not, and whether if contributory the formation of an immense national fund was necessary. Opinions were sharply divided, and many intending speakers were crowded out for want of time. In the discussion on "The Course of the Rate of Mortality amongst Assured Persons since 1800", attention was almost universally called to the fact that the improvement in the rate of mortality was very marked in the younger ages, but that after age 55 or so it was a doubtful point as to whether any improvement had been shown.

The final discussion was devoted to the subject of the Calculation and Loading of Office Premiums, and at the close an invitation to hold the next Congress at St. Petersburg was given by the Russian delegates. Throughout the week the members of the Congress were entertained in every possible way ; our Dutch hosts exerted themselves in a wonderful manner. During the business sessions the visiting ladies were conducted round the sights of Amsterdam and the neighbourhood by the wives and daughters of the entertainment committee. On the Thursday there was no business meeting, but excursions, involving the provision of special trains and steamers, were provided for all the members of the Congress and their friends to visit different parts of Holland. Not only on this occasion, but on every day of the week, was some wonderful entertainment or banquet prepared for our edification, and we who enjoyed the hospitality know what thought and trouble were expended on our behalf and are desirous of expressing our thanks.

I think, however, that the indirect benefits of an International Congress may be said to be greater than the direct. The throwing together of men of all nationalities from all parts of the world, the giving them opportunities of exchanging ideas and gaining information is of more importance, probably, than the discussions, or even than the entertainments. New light is thrown upon international topics, real information is obtained as to the manners and customs in foreign lands, and insular prejudices are rounded off. We obtain our knowledge of what is happening in other countries from our newspapers, we have no time to do otherwise ; but this source of information is no guide to the real condition and habits of a people. As an example, in talking over this point with an American at the Congress, he remarked that if he had to judge the people of the United States from what he read in the European newspapers, he should think that they were

generally given over to murder and divorce, for he had read of little else in the American news as telegraphed from New York to these papers.

We also learn to see ourselves as others see us. When talking over the question of the rate of interest on investments with representatives of other countries, and mentioning the possible danger of loss in the future, through hostile legislation, sharp practice bordering on dishonesty, or unrest and possible insurrections, which I, as an Englishman, thought foreign investments were in some degree subject to, I was surprised to find that they were of opinion that we, in the United Kingdom, were quite as liable to have our home investments disturbed from these causes as they were in their countries. In fact, many thought that so far as the first and last causes were concerned, we were more liable.

I must not close without some reference to the personal and professional side of the Institute. Although it is true that the number of Life Assurance Companies is smaller than in years gone by, thus curtailing the number of actuaryships of companies which are open to the members of the Institute of Actuaries, I think that the openings in other fields fully compensate for the reduction in the number of premier positions in Life Assurance Companies. The National Insurance Act opens up new channels for actuarial activity, of which it is difficult to see the end. In these days when pension funds are required for large establishments in nearly all branches of commerce and industry, it is of the utmost importance that they should be founded upon sound actuarial advice given by a Fellow of our Institute, and periodically looked into and valued as the years go by. An actuarial training should also be of great value in advising on any profit-sharing scheme. In such schemes questions of extreme complexity involving points which require careful calculations founded upon the past history of the concern often arise. Such past history should be analysed by skilled hands, and the results obtained clearly set forth in an understandable way. This brings me to a point which I much desire to impress upon our younger members and students; it is that forty years' experience has taught me that actuarial reports should be set forth in a manner which can be understood by intelligent men. The day for a report which gives results without the reasons for arriving at those results has, in my opinion, gone by. It does not follow because the readers of reports are unable to make the actuarial

calculations involved in the investigations, that they are unable to understand the reasons for, and verbal explanations of, what has been done. It is so manifestly to the interest of the modern actuary that he should not be a man of mystery, that he should be a man conversant with men and their doings, on no pedestal of superior knowledge, but always willing to discuss with others the why and wherefore of technical subjects, not to be tempted to say to non-actuarial friends that it is no good explaining the point because it would not be understood if it was explained.

Actuarial training should help men to success in all branches of insurance work, and it is with satisfaction we note that this is being recognized both in this country and in America ; a man should not be out of the running for what is termed a "non-actuarial" post, because he has had an actuarial training ; his claims should be admitted and considered with others. No preference is asked for, but only that a good all round man with actuarial knowledge should have a chance equally with the good all round man who does not possess that knowledge.

On reading over this Address I am painfully aware of the sketchy and terse nature of what I have written ; in it I have endeavoured to bring before you some of the modern problems which face the actuary of to-day, knowing that I am in possession of your kind consideration and good will, and throughout imbued with the fact that I am a member of a profession which has great possibilities and opportunities confronting all its members.

*Some Observations on Currency and Credit and their Influence on Trade and Exchange. By WALTER THOMAS MAY, F.I.A., of the Liverpool and London and Globe Insurance Company, Limited.*

[Read before the Institute, 16 December 1912.]

THE subject of my paper to-night is, I fear, outside the range of problems usually discussed at the Institute, but I do not ask any indulgence on this account as I am sure we all feel the importance of the question and its intimate bearing on the prosperity of the country. I only wish it was in more able hands.

I do not think it would be an exaggeration to say that the dawn of civilization was heralded by the advent of a medium of

exchange, and I suppose that one of the greatest boons to mankind that has happened since has been the further simplification of the exchange of commodities made possible by the use of credit.

I propose to divide the subject up under several headings, and to consider within the limits of time available the principal functions money has to fulfil both as the measure and standard of value, and as the medium of exchange, and in order to avoid ambiguity when using the word money, I will endeavour to confine my reference to coin and instruments of exchange which actually pass "current" from hand to hand without limitations, such as bank notes.

#### MONEY AS A MEASURE OF VALUE AND STANDARD FOR DEFERRED PAYMENTS.

It does not seem possible to conceive of a time, even in the days when commodities were exchanged without the intervention of a medium, in which some standard substance was not used, perhaps unconsciously, to estimate the *quid pro quo* to be demanded in exchange for an article and, when the system of barter was displaced and a medium of exchange introduced, it was only natural that commodities should be thought of in terms of the medium, and it should become the measure of their value. As civilization advanced it was not long before contracts were entered into and debts incurred where it was not possible to repay the exact thing originally lent, and it was necessary to have some substance of value to all and enter into a contract to repay the equivalent of the debt in terms of it.

A difficulty that besets the student of this subject at the outset lies in the fact that from its nature it is not possible to have any fixed unit of value against which to measure commodities in the way that you have a fixed unit of length. The value of anything depends on the supply of it and the demand for it, and these are constantly varying, so that in choosing a standard the utmost we can do is to select some commodity or combination of commodities, the value of a unit of which may be expected to remain fairly uniform in relation to the value of the average commodity.

How necessary it is for the standard substance not to fluctuate wildly or vary greatly in purchasing power, if it is to carry out its function properly, is apparent when we consider that a rise in the purchasing power of the standard places all debtors in the position of having to repay more real value than the amount

borrowed, while creditors receive more than their just due, while if a fall occurs the positions of debtor and creditor are reversed.

We can the more readily appreciate the importance of this function of the standard of value when we consider the question of long deferred contracts. In fact I have found the consideration of the possibilities of the purchasing power of the standard 100 years hence—say, on the maturity of a leasehold redemption policy for that term—quite an interesting speculation.

It might be possible partially to meet this difficulty regarding the fluctuation of the standard and to do rough justice to both parties if a composite standard based on the purchasing power of the medium with regard to staple commodities was set up. An average of the prices of the commodities in some fixed substance would have to be taken at intervals and all contracts due would fall to be settled in accordance with the purchasing power of the standard substance at the time of settlement.

Unfortunately, however, the difficulty, which is experienced in compiling any series of index numbers, of giving the requisite weight to each commodity, and the complexity of such a standard would so seriously inconvenience business and weaken the stability of contracts that it is not in any way a practical proposition.

Coming to the actual standards used by all nations—the precious metals gold and silver, either alone or in combination—I wish to consider for a few moments some of the advantages and disadvantages of single and double standards of value.

In the first place let us take the joint or Bimetallic Standard, which in some form or other was in use for the last five or six centuries in Europe, and by which I mean the system whereby both metals are freely received at the Mint and coined, circulating together as legal tender at a fixed ratio for unlimited amounts. In earlier times the price of silver seems to have been about 1:11 of that of gold, but during the last two or three centuries it has oscillated between 1:14 and 1:16, and has approximated for the greater part of this time fairly closely to the ratio 1:15 $\frac{1}{2}$  fixed by France in 1785 (and confirmed in 1803) for receiving both the metals freely at her Mint for coinage.

This joint or Bimetallic Standard was, as we know, suspended in 1873, owing to fears that the supply of silver was to become very plentiful, while it was felt that the demand was bound to slacken on the abandonment by Germany of silver as her standard of value, and there is no doubt that it had become almost

impossible for France and the other nations associated with her in the Latin Monetary Union to maintain their position as the make-weight between the two metals as they, or rather France, had done so successfully in the past, so long as England adhered to her gold standard and Germany determined to follow England.

In looking back over the long period before 1873, no student of monetary science will pretend that there were not grave disadvantages attached to the old system, owing mainly in the earlier part of the time to the cloud of ignorance which overspread all monetary questions combined with the cupidity of Monarchs endeavouring to make money out of their currency monopoly, but one result stands out pretty clearly through the last 200 years of the period, and that is gold and silver had a relation to one another of approximately  $15\frac{1}{2}:1$ , and for at least 40 years before 1873 the price of silver in the market did not vary more than 2*d.* in either direction from 60·84*d.* per oz., which accords with the Mint ratio fixed by France.

During all that time the law first propounded by Nicholas Oresme, Count-Bishop of Lisieux in 1366, and commonly known as Gresham's law, was, of course, in operation and the over-valued or cheaper metal really performed to a large extent the function of the exchange medium, driving the undervalued or dearer metal out of circulation for a time, and by so doing enlarging its own field of work and thereby becoming itself scarce and being driven out of circulation by its rival.

For a Bimetallic standard it is claimed that it has the advantage of greater stability as compared with a system based on a single standard, and although it is not possible to prove historically whether this is so or not owing to the want of Index Numbers over a sufficiently long period, and owing to changed industrial conditions of modern times, it seems reasonable to suppose that the source of coinage material being two metals, the total supplies of them at least are not likely to vary as extensively as if only one were used.

It is also claimed for a Bimetallic system, and I think with justice, that it provides a broader basis for the support of credit.

It is, however, acknowledged that, with the intimate commercial connection existing between nations nowadays the system is impracticable unless supported by an agreement between the great commercial powers. Bimetallists, however, repudiate, and I think rightly, the assertion of some Monometallists that owing to the law of supply and demand no Government or

Association of Governments could maintain a fixed ratio between two commodities, whatever they be.

As so large a proportion of the precious metals mined is used in coinage, I think it may be said with truth that Governments do already by their Legal Tenders Acts regulate to a very large extent the demand for gold and silver, and could easily regulate the price ruling between them if the commercial nations were united on this point, and the ratio between the metals suitably fixed. I cannot follow the late Sir Robert Giffen in his statement in his book, "Bimetallism", that "the selection of the standard gives no sort of clue to the extent of the consequent demand of the standard substance (if any)." This might be true if a theoretical standard, such as the commodity standard, was adopted, but it does not appear to hold when the standard substance is also the medium of exchange, as the decline in the price of silver, since it ceased to be a standard of value in France and other countries, so eloquently testifies. (*See Table I.*)

We are told in the Holy Scriptures that silver was nothing accounted of in the days of Solomon, King of Israel, but I would hazard a guess that it was not so cheap, relatively, then as it is to-day, 2s. 4d. an oz., and we must remember that this is part of the price the nations of the world have had to pay for the advantages which they are conferring upon themselves by adopting a single gold standard of value.

While nothing can upset the economic law that the value of a thing consists in the demand for it, I think we shall readily understand why a Union of Nations has maintained and could maintain again a ratio between the two metals if we consider what the so-called Bimetallic tie really consists in. It seems that the nations constituting a strong Bimetallic Union being (*ipso facto* if the Union is to be stable) the largest purchasers of gold and silver really offer to buy and sell gold and silver at certain prices fixed in relation to each other, and their influence in the market being paramount the relation of the prices corresponds to the decree of the principal purchasers.\* The prices of both metals, when used as legal tender, being always well above what is called the marginal value, the cost of production does not enter into the question.

\* e.g.—Suppose that cabbages and brussels sprouts were each of them always of uniform size, and that 15 brussels sprouts were exactly equivalent in the eyes of householders to 1 cabbage, then a large increase in the supply of cabbages would not decrease their price in relation to sprouts.

The alternative to a joint standard seems to be an almost universal single gold standard of value, and it seems to me that it may and probably will eventually lead us into great difficulties. The difficulties of trade between countries using different standards are, we know, very great indeed, and this no doubt accounts for so many nations following one another in the adoption of the gold standard, but it is open to very serious question if we should allow the comparatively successful experience with gold during the last 30 or 40 years to weigh against the experience of centuries.

In the use of gold as the sole standard of value and legal tender medium of exchange England led the way under Lord Liverpool in 1816, when silver was made token money by reducing the weight of the shilling 6 per-cent, and if the world is poorer to-day through the fall in the price of silver, which has been persistent and continuous since 1873 (and seems now hardly to have touched the bottom), we must, I fear, take the major share of the blame. After a lapse of a little more than half a century we find the example set by England followed by the other great powers, and with such alarming rapidity have the nations of the world followed one another in adopting a single standard of value-gold, that it is difficult to find a first class power, except China, using silver both as the standard of value and the principal medium of exchange.

Though the protests of eminent economists have been loud and prolonged against the abandonment of the joint standard, mainly on account of the strain that would be thrown on gold, yet, notwithstanding the enormous increase in the amount of that metal employed in the currency and in banking reserves, as well as in the arts, we have in the last few years been face to face with all the signs of abundance of gold:—Prices rising rapidly, industrial agitation because wages do not go so far as they used to, strikes and other such effects, and I wish to dwell for a moment on the reasons for such a paradox to what one would have expected.

Down to 1898 the predictions of the upholders of a joint standard were almost completely fulfilled—the course of gold prices having fallen continuously since 1873 when the Latin Union commenced to abandon the Bimetallic system.

The economical history of this period of falling prices will no doubt be fresh in the memories of many of the members, and they will not have forgotten how heavily the burden pressed on industry

and how a Royal Commission on the Depression of Trade sat to investigate the causes of the fall.

However, for the last 14 years a glance at the index numbers will show that prices have continuously risen and apparently contradicted the Bimetallists' predictions. Many causes, such as the South African War have no doubt contributed to this change from a prolonged fall to a steady rise in prices, but it would appear to me that by far the most important factor in bringing about the recent rise is to be found in the phenomenal production of gold in recent years.

In all the older books on this subject it is pressed home to us that in regard to the precious metals as currency and their effect on prices, the annual production of gold and silver, being very small in comparison with the total amount of the coin and bullion in existence, does not have great weight in assessing their value as compared with other commodities, and, although this has, of course, always depended on the proportion of new to existing metal, I think we shall find that the very rapid increase in the production of gold has made the annual increment to the supply a very weighty factor nowadays.

During the years of falling prices from 1873 we find the annual production of gold almost stationary up to 1890, and amounting on the average to about £22–£23 millions per annum (*vide* Table II appended), whereas at the present time, 20 years later, we find that the annual production has increased to nearly £100,000,000, the production for the years 1900–1910 being in a rapidly increasing ratio as follows :—

£53,000,000 ...	...	...	1900	approximately
54,000,000 ...	...	...	1901	"
61,000,000 ...	...	...	1902	"
67,000,000 ...	...	...	1903	"
71,000,000 ...	...	...	1904	"
78,000,000 ...	...	...	1905	"
83,000,000 ...	...	...	1906	"
85,000,000 ...	...	...	1907	"
91,000,000 ...	...	...	1908	"
94,000,000 ...	...	...	1909	"
95,000,000 ...	...	...	1910	"

It therefore seems that although the comparatively small annual production of gold up to 1897 was not sufficient to meet

the increased demands made upon it owing amongst other things to the demonetization of silver without appreciating in value, yet the larger output of the last 15 years has augmented the supply in existence to such an extent that not only has it been able to meet the further demands that have arisen and facilitate a large increase in trade, but the ratio of its value to that of other commodities has fallen.

In an interesting article in *Le Journal des Economistes* for November 1911, by M. Yves Guyot, estimates are given showing that the value of gold produced from 1493–1850 amounted to 16,367 millions of francs, from 1851–1875 inclusive 16,448 millions, and from 1876–1910 39,000 millions. Thus, it is estimated that as much gold was produced in the 25 years from 1851–1875 as in three and a half centuries previously, and the production for the 35 years previous to 1910 was 18 per-cent more than the total previous production since 1493. Thus, it seems to me that fortune is favouring the monometallists, but, although we look out upon a world given over to the gold standard and flourishing under it, notwithstanding the tremendous break with the past caused by the changes made in Germany, Holland, France, Austria, the United States, &c., we must remember that the unnatural strain that is being placed on one metal is creating for it a vast field which it may need the discovery of another “Rand” to cope with.

The important bearing that the variations in the standard of value have upon the life of the community has been brought home to us very clearly by the industrial unrest of the last few years, to a large extent caused by the rise in the gold prices of commodities, but great as is this evil of rapidly rising prices, it is when this condition changes and gold becomes scarce and prices fall that the chief disadvantage of a single standard will be felt, and it behoves farsighted statesmen to look ahead and weigh the forces which may make gold scarce, and by so doing hamper industry and decrease production. On the one hand a very large amount of gold is being mined and increasing economies are being made in its use ; but, on the other hand, it cannot be denied that the adoption of it as a standard of value and medium of exchange by the large majority of nations combined with the outlook in the East may easily make us apprehensive as to its ability to satisfy the currency wants of the world.

It is interesting to note that the Comptroller of the United States Mint, who is in an excellent position to judge, seems to

think that we have reached about the high water mark of gold production.

A factor which appears to be equalling the Rand in importance in the question of the gold supply, only in the opposite direction, is India. A glance at the dispatches of Lord Dufferin and other Governors-General is sufficient to show the alarm which the fall in the price of silver in the Eighties caused to the Indian Government, and, although the fall in the rupee has been stopped by closing the mints to the free coinage of silver (in 1893) and thereby reducing the supply and artificially raising the price of the rupee to 1s. 4d., we must not forget that with silver at 2s. 4d. an oz., it is not intrinsically worth more than 10d., and this cannot be considered satisfactory.

The silver coinage of India, where, of course, the rupee is equally with the sovereign unlimited legal tender, is really equivalent to a well regulated inconvertible paper currency; and, although the device adopted has done its work in relieving the intolerable strain on the Government of India, in finding the interest on its gold loans, caused by the fall in the rupee, we must remember we are contradicting the canons of sound currency finance, and shall inevitably be called upon to pay the price of so doing. The relief to India has, no doubt, been great in being able to sell 15 of her rupees for £1—but the fact remains that the rupee is at this price because the supply has been reduced. Besides its artificiality the system, like inconvertible paper, or any restricted form of currency, suffers from the grave drawback that it is quite inelastic, and there is no doubt that at intervals the scarcity of rupees is very acutely felt. It is a preliminary principle of sound money that the standard substance may be coined freely so that the circulating medium increases or decreases according to the laws of supply and demand. In fact, the whole idea of imprinting the sovereign's head on a coin is to certify the weight and fineness of a piece of the standard substance and it is a farce to go through this process and at the same time raise the value of the stamped pieces by keeping down their number. In the past the possession of silver in the form of bars, ornaments, &c., was considered by the people as a safeguard, and in times of famine could be sent to the Mints and coined, but now this is not so, and silver having failed them in the recent famines, the natives are turning their attention to the hoarding of gold—to what extent is not known, but from the number of small gold bars exported (which are, we are told, specially in

demand for this purpose), it would appear to be carried on to a very large extent.

The latest figures I have been able to obtain as to the total gold imported into India in the last two or three years were given by Mr. Montagu, Under-Secretary of State, in reply to a question by Mr. Touche in the House of Commons on the 24 April 1912. They are as follow :—

1909	...	...	...	...	£10,077,472
1910	...	...	...	...	£18,028,008
1911	...	...	...	...	£23,366,814
1912 (January only)	...	...	...	...	£3,657,688*

The rate of increase in these amounts is truly astounding and is enough to upset the calculations of many Monometallists who looked to the East to absorb the silver demonetized in the West, but did not calculate upon it as a competitor for the dearer metal. For instance, the late Professor Jevons in his book "Money", in considering M. Wolowski's warnings to Europe against demonetizing silver says :

"M. Wolowski seems to forget that the nations of Europe constitute only a small part of the population of the world. The hundreds of millions who inhabit India and China and other parts of the Eastern and tropical regions employ a silver currency, and there is not the least fear they will make any sudden change in their habits."

The question whether the purchasing power of the standard is likely to vary as intensely under a Bimetallic system as under gold Monometallism is one that it is very difficult to determine as the prices of commodities depend on so many factors. In order that we may see how great these variations are, I have appended (Table I) the longest list of Index Numbers I could find—those published by Mr. Sauerbeck—from which you will see that the prices of commodities in relation to gold do not tend to become very stable. It used to be held by a number of people that the value of silver rose and fell with the values of commodities, and that on that account silver was really a heaven-sent standard of value, but this connection was, I think, only a coincidence of the time when the controversy regarding the standards was raging.

\* This must not be taken as a representative month as it is a time of the year when trade movements are very brisk.

I suppose there is no doubt that if Great Britain had thrown her weight into the scale in favour of maintaining the Bimetallic tie, the brunt of the effort to maintain which had for years fallen on France, silver would not have fallen as it has done, and though it could hardly be expected that we should abandon our Monometallic gold coinage that we had had for so long to help silver-using nations out of their difficulties, yet there is no doubt that one result would have been that we should have saved India millions in the last 40 years, and the trouble that seems almost bound to come up for settlement in the future unless silver regains some of its price.

After considering the question of the choice of a standard from many points of view my personal opinion is that so long as we were the one important gold Monometallic nation, and others took upon themselves the responsibility for the maintenance of the ratio between gold and silver, no system of money could have been more perfect than our own, but when it came to the question of adopting some form of Bimetallism or of our maintaining our gold standard and allowing the Bimetallic tie to be broken, I think that it would have been wiser, especially considering our interests in India, to have thrown our weight into the scale which would probably have had the effect of stopping the fall in silver. I am perfectly well aware that the late Sir R. Giffen and other eminent economists have asserted that Bimetallism is a sort of fetish, and that distinguished statistician thought that even some of the Monometallic Members of the Gold and Silver Commission were very unsound because they accepted some of the propositions of the Bimetallists, but I am myself convinced it is to a very large extent luck, and the Rand Mines\* that have favoured the Monometallists and not landed us in the slough of despond with a rapidly rising standard of value, and falling prices causing curtailment of production and general stagnation in trade.

The question whether we should try to get back to Bimetallism is one which, owing to the circumstances I have mentioned, is likely to be kept in the background for some time. Any change in the standard of value is bound to be fraught with hardship to some and benefit to others (though any benefit conferred on

\* N.B.—The portion of the world's gold supply produced from the mines of the Witwatersrand has steadily risen from £23,600,000 in 1906 to £35,000,000 in 1911, whereas the production from the rest of the world has only increased from £59,700,000 to £62,200,000 in the same period.

the silver interest would only be a return to them of what has been taken away), and I would not myself advocate any attempt at a change at the present time, when, owing to the enormous production of gold that metal is easily able to carry out its exchange function. I do not doubt, nevertheless, that the question will come up for solution at no very distant date.

#### MONEY AS THE MEDIUM OF EXCHANGE.

I have mentioned above that the chief quality one requires in money as the standard of value is stability of value in relation to average commodities, and I suppose the requisite principally demanded in money as a medium of exchange is that it should be in demand by all parties, and that all should be willing to receive it in exchange. In fact that there should be no question as to its purchasing power, and this is, no doubt, what originally signalled out the precious metals for the purpose. Their beauty, non-corrosiveness and comparative scarcity made them to be desired by all, and despite their drawbacks the weight of sound opinion seems to be on the side of maintaining them (or one of them) in their position as the principal medium of exchange, supplemented by paper and token coin of inferior metals which are kept at a certain value relative to the principal medium because they carry a right to a certain amount of it.

There are, of course, many people who consider that in the present state of civilization it is unnecessary and wasteful to use metals so valuable for other purposes as the exchange medium, and they think that for internal commerce use should be made of the fact that when paper or any similar substance of little or no value apart from its currency function circulates freely in a country it can be maintained at a certain value, owing to the constant demand for a medium of exchange, if the supply is properly regulated.

While the proposition with the two provisoes as stated is true enough, the abandonment of a medium valuable in itself, apart from its currency function, would in my opinion be very inexpedient. The risks attaching to all purchases and sales would be increased by the human element, and this, combined with the difficulties of regulating the supply of money for internal use and the provision of gold for purposes of foreign trade would, in my opinion, make the experiment a highly dangerous one, and the shock to commerce would be far more deleterious than the saving of gold, &c., would be of profit to the country.

I will, therefore, with your permission, leave the question of an unbacked paper currency until the millennium when the morality of the human element will, no doubt, have immeasurably improved and pass on to the consideration of another problem.

#### QUANTITATIVE THEORY OF PRICES.

In his Treatise on Monetary Problems, Professor Nicholson says: "By far the most important proposition laid down respecting money is that other things being the same its value depends in its quantity", and, therefore, I wish to dwell for a few moments on the general relation between the supply of money and prices known as the Quantitative Theory. This theory affirms that the value of money varies inversely as the ratio of the quantity of the medium circulating to the amount of commodities against which it is to be exchanged, regard being also paid to what Mill calls the efficiency of money, *i.e.*, its rapidity of circulation. In its simplest form the theory is axiomatic, and undoubtedly holds where the only money used is standard gold or silver (the price of a thing being the ratio of the value of a unit of it to the value of a unit of the standard) but now that credit enters so largely into exchange, doubts have been thrown on the theory, and although it is subject to modification on this account, I cannot myself see that it has in any sense broken down.

It is, of course, true that only a small minority of exchanges now consist of the purchasing of money with goods and goods with money, whereas in the overwhelming majority of cases goods are exchanged against each other by means of book entries and, therefore, it is not so obvious nowadays that an increase in the supply of the standard substance used as money will cause a fall in its purchasing power and increase the gold prices of goods, as under more primitive conditions. If, however, we consider that credit is only effective in settling debts because it consists in the right to receive standard money, I think we shall see that although standard money does not enter directly into all exchanges they do all ultimately rest upon its foundation, and an increase or contraction of its supply must affect average prices. We are often told that the quantitative theory exaggerates the effect on prices of an increase or decrease of metallic money, and that as prices are measured against money and credit together the latter which can be indefinitely (?) expanded is as important a factor in the equation as the metal it represents, and while it is

true that commodities are measured against the total amount of the medium circulating, a fallacy would seem to underlie the suggestion that a contraction or expansion of credit would have the same effect on prices as a contraction or expansion of gold. The reason being that if a quantity of gold which supports many times its value in credit were permanently exported not only would the medium of exchange be contracted to the extent of the gold exported, but also to the extent of the credit based upon it. This would not hold in regard to the cancelment of a credit of the same amount as the gold, the medium being contracted in the one case to an extent many times greater than in the other.

Further, the course of prices in the nineteenth century and, in fact, throughout history, clearly shows that a large increase in the production of the precious metals, when used as standard money, has invariably caused prices to rise, and thereby has had a stimulating effect on trade whereas a scarcity of them has had the opposite effect.

I do not mean to contend that an increase in the supply of metallic money can in itself directly increase the real wealth of the world to an extent greater than the value of the metals extracted from the earth, but I hold very strongly that indirectly a relative increase in the supply of money in whatever form does have a very important effect on industry by raising prices and quickening effort of all kinds, thus for a time at any rate causing an addition to be made to real wealth. (*Vide Table II.* attached.)

We all know the stimulus that was given to trade by the gold discoveries in California in 1848, and Australia in 1851, and how they were followed by a period of high prices and great commercial activity up to 1873. From this time prices fell almost continuously until 1898, and since that year have again risen fairly regularly up to the present time, and, as I have mentioned in an earlier part of the paper, these rises and falls have a close correspondence with the increases and decreases in the production of gold.

In any case whatever action a large expansion or contraction in the supply of money may ultimately have on aggregate wealth, the importance of its action on the distribution of wealth cannot be disputed. Major Leonard Darwin has shown very conclusively that prices move both in an upward and a downward direction far more quickly than wages, and although it is, I know, popular to speak of money as merely a counter in connection with exchange, yet the fact remains that we are, most of us, paid a

certain number of those counters for our services and feel it very much when their purchasing power falls.

On the other hand although the working and annuitant classes may derive some benefit when the purchasing power of money rises, and prices fall, the burden on industry of the charges for interest and wages which it is difficult all at once to reduce proportionately must soon decrease the profit on business and by reducing the capital in the hands of the producing classes curtail production and decrease the national dividend. This and other results of the variation in the standard are so important to the welfare of the community that no effort should be spared to minimize these variations as much as possible.

#### PRICE OF COMMODITIES AND THE RATE OF INTEREST.

Before leaving the subject of the relation of money to prices, I wish to refer to a mistake that is sometimes made in measuring the plentifullness or scarcity of the standard medium of taking into consideration the yield on investments and also the price of money in the market. One or two witnesses before the Gold and Silver Commission in 1888 remarked that they could not see any signs of the scarcity of gold as the rate of discount continued low, and notwithstanding the general fall in prices they did not believe that such scarcity existed. It would appear that what is known in the money market as the price of money (*i.e.*, the rate at which credit can be borrowed or lent) is the resultant of different factors from those which determine the purchasing power of gold in relation to commodities. The former is governed in the first place by the profitableness of trade, modified by local conditions affecting the permanent or temporary employment of capital, and it would appear to be natural to expect that, when gold is rising in value as compared with commodities and prices are falling trade would be inclined to slacken, and the outlets for the employment of loanable capital would be fewer, and in such a case a scarcity of gold all over the world would co-exist with a plentifullness of loanable capital and low rates of interest.

In order to test this I have taken the average of the rates of discount of the Bank of England for the years of good trade and high prices from 1853 to 1873, and from 1899 to 1911 inclusive and compared them with the average of the rates from 1874 to 1898, the years of falling prices, and find the rate for the period first mentioned approximately £1. 14s. per-cent higher than that for 1874 to 1898, and the rate for the period 1899 to 1911 about

10s. 9d. per-cent higher than that for 1874 to 1898. It should be noted that although prices are rising rapidly now they are not at so high a level as they were in 1873.

There are, of course, a great many factors governing the "price of money" in different localities, and I do not wish to lay too much stress on the above comparisons, nor do I for a moment suggest that the loanable price of money will always be cheaper everywhere when gold is scarce, but what I wish to emphasize is that the scarcity of gold as a commodity has little or no connection with the plentifulness or scarcity of loanable capital. For instance, the ratio of gold to commodities was very low in the years 1894–5–6, Mr. Sauerbeck's average index number for those years being about 62, while the mean bank rate was £2. 4s. per-cent and the market rate £1. 4s. 9d. per-cent, the lowest average for any three years since the Bank Act.

I have dwelt on this point at some length as the error of supposing that an increase in the supply of money and credit will reduce the rate of interest at which capital can be borrowed is a constantly recurring one, notwithstanding the fact that it has been refuted by every economist of note from Mill to Marshall, and, as it is the basis of some plausible theories, it is as well to lose no opportunity of exposing it.

#### MINIMUM MONETARY WAGE.

Akin to the relation of money to prices is the effect of variations in the purchasing power of the standard of value on wages, and while the general relation of money and the prices of commodities with wages is beyond the scope of this paper, I should like to say a word on the subject of a fixed minimum monetary wage, of which we have heard so much recently.

We know the attempts that were made in the Middle Ages to impose a limit on the wages labourers were permitted to earn, and that although heavy penalties were imposed on persons giving or receiving more than the statutory limit, those employers who, when the demand for labour at the maximum wage exceeded the supply, would not disobey the statutory and obey the economic law could not get labour.

The tendency now is in the opposite direction and minimum wages have been established in certain trades. While these may hold for a time in a closely organized trade the entrance to which is made difficult, I think Economists would agree that any such convention would assuredly be broken down under any other

condition should the supply of labour at the minimum rate exceed the demand. A further difficulty with which we are now more directly concerned confronts us when we consider that the minimum wage is fixed in money, and what might be fair and reasonable in a time of high prices like the present would become entirely unreasonable when prices fall.

In an article in the *Economic Journal* by the Honourable Evelyn Hubbard we read that the question of minimum wages arose in the early part of the nineteenth century, and a Society existed until 1827 to promote the enforcement of a minimum wage until it received the following rebuke from the Select Committee on Emigration, after which it seems to have ceased to exist—

“It is from an entire ignorance of the universal operation of the principle of supply and demand regulating the rate of wages that all these extravagant propositions are advanced.” *Tempora mutantur.*

Having briefly touched on the general relation of money to prices we pass on to consider one or two points in connection with our own monetary system and incidentally to see how it differs from that of other countries and also some modifications and improvements that have been suggested.

#### CENTRAL RESERVE.

It is estimated that the total amount of legal tender in this country (*i.e.*, gold in the form of coin and bullion and Bank of England notes issued against security) does not exceed £150,000,000, 30 to 40 millions of which is kept in the Bank nominally to redeem notes and meet the cheques of their customers. This small amount is, however, in reality, the pivot on which not only the commerce of England but the gold supply of the world rests, and when we consider the thousands of millions of debts, contracts, &c., expressed to be payable in it, often on demand, we see part of the great work it has to perform.

For instance, the cheques passing through the London Bankers' Clearing House in 1911 amounted to over £14,600,000,000, and the deposit and current accounts of the Bank of England and the Joint Stock and Private Banks of the United Kingdom amounted to very close upon £1,000,000,000.

This raises the question of our supply of what is after all the only ultimate legal tender medium of exchange, and I find, in looking over the statistics of gold production in the last few

years, that although a large proportion of the £100,000,000 now being produced annually passes through our hands there is not much evidence that the central supply in the country is increasing.

In the *Bankers' Magazine* for March, 1912, we find that the net import of gold into the United Kingdom for

1909 was £6,500,000

1910   ,,   £7,000,000

1911   ,,   £9,800,000

and the Deputy Master of the Mint estimates that £13,000,000 has been added to the amount of gold coin in the hands of the Joint Stock and Private Banks in the seven years from 1903 to 1910, and also that a large increase has been made in the general circulation. yet the average amounts of gold in the Bank of England for

1900	...	...	...	£33,321,000
1909	...	...	...	£32,538,000
1910	...	...	...	£32,081,000
1911	...	...	...	£33,309,000

do not show any sign of it remaining with that Institution, though advantage has been taken by the other nations of the comparative plentifullness of gold to increase their central supplies as follows :—

Gold held	Bk. of France.	Imperial Bank	United States.
31/12/1906	106,876,000	24,069,000	294,000,000
	rise 22 %	rise 37 %	rise 11 %
31/12/1910	130,524,000	33,052,000	327,000,000

I am not myself in favour of the plan adopted by some countries of keeping a large national hoard of gold. It savours too much of the old dead and buried mercantile theory that a nation's wealth is measured by its stock of the precious metals, and when one thinks how much of the proceeds of labour and life-blood as well as of treasure must be exported from a country to pay for an import of (say) £100,000,000 in gold, one may well pause and consider the cost before heaping up useless metal. What is required would rather seem to be a sufficiency to underpin a sound system of credit, though it would seem as if we trade on too small a margin.

## FREE GOLD MARKET.

Turning now to the consideration of the English system of banking and exchange, we find the foundation of the system embedded in the fact brought out very forcefully by Mr. Hartley Withers in his book, "The Meaning of Money", that London is for all practical purposes the only absolutely free market for gold in the world, and it is held by many people, bankers and others, that our predominance in the world's money market depends on the fact that anyone with undoubted credit or possessed of undoubted security is able in London at once to turn it into gold. Historically it would seem that the centre of the world's money market goes to the nation with the preponderating overseas carrying trade—the transfers of the centre from Venice to Amsterdam, and from Amsterdam to London being obviously due to the rise in turn of the Netherlands and England to commercial supremacy on the seas—and I do not myself think that any modification of the gold standard would wrest our position from us. It cannot, however, be denied that credit in London is more valuable than credit in other places, and the convertibility of that credit all important if we are to maintain our supremacy.

A result of our system which has come in for criticism, which seems to be justified to some extent, is that although our free gold market has been an excellent thing for Lombard Street, and the interests dependent upon or connected with it—and these interests are a large factor in the national wealth—yet it subjects us to the disadvantage that the Bank of England is obliged not only to buy, but also to sell gold freely and yet maintain the requisite reserve for safety. It is, therefore, often compelled to raise the price of money or rather to make credit dearer and more difficult to obtain in order to attract sufficient gold from abroad to replace an amount lost. If the bank's efforts are successful, and money begins to flow in, we find the bank rate falling roughly in proportion to the influx, Mr. Clare having estimated that an influx of £1,000,000 corresponds with a fall of £1 per-cent in the rate.

The action of the Bank of England is necessitated by the fact that scarcity of gold in any country has to be made up by us in the first instance, to be recovered by the natural attraction of money to its centre, or by making it more profitable to have credit or cash in London than elsewhere, and by so doing turn the exchanges in our favour so that gold is shipped to our shores.

As an instance of how widely the scope of our influence extends, Mr. Hartley Withers informs us that London drew gold from 17 centres for New York at the time of the last American Money Panic in 1907.

Although the system works remarkably smoothly, there is no doubt that the continual variation in the price of loanable capital—the Bank of England rate varying nearly four times as often as that of the Bank of France—is harassing to the commercial classes who are interested in money and credit being cheap and easy to obtain and above all regular in price.

#### RESTRICTIVE MEASURES OF GERMANY AND FRANCE IN RELATION TO GOLD SUPPLY.

While on the subject of London as the international market for gold, I should like to explain the reason why Germany which has a gold Monometallic currency and central bank modelled to a considerable extent on the Bank of England does not share with us the duty (or privilege) of supplying gold to the world.

The chief reason for this would seem to lie in the control exercised by the Reichsbank over the other financial houses in the country. The Imperial Bank, although a Company, is completely under Government control and its immense resources, large note issue and system of branches throughout the Empire give it an influence which is paramount in the money market of that country.

In this connection we may remark that the close and intimate connection of so many Commercial Joint Stock undertakings with, and their dependence on, one or other of what we may call the Commercial Banks to distinguish them from the Reichsbank, though of undoubtedly help in the development of commerce is often a source of weakness to the banks. The Chief Director of the bank being often on the Board of a number of the companies floated under its auspices and the intimate association of the finances of the company with those of the bank prevent that freedom of action in reducing their discounts and curtailing accommodation, the exercise of which has so salutary an effect in enabling an English bank to strengthen its position when there are indications of a scarcity of money in the market. This is a serious weakness in the banking system of the country, and taken in conjunction with their method of settling so many transactions at the ends of the quarters it causes very violent demands for legal tender to arise at certain times, an increase of £30,000,000

in a week in the note issue of the Reichsbank being not unknown, and makes the great banks very dependent on the Reichsbank and compels them to do its bidding in such matters as withdrawals of gold. When, therefore, the Imperial Bank lets it be known that it does not wish to part with any considerable quantity of gold for export it can make its wishes felt by the other banks, and in consequence persons desiring gold do not choose Berlin to get it from but always London.

In the *Economist* for 21 October 1911, there is an excellent article illustrating the difficulties of the close association of the banking and commercial interests of the country, showing what actually happened during the crisis over the Moroccan affair earlier in that year. The balance sheets of eight important Berlin banks are shown, and it is clearly seen to how small a margin their cash resources were reduced, through the tension caused by the political outlook. From 30 June to the 31 August the cash reserves fell no less than 26 per-cent to £10,400,000 as compared with their liabilities to creditors, depositors, and under acceptances amounting in all to £313,000,000, while discounts and advances were increased by £10,000,000 beyond what they were on 30 June, which, it must be remembered, is a quarter day. It was thus shown pretty plainly that under a severe crisis such as a war, the system might be strained almost to breaking point.

In contradistinction to the small margins on which the German commercial banks trade, we must put the exceptionally strong position occupied by the Imperial Bank, which is compelled by law to hold discounted bills, a large proportion of which are at short dates, against all the notes that are not covered by gold or notes of the Empire. It is further provided that the gold and notes of the Empire held should not fall below one-third in amount of the notes issued, though within the limits of that provision notes may be issued in excess of statutory number against security provided 5 per-cent on the excess issue is paid to the Government.

As an additional line of defence the Reichsbank makes a practice of holding a certain number of English and other foreign bills which gives it a command over the exchanges with the countries on which they are drawn and enables the bank if necessary to replenish its supply of gold.

Although the State has not given a monopoly in the matter of note issue to the Reichsbank, there seems to be a tendency to curtail the rights of the other Notenbanken. Their total issues

are small as compared with those of the Imperial Bank, and they are forbidden by law to discount below the Reichsbank's rate when it is 4 per-cent or more, while at other times they may discount at a rate not more than  $\frac{1}{4}$  per-cent below it. If, however, a notification appears in the *Gazette* to the effect that the Reichsbank is itself discounting below its official rate, they may discount at not more than  $\frac{1}{8}$  per-cent below the reduced rate.

France is another country with an enormous stock of gold, but the Bank of France does not always part with it easily, and having the right to pay its notes in silver often puts a premium on gold when it is known that it is required for export or offers coin or bullion in some form which is not the form desired.

In both these cases the chief functions of these central banks would appear to be to act as nursing fathers of the commerce of their countries, and they are not so much concerned with international finance as the Bank of England. Their cases are the opposite in every respect, Germany being a progressive country and carrying on an immense and growing trade with a small margin of capital, which has been supplied to a very large extent from abroad, and her banks under the leadership of the Reichsbank look upon it as their duty to do their utmost even to the extent of straining their resources in order that commerce may not suffer owing to lack of monetary accommodation. France is a wealthy creditor country with an immense amount of accumulated savings, and a stationary population which has laid a large part of the world under tribute to her in consideration of money lent in the past.

The Bank of France is considered by many to be the finest example of what a national bank should be. As a commercial undertaking it pays its stockholders a good rate of dividend, and its shares stand at 350 per-cent premium, notwithstanding the concessions that are being continually obtained from it by the Government. Its management are largely nominated by the Government, but apart from that it is not tied down or restricted by Statute like the banks of England and Germany, and has practically an unlimited note issue. It lends the Government £8,000,000 free of interest, and is compelled to discount French paper in the interests of foreign trade. It also has to its credit the masterly management of the French finances during the Franco-German War when the paper money it issued, though inconvertible for a considerable time, never fell to an appreciable

discount. Last year (1911) the State took in the form of taxes and dues 74 per-cent of the net available profits of the Bank.

#### BANK CHARTER ACT.

Leaving the question of a free gold market which we have maintained since the Act of 1816, establishing the gold standard, (with the exception of the three years following that date when the bank was prevented by law from paying its notes in gold) I want to consider what was in the minds of the framers of the Bank Charter Act of 1844, and the changes it brought about in our monetary system.

Up to the time of the passing of that Act the issuing of promissory notes was by far the most important function of a banker, the amount of deposits being small in comparison with the notes. The issues were regulated solely by the number of notes the banker could keep in circulation and the principle of the Act seems to have been the assertion of the right of the Government to interfere with this free issue, on the ground that bank notes were quite distinct from other credit instruments such as bills and cheques and really formed part of the money of the country, which had always been within the prerogative of the Sovereign or Parliament to control.

Although I am aware that this right of interference had been hotly contested, and I do not myself think the premises which led up to the regulation of notes were proved, my feeling is that Sir Robert Peel was right in asserting the responsibility of the Government to safeguard the holders of notes, and to see that every reasonable precaution was taken to insure their being redeemed when presented; but in limiting the supply of notes as the Act has always tended to do the Government seem to have had another reason for their action than the assurance of their convertibility.

There is no doubt that Sir R. Peel was guided in this matter by Mr. Samuel Jones Loyd, afterwards Lord Overstone, the greatest authority on banking of his day, and they both held what Mr. Tooke calls the "currency principle" with regard to note issues. The supporters of this doctrine held that it was not only necessary for notes to be convertible on demand, but that the ideal system of money was a purely metallic one, notes being required merely to take the place of the same sum in gold, their total amount to be equal in value to the gold against which they are issued. This is, of course, the extreme case, and although

the Bank Act did not attempt to limit the issue of notes to those with a gold backing, its authors held that a large expansion or contraction in the note issues of the country, even when the same were strictly convertible, was undesirable for the reasons given by Lord Overstone in his evidence before the Commission on the Banks of Issue that "If your circulation is subject either to depreciate owing to the excess of its amount or to violent fluctuation of amount, then undoubtedly that will be followed by corresponding effects upon confidence, upon credit, upon prices, upon banking, and so forth." They, therefore, decided strictly to limit issues of notes for other reasons beyond the fact that they should be convertible.

At the time of the passing of the Act it was considered that issues of bank notes had a very important effect on the foreign exchanges between this and other countries, and although the effect was undoubtedly greater then than now, notes being at that time a more important part of the circulating medium, I cannot help thinking that the framers of the Act did not wholly distinguish between the effects of inconvertible paper, and paper redeemable on demand, in driving specie out of the country. They seem also to have been of opinion that where an unfavourable exchange was brought about by excessive speculation inducing high prices, it was absolutely necessary to curtail the circulating medium, and by so doing reduce prices, if you were to restore the favourable exchange and stop the drain of gold.

We know now that a much more powerful influence in its effect on the foreign exchanges is that exercised by the rate of discount, and that it is only necessary to raise the "price of money" slightly above that ruling in foreign countries to draw gold to our shores.

The Act also attempted to separate the issue from the banking department, and to make notes absolutely secure by strictly limiting their issue against security, while leaving the bank directors a free hand to deal with the business of deposit banking, but sufficient weight does not seem to have been given to the fact that it is always possible to attack the bank's reserve via the banking department, and until the proper method of protecting that reserve, that I have just referred to, was adopted, it was generally found both before and after the passing of the Act that restrictive measures in regard to notes resulted in heavy withdrawals of deposits in several cases amounting to a run on the banking reserve which was only stopped when it was

known that further issues of notes were to be made. In fact, as Mr. Macleod tells us, the first effect of any excessive restriction on credit is to produce and cause a run on gold. This, I think, stands to reason the exchanges of commodities must go on and a medium of some kind is essential.

The contrast between the absolute liberty of the banker regarding the reserve he chooses to hold against deposits and the restrictive measures taken to enforce the holding of special reserves against notes was not so difficult to explain in 1844 when deposit banking was in its infancy as it is now. It must, however, be patent to all students of this subject that in connection with our present system of one centralized reserve, the factor on which everything depends is the sufficiency of the banking reserve. Great as would be the calamity if the issue department was unable to pay, or was prevented by law temporarily from paying its notes in gold this, to my mind, pales into insignificance compared with the disaster that would be caused if the banking department could not meet the cheques presented to it. The whole system of banks and changing houses throughout the Kingdom would be broken down and trade and exchange would be completely paralysed, while, on the other hand, a failure to redeem the notes by the issue department would only lead to the return, for a time, to an inconvertible paper currency, a thing, of course, to be avoided if at all possible, but which most nations have had to put up with at one time or another.

An important effect of the Act was, therefore, to tie the contraction and expansion of legal tender down strictly to gold.

If the chief object of the Act was the suppression of the country note, and the making of the Bank of England note as good as gold it has no doubt succeeded well, the authorized issue of private and joint stocks having fallen from £8,631,647 in 1844 to £617,865 in 1912, while the average circulation of notes other than those of the Bank of England is at the present time roughly £150,000, but it must not be forgotten that credit is the life of commerce, and had it not been for the fact that another form of credit—the cheque and deposit system—was ripe for expansion, there is no doubt that the vast increase of trade and commerce of the last 70 years must have been severely handicapped and restricted.

#### EFFECTS OF THE BANK ACT.

In my opinion the absorption of the country banks and the reduction of the total circulation of notes have not been on the

whole beneficial to the country. In reference to the first of these I have no doubt myself that agriculture has suffered through the country banks being taken over by large London firms. The country bankers were country men, they knew the wants of their counties and the men who lived on the land, and also the risks they were taking when they lent money on crops, &c., to a greater extent than many branch managers of London banks would be likely to do, and if these local banks had remained and been strengthened, I do not think we should hear so much of the depression in agriculture and the demand for the establishment of some land or agricultural banks to help in its development.

Regarding the reduction in notes it is remarkable that while the total note issue of the United Kingdom has remained stationary for the last 40 years—that for England and Wales showing a decrease of two to three millions—the circulations of France and Germany have increased by leaps and bounds.

We have, of course, the advantage of a far more highly developed system of cheques and clearing houses than our neighbours in France and Germany, yet cheques cannot and do not take the place and perform the functions of notes which if carefully regulated are capable of doing great service to the community, and of giving their makers at the same time a good profit. I am well aware that in many cases where we use cheques our neighbours in France and Germany use notes in cases where they are not so convenient and simply because they have not yet got used to cheques, but the enormous note circulations of France, £200–250,000,000, and Germany (say) £80–120,000,000, are not wholly accounted for by such transactions, but to a large extent by the fact that the wasteful habit of carrying about gold as change is not indulged in to the same extent by them. Moreover, the cheque is in the nature of a special credit between two parties, and it is necessary for the personal credit of the drawer to be good with the payee, whereas with notes the standing of the maker of an acceptable note is a matter of public notoriety, and they can be used in the far more frequent occasions when exchanges take place between two persons whose respective credit is unknown the one to the other.

Again the cheque is only capable as a rule of settling one transaction, whereas a note possesses almost all the functions of coined money, and is nearly as effective in settling exchanges.

## INELASTICITY OF OUR SYSTEM.

Another and a weightier reason why we should regret a diminution in the note issue of the country lies in the fact that the effect of the Legal Tender Acts of 1816, 1833, and 1870 (which make gold and Bank of England notes the only unlimited legal tender for the satisfaction of debts) and of the Bank Act is to curtail the elasticity of legal tender. I do not, of course, mean by this that our system is not economically quite sound, the coinage of the standard metal being perfectly free, and the quantity in circulation being regulated by the law of supply and demand, but as we have seen above the market for gold is to some extent restricted by other holders of the precious metal, and it is not always easy to secure a supply immediately to meet any extraordinary demand. Although, therefore, we may claim to have cured the disease of over-production of paper we must remember that an equally painful disease to the body politic is to be found in the scarcity of legal tender which unhappily dislocates our currency machine from time to time.

The number of exchanges being carried out in the country is continually varying, and although they are largely carried on by means of paper instruments, there is a certain relation between the amount of paper employed, and the bullion and legal tender money at the back of it (for instance, the joint stock banks have now for a considerable time maintained on the average about 16 per-cent of their liabilities in cash in hand, and at the Bank of England), and in times of great trade activity or some other extraordinary demand for the medium of exchange an inroad is made on the supply of legal tender, and there should be means whereby such a temporary demand could be met without hardship being immediately caused by the charge for accommodation being raised to a high level. The cause of the demand is often well known, and there should be power with the Directors of the Bank to increase temporarily the amount of legal tender in circulation should they with the special knowledge at their disposal think right to do so. This could be done more or less effectively in any one of three ways—(1) by restoring to the Directors of the Bank the right of free issue. (This method is not likely to be adopted owing to the bureaucratic tendency of the age unless the independence of the Bank is first curtailed.) (2) by adopting the device of our German neighbours of allowing their central bank to exceed the authorized note issue against securities, but not to profit by the excess issue as previously explained, or (3)

by amending the Act of 1844 so that it shall not be necessary for so many grains of gold to be held against each note issued so long as the average issue for, say, a month, or even two months, does not exceed the average amount of gold in the issue department, for the issue in excess of that against securities, over the same period, as is done in the case of the Scottish and Irish issuing banks.

As it is, notes are, of course, always in circulation in excess of the legal limit against security, and therefore a fresh issue does not increase the amount of legal tender in circulation, as the equivalent in gold is simultaneously placed in the issue department, and no means of any kind exist for temporarily increasing the supply of legal tender other than the attraction of gold to the Bank.

A safety valve is even more necessary where the demand on the supply of legal tender is caused by any feeling of uncertainty, such as is brought on by a bank failure, &c., as in that case, when other forms of commercial credit are almost valueless, the cry is for legal tender, and past experience has shown that panics soon subside on the promise of further issues of notes, and that in past crises there has been no eagerness to encash them. The demand is often accentuated by the banks withdrawing their call money from the bill brokers, thereby forcing the latter to go to the Bank of England and to rediscount bills with it or borrow on security. The division of labour between the banks and the bill brokers, whereby the latter specialize in bills and relieve the banker of this trouble, is considered by Sir Robert H. Inglis Palgrave, Bart., the greatest living authority on this subject, to be a weak spot in our system. The bill broker trades on the difference between the rate he pays the banker for call money, and the rate he makes on the bills, and, as he has to pay more on the average for money than the banker, the margin for profit is small, and he cannot afford to keep any reserves uninvested. As he has to meet his obligations at once he has no option in the matter when his call money is withdrawn, and be the price what it may, he must have immediate accommodation, and this he can only get from the Old Lady. The authority I have quoted is of opinion that the Continental system whereby the banker himself deals directly in bills and habitually rediscounts with a central institution is preferable.

The credit system is so sensitive that a comparatively small shock to confidence is sufficient to create an extraordinary demand

for legal tender, and it speaks volumes for the wisdom and foresight of the Bank of England Directors when times of difficulty and scarcity have been upon us that we have not had to consider the supplying of the market with what may be described as illegal tender by suspending or arranging to suspend the Bank Act for more than forty years.

I have said above that I consider our central supply of gold is rather a small one, and though it is sufficient for its main purpose owing to the watchfulness of the Bank of England Directors in protecting it, we have seen that its protection by operation on the rate of discount is costly and somewhat harassing to the commercial community.

#### REMEDIES FOR INCREASING GOLD SUPPLY.

Before considering the suggestions that have been put forward for increasing the supply of gold in the Bank, I should like to say that the fact of the withdrawal of even a small amount of gold having so great an effect on the price of credit is not due solely to the smallness of the central supply, but is considered by authorities to be also owing in part to the fixed nature of so large a portion of the Bank's assets. We do not know any more about the securities held by the banking department than that there is a certain amount in Government securities and a certain amount in other securities, but it is the general belief that the bank does not hold a large proportion of its assets in bills and short loans like the Bank of Germany, which might be a great help in enabling it to strengthen its position at any time by increasing the ratio of the reserve to the deposits.

The first suggestion as to the increase of the Bank's supply of gold with which I wish to deal is that the Government should repay the debt to the Bank and reduce the notes issued against security by £11,000,000, which would go a long way towards giving the notes an all gold backing. Against this suggestion it is urged, and I think rightly, that it would be a very expensive proceeding, and quite unnecessary, the convertibility of the note not having been in question since the close of the period of restriction in 1819.

Another suggestion, which, if put into operation, would, in my opinion, not only have the desired effect of increasing the gold reserve, but would also make for greater elasticity and a wider basis for credit, is that for issuing notes of smaller denominations than £5—and especially £1 notes. If these were issued I feel

sure they would speedily become popular in the country, and one of the most important of the effects of their issue would be that gold would find its way into the Bank of England.

The efficiency of a note as currency depends on its amount, coupled with its rapidity of circulation and the time it remains outstanding, and it is interesting to note that in a letter to the *Bankers' Magazine* for July 1912, Mr. Henry Meulen gives the average time the notes of various denominations remain in circulation as follows :—

£5 notes ...	...	...	...	...	70 days.
£10 notes ...	...	...	...	...	58 days.
£20, £50, or £100	...	...	...	...	27 days.
£200, £300, or £500	...	...	...	...	9 days.
£1,000 ...	...	...	...	...	7 days.

From this we can reasonably infer that £1 notes would remain outstanding for a considerable time longer on the average than any of those at present issued, and would probably circulate more rapidly and form a very efficient currency instrument.

Now let us consider what would happen if the total notes issued by the Bank to the public were largely increased. In the first place the proportion of the issue with a gold backing would be greatly augmented, and it would in such a case be quite safe to allow a permanent increase in the issue against securities over and above £18,450,000 (the profit on which would be a matter of arrangement between the Government and the Bank) and also to give some latitude in the issue against securities to meet any special demands for legal tender.

I know that one of the reasons urged against the adoption of notes of small denominations is that their issue would drive some of the gold in circulation out of the country, but if some more was driven into the Bank of England I do not think any harm would be done. When there is a demand for gold, the fact that there is a large amount scattered over the pockets of the community does not help matters while half the sum in the Bank of England might be of the greatest assistance.

If anyone doubts whether notes of, say, £1 would be taken up I would refer him to the cases of Scotland and Ireland, and also to the case of France, with regard to the latter of which, Mr. Birch, in an appendix to the Report of the Gold and Silver Commission in 1888, makes the following quotation from the report of the Governor of the Bank of France for 1881, "that the

stock of gold in the Bank was largely increased by issues of small notes." Although the circumstances are not quite the same, the French people having got used to notes during the period of restriction at the time of the Franco-German War, I am convinced that small notes would soon create a field for themselves, as in the cases above mentioned and of those of America, Austria, and many other countries.

It is often said that the Anglo Saxon people prefer to carry gold to notes, but in England at any rate I fail to see that they have had a chance of answering the question. The first requisition of a medium of exchange is that it should be valuable to all, so that all are willing to receive it in exchange, and this cannot be said of the bank notes. The legend "no Bank Notes, &c., changed here" is still often seen in shops, restaurants, &c., and though it might not be advisable to alter the law and compel everyone to give change for notes, yet if £1 notes or ones of even smaller denomination were issued they could be used jointly with token coin to settle debts, and if made legal tender could not be refused for payments over, say, £1. It may seem incongruous that paper which has not been at a discount for nearly a century should be so unpopular with all except the higher commercial and banking classes in this country, but the denomination of the notes, combined with the fact that the Bank has so few branches, do not make them a very useful medium in a cheque-using country. Moreover, the public is very curious in its likings and dislikings of money, an example of which was shown in the long-continued coinage by the Austrian Mint of the Marie-Thérèse thaler on account of its popularity in the near East and the Levant, and the reply of the Somersetshire cattle dealer when offered a Bank of England note "Give I Stuckey" will recur to most of us.

I do not propose to say anything about the dream that all good economists are expected to pray for, namely, the advent of an international medium of exchange, combined perhaps with an international clearing house at the Hague. The tendency of the time seems to me to be in favour of separation and not combination among nations, and although the late Mr. Lecky was of opinion that creeds and systems and religion and morals could be stamped out by persecution provided it was sufficiently severe, it does not seem possible to eradicate national feeling, and notwithstanding the truths brought out by Mr. Norman Angell's book "The Great Illusion", some of us cannot help

feeling that the lesson of national economic interdependence will take ages and ages to learn, even though we are taught with the lash of war, and sentiment being stronger than mental conviction it will soon be forgotten (if ever learnt) in times when the national honour and dignity seem in danger. Rather useful would it seem to me than to attempt to teach so difficult a lesson to a democratic people to strengthen our own currency system in any places where it may be weak and endeavour to increase its elasticity, so that if troubles come upon us we shall not have to reproach ourselves that we did not do what we could in times of peace and prosperity to widen the margin of safety and make provision for our supply of legal tender being not only sufficient in times of quietness but capable of expansion and contraction to meet the exigencies of trade and exchange.

In conclusion, I wish to disclaim any credit for originality in the production of this paper. Where opinions are expressed they are those which have been borne in on me after a study of the history of the subject and the writings of those great thinkers, past and present, who have given us the benefit of their views and to whom we are under a deep debt of gratitude.

I had always determined that I would not write a paper for the Institute until I could make some discovery or produce some instrument which would be of real benefit to actuaries in their daily work, but as this would have indefinitely postponed my appearance as a contributor to your proceedings, I have abandoned it, though I am quite aware that before many minutes have elapsed, I may repent having done so in dust and ashes.

TABLE I.

Mr. Sauerbeck's Index Nos. of the prices of 45 commodities, 1867-1877, being taken as equivalent to 100	Year	Ratio of the average price of Silver in London to 60-Sid. per oz.—the price corresponding to the ratio of Gold to Silver 15½ : 1 (Ratios down to 1885 taken from Mr. Sauerbeck's paper before the Gold and Silver Commission)	Mr. Sauerbeck's Index Nos. of the prices of 45 commodities, 1867-1877, being taken as equivalent to 100	Year	Ratio of the average price of Silver in London to 60-Sid. per oz.—the price corresponding to the ratio of Gold to Silver 15½ : 1 (Ratios down to 1885 taken from Mr. Sauerbeck's paper before the Gold and Silver Commission)
89	1846	97·5	83	1879	84·2
95	7	98·1	88	80	85·9
78	8	97·8	85	1	85·6
74	9	98·2	84	2	84·9
77	50	98·7	82	3	83·1
75	1	99·9	76	4	83·3
78	2	99·9	72	5	79·9
95	3	101·2	69	6	74·6
102	4	101·1	68	7	73·4
101	5	100·7	70	8	70·5
101	6	101	72	9	70·2
105	7	101·5	72	90	78·5
91	8	101	72	1	74·1
94	9	102	68	2	65·4
99	60	101·4	68	3	58·6
98	1	99·9	63	4	47·7
101	2	100·9	62	5	49·1
103	3	101·1	61	6	50·5
105	4	100·9	62	7	45·3
101	5	100·3	64	8	44·3
102	6	100·5	68	9	45·1
100	7	99·7	75	1900	46·4
99	8	99·6	70	1	44·7
98	9	99·6	69	2	39·7
96	70	99·6	69	3	40·7
100	1	99·7	70	4	43·4
109	2	99·2	72	5	45·7
111	3	97·4	77	6	50·8
102	4	95·8	80	7	49·6
96	5	93·3	73	8	40·1
95	6	86·7	74	9	38·9
94	7	90·2	78	10	40·6
87	8	86·4	80	11	—

TABLE II

Years (inclusive)	Value of Gold produced (Millions of Pounds)	Remarks	Year	Total Exports and Imports (less re-exports) for the United Kingdom (Millions of Pounds)	Difference	Per-cent increase (on the earlier year) during decade
1861-1870	259	Production of Gold on a much increased scale since 1850. Prices very high ...	1861	308	186	60·4 %
1871-1880	232	Decrease in Gold production. Prices falling ... ... ...	1871	494	74	15·0 %
1881-1890	221	Further decrease in Gold production. Prices falling lower ... ... ...	1881	568	53	9·3 %
1891-1901	431	Gold production increasing, but prices still falling in the earlier part of the period as the Gold supply has considerable lee-way to make up ...	1891	621	113	18·2 %
1901-1911	778	Gold production rapidly increasing. Prices rapidly rising ... ... ...	1901	734	298	40·6 %
			1911	1,032		

## ABSTRACT OF THE DISCUSSION.

Mr. G. M. REEVE said that the writer of the paper advocated Bimetallism, but had not brought forward any fresh arguments in support of the theory, or modified the old arguments to suit the altered facts. He thought it would be admitted that the events of late years had been in direct opposition to Mr. May's general theory. Bimetallism had been put forward as a remedy for curing the alleged evil effects of an alleged scarcity of gold, and the consequent falling prices which, it was said, ensued, and yet the statistics given in the paper showed that gold production had enormously increased, and that prices in the last fourteen years had risen very considerably. With regard to the question of falling prices, he thought Mr. May should have mentioned the fact that falling prices were only injurious to commerce if they were due to an appreciating gold standard. If falling prices were due to production at a lower cost of capital and labour, it was clear that commerce, as a whole, was benefited and not injured. The only fact which supported the assumption that industry had suffered was the appointment in 1885 of the Royal Commission on the Depression of Trade ; but he thought he was right in saying that that Commission reported that the alleged depression in trade was merely a re-adjustment or re-distribution of wealth. Profits had fallen, but wages had risen, and consequently the manufacturers thought that trade was depressed, but he could not gather that the same opinion was held by the workmen. The first Fiscal Blue Book showed that at the time when the bimetallist controversy arose, wages actually rose slightly, and he could not conceive why wages should rise in a period of depression. Further than that, the ratio of paupers to population in the same period fell very considerably. He thought the fact that wages had remained steady was almost a sufficient refutation of the allegation that the gold standard was not steady, for when it came to the last analysis it would be found that a unit of currency had a value because it enabled one to command the services of a certain amount of capital and labour, and if wages were practically steady over a given period he thought it proved that gold commanded a constant amount of labour during that period. Therefore, taking labour, and he thought rightly taking it, as the final test of value, he did not think it was correct to say that gold had depreciated over that period.

The same inference followed from consideration of the commodities comprised in the index numbers, for he believed it could be shown that, taking each of those commodities individually and separately, the falling price was due to improved methods of production, greater quickness of transit, or an opening up of a wider area of supply. That at once raised the question—If gold really remained steady, how was it that the extra work thrown on gold owing to monetary changes in and about 1873 did not cause it to appreciate, as, theoretically, it ought to have done ? The answer was that gold had remained steady in spite of the greater work thrown upon it,

first, because of the growth in the production of the precious metals (the average production of gold and silver in the period from 1866 to 1873 was £38,000,000, whereas in 1896, when the bimetallist controversy was at its height, the production of gold alone was more than that); and, secondly, because of the greater use of credit instruments and the increased efficiency of the Clearing House system. It was clear that that amounted to saying that the additional work thrown on gold was only a check on its depreciation, which really had a tendency to go on all the time, but since the additional work thrown on gold could only, from its very nature, be an evanescent check, it followed, if the argument of the monometallist was right, that as soon as that check had evaporated, as it were, then gold would start depreciating and prices would go up. And since that had obviously been the case in the last fourteen years, he claimed that the monometallists were entirely justified in their interpretation of the facts.

He now desired to examine some of Mr. May's statements with regard to the prices of silver. With regard to the suggestion that the United Kingdom was mainly to blame for the fall in the price of silver, he thought Mr. May might have mentioned, as at least a contributing cause, the fact that the production of gold rose from 63 million ounces of silver troy in 1873 to 162 million ounces in 1893. He thought that alone must have contributed largely to the fall. But he did not deny that if the bimetallist system had been put in operation in 1873 by all the leading commercial nations that fall might have been arrested. He did not, however, see that that implied any blame upon this country, or that matters would have been at all remedied. The effect of linking up, as it were, silver to gold would have been to have brought down gold with silver. He had proved, to his own satisfaction at any rate, that gold was tending to depreciate in 1873, and if it had been linked up with another depreciating metal then both would have depreciated together, and the rise in prices which started in 1898 or thereabouts would have been anticipated by about 25 years. Mr. May might have argued that the effect of the Latin Union being broken up was to expedite the fall, and there he would agree with him, but what was actually happening in the Latin Union was that France and other countries making up the Union were becoming monometallic on a silver basis, and when that had happened the bimetallic tie would have no further restraining power on the fall in silver.

But if he demurred to that particular statement, he took still stronger exception to the statement that "Any change in the standard of value is bound to be fraught with hardship to some and benefit to others (though any benefit conferred on the silver interest would only be a return to them of what has been taken away)." He thought he was justified in characterizing the statement in brackets as extraordinary. If traders were to be allowed to consider they had a claim on the Government for variations disadvantageous to them, in the price of the articles in which they dealt, it was difficult to see where that principle could end. It had been

found in America that a very similar sort of principle led to a rather unusual application of the Biblical doctrine. "To him that hath shall be given, and from him that hath not shall be taken away even that which he hath."

With regard to Indian currency, Mr. May was a believer in the quantitative theory of money, and yet he said that the natives of India had suffered loss through the demonetization of silver, the argument being that formerly they could take their bangles and ornaments to the mints and have them coined into standard rupees. But if he was really a believer in the quantitative theory of money he must admit that if silver could be brought to the Indian mints and coined, since the market price of the silver in a rupee was now one-twenty-fourth instead of one-fifteenth of a £, the value of a rupee in free coinage would sink from 1s. 4d. to 10d. Consequently, while the poor native would get 60 per-cent more rupees than he got before, he would find the prices had risen against him to the same extent—perhaps not quite so much, because more silver would be used in the coinage, and that would be a very small cause in the other direction. Mr. May also said that the Eastern demand for silver was not now so great as it was, but it had still a very important bearing on the whole problem of bimetallism. The argument was as follows.—If two metals such as gold and silver were linked together by a legal ratio accepted by all the gold-using countries of any importance, then if silver was produced more cheaply the only thing that happened was that gold was driven out of use to some extent in favour of silver. That could be shown very easily by an illustration. If it were supposed that gold obtained from alluvial deposits and gold obtained from rock were different substances, although everybody consented to define them both as gold, then if improved methods of production were found by which gold from rock could be obtained more easily, then the value of all gold would fall, and the men who were working alluvial deposits would find that they were not getting so large a return for their money. Consequently the production of gold from alluvial deposits would contract, whereas the production of gold from rock would expand, and the consequence would be that more gold coins would be manufactured of gold extracted from rock and fewer of gold extracted from sand. Exactly the same thing would happen if silver and gold were both considered legal tender at a fixed ratio, and silver was produced more cheaply. The only thing that would happen would be that gold would be driven out of circulation and its place taken by silver. Turning that argument the other way, if it were gold that was produced more cheaply to any considerable extent, the result would be that silver would be driven out of circulation, but if that went on to such an extent that prices rose so that silver prices under bimetallic conditions were greater than silver prices would be if silver were the only legal tender, then at once Gresham's law would come into operation, and silver would depart from the gold-using countries—from the supposed bimetallist countries—and go to China, and consequently the bimetallic ratio would break

down. The practical meaning of that was if any bimetallist ratio was adopted it would have to be a ratio that would over-value silver, and taking into consideration, among other things, the fact that the ratio of value of gold coins to silver in India was about 16 to 1, it might have to be about that ratio. The effect of that would be to introduce an enormous amount of silver coins into the currency, and to send prices violently up, not only here, but all over the world, because, of course, silver-using countries would be equally affected, and the consequence would be to precipitate a financial crisis of the first magnitude. He thought that argument alone was sufficient to justify actuaries in refusing to give any further consideration to the theories of bimetallism.

Mr. HARTLEY WITHERS (a visitor) considered it a great honour to be asked to take part in the discussion. When he read Mr. May's paper the first thing that occurred to him was that the result of it would be inevitably a debate on bimetallism, and so far his apprehension had come true. He thought Mr. May's remarks on the subject were of much interest, and the criticism upon them was, perhaps, even of more interest; but he questioned whether it was worth the while of the Institute to give much time at present to debating the subject of bimetallism. He thought Mr. May would agree with him that at present the matter was an entirely academic one. A subject of more practical interest was raised by Mr. May's strong belief in the quantitative theory of money and by his opinion that the present high range in the price of commodities was entirely due to the large recent increase in the production of gold. He thought that the quantitative theory had to be taken with a very great deal of reserve. Of course, other things being equal, if this country suddenly had twice as much gold to-morrow he supposed the price of everything would necessarily have to be roughly double, but it had to be remembered that other things very seldom were equal. They must take into account the efficiency of the monetary standard whatever it might be. He could not follow the argument that the supply of metallic money—to which he rather thought that Mr. May had pinned his faith—was the most important consideration. It seemed to him that a man who had a cheque which would be accepted in payment was as efficient in increasing the prices of the things he wished to buy as a man who had a sovereign. Therefore he thought all credit instruments, not only Bank of England notes and legal tender notes, but all credit instruments which people would accept had to be taken into account. He quite agreed with Mr. May that there was a rough agreement between the volume of those credit instruments and the gold backing, at any rate in this country, but he thought what had to be remembered was that rapidity of circulation was a thing which could not be calculated, except very roughly in the bankers' clearing returns; if a shilling stayed in a man's pocket for a week it had no effect on prices during that time, whereas if it passed from hand to hand ten times a day it was having considerable effect every day during that week.

He ventured to think that there was good reason for questioning whether the recent rise in the price of commodities had been due to the increase in the production of gold ; in fact, he was prepared to go further and to say that he rather doubted whether there had been any effective increase in the production of gold. When one spoke of an increase in a thing one must mean, he thought, an increase in it in comparison to the demand for it ; and in that sense he doubted whether there had been any increase in the supply of gold. As Mr. May had shown, the Continental Banks had been heaping up a hoard of gold in their vaults, making no use of it. The same thing had been done in Argentina and Brazil, and to a small extent by Japan. All over the world banks of issue had been heaping up stocks of gold wherever they could. In this country we had not increased our stock in the Bank of England, but Mr. May had shown that about 13 millions had gone either into circulation or into the vaults of other banks, and it had to be remembered that the extent to which the other banks held cash reserves outside the Bank of England was a very important matter in these days. He thought if those things were taken into consideration there would be some reason to question whether there had been any real increase in the supply of gold ; and he was inclined to think that the high range of the rate of interest was another argument on the same side. Mr. May rather poured contempt on anybody who thought there was any connection between the rate of interest and the supply of gold, and had said that that old error had been refuted by all economists, from Mill down to Marshall. He supposed it had been, but nevertheless he was not quite so sure that it was dead, and he rather thought that Mr. May himself might be called upon as a witness in its favour, because he said that a small amount of gold going away had an immediate effect on the price of credit. He (Mr. Hartley Withers) maintained, in spite of Mill, Marshall and May, that the supply of credit was roughly based on the amount of gold. If ten millions of gold were taken out of the Bank of England to-morrow, there would be a startling jump in discount rates and the price of money all over the country would be multiplied perhaps two or three times.

What Mill and Marshall had worked on was the fact that when an increase in the amount of gold and a consequent increase of credit occurred (with the result that credit was cheap) the ultimate result was a great stimulus to trade and a rise in the price of commodities, which meant that more credit was required for financing trade, and consequently the price of money would ultimately rise. But to say that there was no connection between the supply of gold and the price of credit was, he ventured to think, to state what was not yet proved.

He would like to say a word concerning the Indian currency arrangements to which Mr. May had taken such strong exception. It was a question of great public interest at the present time. England had been accused, he was sorry to say by people who ought to know better, of deliberately giving India a currency which was

not sound, in order to fatten Lombard Street and the London money market. He thought that was a disgraceful accusation. Mr. May, of course, did not support that contention, but he thought anybody who understood such matters ought to give attention to the question of Indian financial arrangements, because it seemed likely to be very shortly a burning political question. Mr. May said the Indian currency arrangements were entirely inelastic. He quite disagreed with that statement. Anybody could take a sovereign to the Indian Government, either in India or in London, and get fifteen rupees for it. The amount of the Indian currency could be increased immediately as the demands of trade required it by that simple method of putting a sovereign down on the table and asking for fifteen rupees. If the opposite occurred, if there was such a pause in Indian trade that there were too many rupees in circulation, what would happen would be that the price of the rupee would be depreciated, and at a certain point—he thought it was 1s. 3 $\frac{1}{2}$ d.—the Indian Government would come forward and sell sovereigns for rupees against its balances in London and so maintain the price of a rupee in India. Consequently, there were easy and readily-working arrangements, both for the provision of rupees when they were wanted and for the redemption of them at a certain rate in sovereigns when there were too many of them. He should have thought that the currency arrangements of India were entirely elastic from that point of view. Mr. May said, in effect, that this country had violated all the sound canons of currency theory. He wished to know what those sound canons were. The currency for a poor country should be, in the first place, elastic—which he thought he had shown that the Indian currency was. In the second place it should be cheap and thus give the Government a profit, which was exactly what the Indian currency arrangements had done most successfully. The Indian Government was able to buy silver at 10d. and coin it into rupees at 1s. 4d., and consequently it had a very handsome credit balance in London, which balance was used to maintain the rupee when necessary, the Indian Government drawing on its balance in London, selling the bills in India, and so maintaining the price of the rupee. The balance kept in London was almost entirely lent to large banks and bill-brokers and stockbrokers of first-rate standing. It was just as good as gold for the purpose of India, and yet it brought into India a certain revenue by being so lent. Yet a gentleman, the other day in the House of Commons, practically asked whether the Indian taxpayer had not been over-taxed in order that the London money market might be fed with cheap money! He hoped the Institute would have something more to say on the subject, because he thought it was a very important one.

He desired to say one word about Mr. May's assertion that this country suffered from periodical lack of legal tender. He would like to know how long it was since any evidence had been shown of that scarcity of legal tender. Speaking from memory, he thought it was more than half-a-century since the Bank Act had been

suspended, and machinery was available at the present time by which the Bank Act could be suspended at any moment, if necessary. Everybody knew that nowadays Bank of England notes were not wanted for commercial purposes. Cheques were drawn. When the commercial community was short of money it could go round to the Bank of England with its securities and in five minutes it could raise two or three million pounds, or ten millions if it wanted it, merely by the Bank of England giving it a credit in its books against which it could draw. He should have thought the perfect ease and elasticity of that system was unrivalled anywhere in the world, and matters were not going to be improved by increasing the available supply of the bank note, which was practically an anachronism in these days.

Mr. H. E. RAYNES quoted a statement by Sir George Paish to the effect that if the quantity of gold in England were trebled at the present time it would have very little effect upon prices. He thought it was quite true that the "Quantitative Theory" was a fallacy throughout. The financial system of this country was based on credit and gold combined, and it was immaterial what proportion of the whole was credit and what proportion was gold so long as confidence was maintained. The system might be compared to a vessel filled with two different liquids of the same specific gravity. So long as the vessel was full the total weight would remain the same, no matter in what proportion the two were mixed. If the Banks actually were able to pay out the whole of their liabilities in gold at once, would this affect the prices of commodities? It seemed to him that it would not. It would simply mean that credit could be changed for gold. So long as the confidence of the country was maintained it did not matter whether the system was kept up by a reserve in gold or in credit.

England had taken very little gold during the last eleven years. That was shown in the Report by Mr. Roberts's, of the United States Mint. England and Germany between them, for their currency system, had taken about 12 millions, and the United States absorbed during the same eleven years 140 millions. England and Germany during that period had maintained a perfect system of credit, while in America there had been the financial panic in 1907, which lasted for three months. These facts indicated that it was not the quantity of gold in a country which measured the price of commodities. He thought that the rise in the prices of commodities during recent years was mainly due to the incidence of the industrial energy of the world. While the energy of the world was being devoted in towns to manufacture, it followed that agricultural work would suffer, and so the necessities of life were in fewer hands and were more difficult to obtain. Consequently they rose in price. He thought that the chief cause of the fluctuation in prices was the variation in the direction towards which the world's industrial energy was applied. It appeared from Mr. Roberts's Report that the agricultural productivity of the United States had actually decreased 2 per-cent during eleven years and that must necessarily have had an effect

upon the prices of the necessities of life. There were also the changing habits and customs of a country. He believed these had an effect on prices. Prices would rise when there was a tendency towards luxury, which, perhaps was the case in England at the present time. When such an industry as the motor trade had developed in recent years with such extraordinary rapidity it had the effect of taking labour away from the primary necessities of life, and therefore the price of the latter would rise. There was also to be considered the extraordinary expenditure on armaments in Europe during recent years which took away labour from productive to unproductive pursuits and directed it, economically, to more or less wasting results. These he thought the real causes in the rise of prices, not the over-production of gold.

Mr. OWEN KENTISH thought that Mr. May, in giving a statement of the amount of gold taken by India, might have put in the last available figures, which were those up to September, 1912. The amount taken this year by India to the end of September was 26½ millions, which was three millions more than it took for the whole of last year. As Mr. May had pointed out, hoarding had always been a practice in the East, and there was no doubt that at the present time a good deal of hoarding was going on among the Indian natives, especially in the little 5-oz. and 10-oz. gold bars, which were very popular in that country. But with the slowness with which habits changed in the East, he thought it was probably doubtful whether that hoarding had reached its maximum, especially with trade as prosperous as it was in the East at the present time. It was a striking fact that, taking the world's production of gold, as Mr. May had done, at 100 millions per annum, and, accepting the high authority quoted, that it had reached the maximum, India, which was a poor country, was absorbing at the present time considerably more than a quarter of the total gold production of the world. It seemed to him that that fact had a direct bearing on the quantitative theory. There was another important fact, namely, the continual drain of sovereigns to India which was going on in an increasing amount. They were required largely to finance the trade requirements of India. The sources from which India drew sovereigns were London, Egypt, and Australia. Formerly the sovereigns which went from London to Egypt each autumn for the purposes of the cotton crop used to return to London, but now, since the closing of the Indian mints, a good many of the sovereigns were re-exported from Egypt to India. He believed that Australian sovereigns were only exported to India when the wool farmers had had a bad year, and the amount of sovereigns was not required to finance their trade. So eventually the continual drain of sovereigns had to come from London.

An interesting point which Mr. May had not touched upon was the question of the export of the old Shield sovereigns, which had now ceased to be minted. He believed in former years they were exported in very large quantities for the purpose of ornament in India, and never returned to this country at all. One man had informed him

that he had recently had a large order from India for Shield sovereigns which he was quite unable to fill—so he supposed the industry might be considered to be at an end. The figures given with reference to the gold export to India did not enable one to tell in what form the gold was exported, and he believed it would be found, if one looked back a few years, that the actual amount of Shield sovereigns which was exported from this country was an appreciable amount of the total, making the absorption of other gold by India all the more striking. There was also the very important question of the Indian Gold Standard Reserve. At the present time the amount of that reserve held in London was probably about 20 millions, but of that 20 millions a vast quantity was held in the form of first-class securities, Consols, Exchequer Bills, and so on. He believed that at the date of the last available return of the 20 millions, something like 16½ millions was in the form of securities, including over 4½ millions in Consols valued at 90, so that, as was pointed out in the House of Commons the other day, the actual depreciation in the Gold Standard Reserve at the present time must be, at the very least, one million, and was probably more. If anything like a failure of the rains or a bad season occurred in India, causing a sudden drain upon that reserve, or anything happened which would lead to realization in London, it was in an extremely unavailable form, and the securities would be thrown upon the market just at the time when it would be impossible to realize them without heavy loss. That question, of course, would have an immediate bearing upon the subject of the central gold supply which was being discussed that night.

Mr. R. W. MATTHEWS (a visitor) thought the members must admire Mr. May's courage in putting into a paper so many controversial subjects. There were three or four topics which would in themselves require a full evening to discuss thoroughly—for instance, that of the issue of £1 notes. There was one portion of the paper which had not been touched upon by previous speakers, namely, the important part which the cheque took in the currency of this country. Mr. May had pointed out that in the event of a severe panic, such as the failure of one of the large banks, credit would be very much shaken, but when the history of this country for the last fifty years was looked at he ventured to submit that the great banking institutions had become a great deal more secure than they were fifty years ago. There had been no panic of any importance since the year 1866. There had been the Baring crisis in 1890, but that had been met in a masterly manner, and at no period in the history of this country since 1866 had its credit been called in question; and taking that fact into consideration, together with the currency of the notes and the gold and the silver which this country already had, it seemed to him that a shortage need not be feared. He had never heard, until he had seen it in Mr. May's paper, that there was such a thing as a shortage of the available currency. He thought it was decidedly to the advantage of the credit of the country that debits and credits should be made as quickly as possible; and it

was the fact that cheques were so rapidly collected and placed to debit and credit that had given confidence in the cheque as part of the currency of the country.

Mr. May had suggested the issue of £1 notes. He maintained there was no necessity for anything of the sort. The cash they had at present was quite sufficient for all requirements, and there were also postal orders. Cheques were a source of revenue to the Exchequer, and he thought it would be a long time before a Chancellor of the Exchequer would substitute a £1 note, on which he would get no return, for the cheque, on every one of which he got a penny. It seemed to him that instead of the Government having to subsidize its currency, by the system at present in vogue the population paid the Government for the privilege of being the currency—if he might so put it. He thought that when once the importance of the cheque was realized it would help to get rid of many doubts and fears as to the future of the credit of this country.

With regard to the old banks having been superseded by large institutions, Mr. May seemed to think that agriculture had suffered in consequence of the old private banks being done away with and the new institutions not coming in contact with clients in the same way that the old banks did. That was perfectly true, but, after all, it was more sentiment than anything else. As far as the bankers themselves were concerned, the change had been exceedingly beneficial. It must be remembered that a bank had a large number of branches. If trade in one part of the country was bad, in another part of the country it might be good. The bank was able to use its resources in that part of the country in which they were required. There was no such thing now as a bank for one particular industry, which must be a very great benefit to the community. Fifty years ago there were banks which relied upon certain industries or certain businesses to support them, but now a bank lent its money in all parts of the country and to all industries. As a result, the banks had not too many beans in one sack, a fact which had added considerably to the stability of this country's currency and banking system.

Mr. G. E. MAY said that the paper fell naturally into three parts. The first part dealt with bimetallism; the second part was a discussion of the relation of money to prices; and the concluding part discussed this country's system—a gold standard—and put forward suggestions for the possible improvement of the system. With regard to the bimetallic portion of the paper, the Author came to the conclusion that the two substances most suitable for a medium of exchange and a standard of value were gold and silver. He thought everybody would agree with that opinion. The chief desideratum for a standard of value was that it should not fluctuate; in other words that its purchasing power should remain more or less constant. The only way in which that could be achieved was by the supply of money more or less approximating to the demand. The Author stated in his paper that the supply of gold had been more than sufficient to meet the demand, and therefore prices of commodities

had risen. That statement had been questioned by some speakers, either rightly or wrongly; but if the theory was right that the supply of gold had been more than sufficient and the rise in prices had been the result, then, surely if this country had supported the Latin Union to maintain a bimetallic standard in 1873, at the time when they ceased to coin silver freely, the result would have been that the amount of money would have been increased by the silver being added to the gold. The silver in existence in the world at that time would have been available and the probability was that a much larger amount would have come to hand, by reason of the fact that a fixed price per ounce could be obtained. Therefore, he could not agree with the Author in thinking that it would have been advantageous to this country or to the countries of the world generally if they had thrown in their lot with France.

Considering the future one had to forecast whether the increase in the amount of gold in existence was likely to be maintained at the same rate. The Author appeared to think that the supply had reached its maximum and would gradually decline, and that it was only due to good fortune and to the discovery of the Rand that a gold standard had been possible. Apart from the point which Mr. Hartley Withers had raised in regard to that subject, he could not see why the gold supply could not be increased in the future—why should there not be some new discovery of gold? Anyhow, it seemed that the Rand and other places would produce large amounts of gold for many years, and there were large unexplored tracts of country all over the world from which he thought there was every reason to believe further amounts of gold would be forthcoming. In fact a very sanguine person could go still further and imagine that the supply of gold produced by new discoveries might be so great as very largely to exceed the demand. Although he had spoken about the bimetallic system he was entirely in agreement with Mr. Hartley Withers that the system was dead for many years to come. Perhaps one might not inaptly refer to it as the "Sleeping Beauty"—the Author certainly seemed to see beauties in the system, as undoubtedly, in theory, there were. Although the paper had been considerably criticized, he thought it of very great value because it was always possible that the time might come when it might be necessary to adopt a bimetallic system, and when that day arrived (if it ever did) it might be that the handsome Prince of the fairy tale would be a member of the Institute, and by such a paper as Mr. May's would wake the "Sleeping Beauty" into life.

Mr. Hartley Withers had expressed a doubt whether, taking the requirements of the world into consideration, there had been any real increase in the supply of gold. He thought that was a point for careful consideration, and was certainly an ingenious suggestion. The question of the coinage in India had also been dealt with by this speaker, who defended the system on the ground of its elasticity, thus differing from the Author, and with this he was entirely in agreement. The paper had referred to the rupee as approximating to an incontrovertible paper currency; probably

the Author had gone further than he intended, because, after all, there was some value in a rupee, for it was anyhow worth at the present time 10*d.*, as compared with 1*s.* 4*d.* its current value. Another point touched upon was the effect of the supply of gold on the rate of interest. When the rate of interest was referred to, it was a question whether the money market rate or the investment rate was meant. As regards the money market there was no doubt that the supply of gold did have an enormous effect, temporarily, at any rate; but if the rate of interest earned on investments was considered, he thought it was open to question whether the effect was very great or not. The Author had referred to the inelasticity of the system in this country. He had always heard it referred to as one of the most elastic in the world. Mr. Raynes had commented on the financial panic in the United States, where the stock of gold was large, and drew some comparisons between it and this country and Germany, where the stock of gold was small, but where there was at the same date no money panic. He did not think that comparison was of very great value, because the conditions prevailing in the two countries were so very different. Mr. Matthews had spoken in a very interesting way about the different Banks, and the regret he felt along with Mr. May at the departure of the smaller country banks, and he had also referred to the suggestion made by many people that certain advantages could be gained by the issue of £1 notes. He was quite in agreement with Mr. Matthews in that respect, for he did not think they were required.

The PRESIDENT was sure all the members would agree with him that they had had a most interesting discussion upon a matter which, as the Author had pointed out, was outside the general range of subjects discussed before the Institute. They all heartily welcomed the contributions to the discussion of Messrs. Hartley Withers and Matthews, especially Mr. Hartley Withers' remarks as to the Indian currency, because, no doubt, at the present time that matter was a very important one. Mr. W. T. May in his paper had laid down no authoritative dictum upon the various points raised, although he apparently had a leaning towards bimetallism. He thought it was the difficulty of arriving at definite or exact conclusions upon such questions as had been discussed that evening which made them attractive to so many minds. Time alone could answer the problems discussed, but valuable information had been placed before them in a concise and readable form.

Mr. W. T. MAY, in reply, said that in taking up a subject like currency and credit one expected to meet with a good deal of criticism, especially when one advocated the unpopular view. Economists were, he believed, for the most part nowadays on the side of bimetallism, theoretically at any rate, while bankers and others in the practical realm of finance were, as a rule, opposed to it. He had not advocated any change at the present time because, for the reasons stated, it was not opportune.

Mr. Reeve had mentioned that the Royal Commission on the Depression in Trade did not connect that depression with the decline

in gold production, and he (Mr. Reeve) did not seem to think trade was depressed at all in the Eighties; but it must not be forgotten that as a result of the recommendations of the Commission another Commission—that on Gold and Silver—was appointed in 1886, to examine into any effects that currency matters, such as the fall in silver, might have had on trade. In one of the reports of this Commission facts were elicited that, in his opinion, showed conclusively that trade was very depressed at the time. For instance, it was stated that during the period from 1874–1886 the increase in the gross amount of property assessable to Income Tax was not quite 15 per-cent whereas in the preceding twelve years (when the production of gold was rapidly increasing) the gross amount of property assessable to Income Tax increased by 56 per-cent. He thought there was no question that the country was very unprosperous during the years when gold was very scarce.

He did not, of course, contend for a moment that the supply of gold was the only factor which affected prices. Improved machinery and methods of production were also of significance, but he thought that consideration of the table at the end of the paper would show that the correspondence between gold production and prices was wonderfully striking. Thus, in the years from 1861–71, after there had been an enormous increase in gold production, the increase of exports and imports (less re-exports) of the United Kingdom was 60·4 per-cent during the ten years. After 1870, when the gold production was falling, the increase in exports and imports had been only 15 per-cent for the decade 1871–81 and only 9·3 per-cent for the decade 1881–91, during which period the gold supply further decreased, and the ratio of the increases did not rise again until after the further large discoveries of gold in the late Nineties and the early part of the present century.

Again the Index number fell from 111 to 61 in the years of falling gold production, 1873–1896, and it seemed incredible to him that the one factor common to all prices, viz., gold, should have remained stationary in value, and that the changes in prices were due only to variations in the other factors.

With regard to the relation between the rate of interest and the supply of gold, he only meant to refute a contention that had been often raised of recent years, that, if the supply of currency in any form was largely increased, the rate of interest would fall and credit would become cheap, and he tried to show that the opposite was more likely to be the case. He was interested in what Mr. Kentish had said as to the amount of gold imported into India up to September, 1912, which showed that another very large increase on the 1911 figure was to be expected. With reference to Mr. Withers' remarks on the Indian currency, while deferring to Mr. Withers' superior judgment, he thought that when goods were exchanged for money in any country, it was a canon of sound currency that the monetary *quid pro quo* should be of the same market value as the goods. This was not the case in India—there the value of the rupee depended on the decree of the Government.

He did not think one could bring up a subject like currency without referring to bimetallism, which did not seem popular at the Institute. Of course, he knew quite well that at the present time the matter was not one of practical politics, but it was as well to remember that there was an alternative to universal monometallism. Mr. Reeve had said something about bimetallism being put forward in 1873, but up to that time it had always existed in some form or other. Sir David Barbour, in his recently-published book on the "Standard of Value", showed that there had been a close correspondence in the relation of the values of gold and silver for nearly 3,000 years, owing, no doubt, to their both being used as currency, and pointed out that gold and silver had a relation to one another of  $13\frac{1}{3} : 1$  from the time of the Babylonian to that of the Persian Empire ; and that in the Roman Empire, for 150 years after the return of Julius Cæsar from Gaul, the ratio was  $12\frac{1}{2} : 1$ , thus proving that the ratio of gold to silver at the present time,  $33 : 1$ , caused by the break of the bimetallic tie, was unprecedented in history.

#### MORTALITY OF GOVERNMENT LIFE ANNUITANTS.

[Return to an Order of the House of Commons dated 2 August 1912 (No. 298). Reprinted by permission of the Controller of His Majesty's Stationery Office.]

*COPY of Report of the Actuary of the National Debt Office, dated the 12th day of October, 1910, on the Mortality of Government Life Annuitants.*

NATIONAL DEBT OFFICE,  
12 October 1910.

SIR,

I have the honour to submit the following report respecting an investigation which has been made into the Mortality Experience of Government Life Annuitants.

The last investigation was made by Mr. A. J. Finlaison, and his report thereon, under date of the 10 February 1883, was published in House of Commons Paper No. 8 of 1884.

The observation on that occasion extended over the long period of 67 years, from 1808 to 1875, and comprised the experience of the nominees of all annuities which had been granted under the Acts 48 Geo. 3. c. 142. and 10 Geo. 4. c. 24.

If variations in rates of mortality were only of a temporary character the inclusion of the experience of a further series of years with that of the period 1808 to 1875 would be desirable in order to obtain as many facts as possible from which the requisite average deductions could be drawn. There may, however, be causes at work in a later period of years, which were either absent or of less force in an earlier one, tending to render these variations permanent.

The conditions of life in this country are now somewhat different from what they were in the earlier years of the nineteenth century.

A knowledge of hygiene is more general, considerable improvements in sanitary matters have been effected by legislation, and great advances have been made in medical and surgical skill; all causes which favour an increase of vitality. On the other hand there may be causes tending to a contrary effect, for instance, a greater density of population and the stress of modern life; the latter cause probably affecting male more than female lives. The duration of life in a special class of lives like annuitants may not be influenced by all the causes mentioned, but that some of them will have an effect upon it seems to be more than probable. The object sought in investigating the mortality experience of a past period is to obtain data from which the probable rates of mortality in future may be deduced. With the view, therefore, of obtaining the experience of lives living under conditions more closely resembling those of the present time it has been decided to commence the present investigation with the experience of the year 1875, and to omit altogether that of previous years. If any later year were taken as the commencement of the observation it would hardly allow of the collection of sufficient data.

The lives comprised in the present experience are accordingly—

- (1) The nominees of annuities granted before 1875 who were living in that year on the anniversary of the date of purchase of the annuity.
- (2) The nominees of annuities granted under the Acts 10 Geo. 4, c. 24 and 51 & 52 Vict. c. 15, between the 1 January 1875 and 31 December 1903.

Where two or more annuities have been granted on the same life only the experience afforded by the one first granted has been included.

The following table gives a summary of the data:—

Government Annuity Experience, 1875-190	SUMMARY OF DATA		
	Males	Females	Total
Number of lives under observation ...	5,504	13,863	19,367
Number of Years of Risk ... ...	57,652	163,378	221,030
Number of Deaths ... ...	4,168	9,333	13,501
Number of Lives existing at the close of the observation ... ...	1,336	4,530	5,866

There does not appear to be any material alteration as regards the ages at which annuities were purchased in the periods before and after 1875 respectively. In both periods about 5 per-cent of the total number of lives were under 40 years of age at the time of purchase and 65 per-cent between the ages of 50 and 70. The

following statement shows the number and ages of the lives on which annuities were purchased during the period 1875 to 1903 :—

Age	No. of Males	No. of Females	Total Number	Per-cent
15-19	5	8	13	...
20-29	35	103	138	1
30-39	134	323	457	4
40-49	351	909	1,260	11
50-59	867	2,473	3,340	29
60-69	1,123	3,122	4,245	36
70-79	601	1,325	1,926	17
80 and above	101	160	261	2
	3,217	8,423	11,640	100

The number of lives included in the experience on which annuities were purchased before 1875 is 7,727.

In order to deduce rates of mortality from the experience of annuitants during the period 1808-1875 Mr. Finlaison arranged the data so that the mortality could be traced through each year of age. For the first year the lives were assumed to come under observation, on the average, four months after their preceding birthday, and the annual rate of mortality based on the experience of the following eight months was taken as the rate for the whole year. After the first year the mortality was traced through exact years of age, that is, from birthday to birthday.

A different method has been followed in the present investigation, viz., that adopted by the Joint Committee of the Institute of Actuaries and the Faculty of Actuaries in the investigation into the mortality experience of annuitants in British Life Offices during the period 1863-1893. According to that method the lives are assumed to come under observation at the age attained upon the birthday nearest to the date of purchase, and the mortality is traced through each year of the duration of the annuity. The accuracy of this method is conditional upon no error of importance being involved in the assumption that the exact age at date of purchase is, on the average, equal to the nearest integral age. With the object of measuring the extent of the error when the method is applied to the case of the Government life annuitants an examination has been made of all cases included in the experience where the actual dates of birth are stated. It was found that the assumption mentioned had the effect of understating the tabular ages at purchase by about 20 days in the case of male lives and about 15 days in the case of female lives, and, in view of these slight differences, the method was considered to be sufficiently exact for the purposes of the present experience.

In all cases where only the date of baptism was available and not the actual date of birth, it was assumed that birth preceded

baptism by a period of 30 days. The average deviation between the tabular and exact ages at purchase, when the birthday was obtained on this assumption, was 18 days for male lives and 16 days for female lives, or practically identical with that shown in the cases where the exact date of birth was known.

The ages at date of purchase having been obtained in the manner described, the elementary facts relating to males and females respectively were tabulated under each age according to the duration of the annuity.

The period each life was under observation began either from the anniversary in 1875 of the date of purchase of the annuity or from the actual date of purchase in the years 1875 to 1903, and continued until the anniversary in 1904 of the date of purchase, or until previous death. The numbers of the lives coming under observation at their contract anniversaries in 1875 are stated in the column "Entered" of the tables opposite the appropriate number of "Years elapsed since purchase." The latter are the number of years between the actual date of purchase and its anniversary in 1875. The lives on which annuities were granted during the years 1875 to 1903 all come under observation from the date of purchase, and the numbers are stated on the first line opposite year "0" as there was no previous duration of the annuities in these cases.

The column "Existing" refers to the lives who were living in 1904 on the anniversary of the date of purchase, and who passed out of observation after their annuities had continued for the number of years given in the column "Years elapsed since purchase," the number between the date of purchase and its anniversary in 1904.

The duration in the case of the "died" was obtained by taking the number of integral years between the date of purchase and the date of death, and represents the number of contract years through which the life had passed previous to the contract year in which death occurred. In some instances the actual date of death was not known, and death could only be presumed owing to the cessation of the usual application for payment of the annuity. The assumption made in these cases was that the nominee of any annuity of which the instalment had remained unclaimed for three or more years could be taken as having died in the middle of the period between the date when the last paid instalment became due and the date of the first unclaimed instalment becoming due.

A series of independent tables for each age at purchase showing the variation in the rate of mortality with the duration since date of purchase are known as Select or Analyzed Tables. A considerable number of annuities are purchased at the National Debt Office under wills, &c., where the nominees do not exercise any option in the matter, but in most cases the nominees themselves are the purchasers. It is unlikely that the latter would choose a mode of investment which involves the risk of losing, in the event of premature death, a considerable part of the money invested, unless they considered themselves to be in a good state of health at the date of purchase. The effect of this self-selection, which corresponds in some measure

to that of the medical examination in the case of assurances against death, is shown by the rates of mortality during the years immediately following the grant of an annuity being lower than for lives of the same age whose annuities have been longer in force. It is usually found, however, that the duration, apart from the age, ceases to have much effect upon the mortality after a few years have elapsed from the date of purchase, and thereafter the rate of mortality may be considered to be dependent upon the age only. Tables constructed upon this basis and formed by grouping together all lives of the same attained age irrespective of the time elapsed since the annuities were purchased are known as "Aggregate Tables." Such tables may be either a complete aggregate of the Select Tables comprising all ages at purchase and all durations, or they may consist of an amalgamation of the Select Tables after excluding the experience relating to a certain number of years immediately following the date of purchase. Tables of the latter class are intended to represent the normal rate of mortality after the effect of the selection which is exercised at purchase is more or less exhausted. There are no means at present available of accurately measuring the full effect of this selection, and no definite limit can therefore be assigned to the period following purchase during which the rate of mortality is affected by its influence. The extent, however, to which the different tables are affected by the varying degrees of selection present in the data upon which they are based may be illustrated by a comparison of the expectations of life. In the case of males the values of  $e_{[x]}$  are given for age groups 50-54 to 70-74 only, owing to the paucity of the select data relating to ages outside these limits. The table also serves to show the considerable differences which exist between the expectations of life of males and females for all the ages at which annuities are usually purchased.

*Table showing the Mean Values of the Expectations of Life for various Groups of Ages according to different Classifications of the Data.*

Group of Ages	MALES				FEMALES				Group of Ages	
	Mean of Five Values of Expectations of Life				Mean of Five Values of Expectations of Life					
	Select Tables	Full Aggregate Table	Aggregate Table excluding First 5 Years	Aggregate Table excluding First 10 Years	Select Tables	Full Aggregate Table	Aggregate Table excluding First 5 Years	Aggregate Table excluding First 10 Years		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
40-44	...	24·97	24·42	24·00	29·01	29·47	28·54	27·52	40-44	
45-49	...	21·78	21·31	21·16	25·17	25·82	24·99	24·42	45-49	
50-54	18·89	19·13	18·69	18·52	22·21	22·23	21·60	21·19	50-54	
55-59	16·15	16·01	15·66	15·23	18·13	18·51	18·04	17·67	55-59	
60-64	12·80	12·89	12·61	12·34	15·42	15·00	14·61	14·37	60-64	
65-69	10·26	10·07	9·83	9·71	12·35	11·78	11·58	11·39	65-69	
70-74	8·58	7·59	7·41	7·27	9·36	8·84	8·73	8·67	70-74	
75-79	...	5·51	5·43	5·39	6·98	6·43	6·38	6·34	75-79	

NOTE.—The expectations of life given in the above Table and elsewhere in this Report are the "curtate" expectations ( $e_x$ ). If comparison is made with the corresponding values of  $\bar{e}_x$  (the "complete" expectations), as given in Mr. A. J. Finlaison's Report, 5 should be deducted from the latter values at all ages.

It will be of interest to compare some of the results brought out by the present investigation with those shown by (1) the Government annuity tables at present in force, based upon the experience of annuitants during the period 1808-1875, and (2) the British Offices annuity tables, based upon the combined experience in respect of life annuities of 43 British Assurance companies during the period 1863-1893.

## MALES.

Table showing, in Decennial Age Groups, the actual Number of Deaths recorded in each of the First Five Years following Purchase and the corresponding Number that would have been expected to occur if the Rate of Mortality had been similar to that shown by (1) the Government Annuity Experience (1808-1875), (2) the British Offices Annuity Experience (1863-1893).

Group of Ages at Purchase	ACTUAL DEATHS (Government Annuity Experience, 1875-1904)				EXPECTED DEATHS (Government Annuity Experience, 1808-1875)				EXPECTED DEATHS (British Offices Annuity Experience, 1863-1893)				Group of Ages at Purchase					
	Number of complete Years elapsed since Purchase				Number of complete Years elapsed since Purchase				Number of complete Years elapsed since Purchase									
	0	1	2	3	0	1	2	3	0	1	2	3	4					
Under 30	...	1	2	3	1	1	1	1	3	1	3	2	1	Under 30				
30-39	5	6	5	8	10	34	3	6	6	7	30	2	5	30-39				
40-49	15	9	19	21	20	84	15	19	23	22	101	10	17	40-49				
50-59	25	47	54	58	36	220	35	38	42	59	209	31	39	50-59				
60-69	23	47	31	55	54	210	27	43	47	49	219	33	43	60-69				
70-79	9	14	11	13	15	62	11	11	15	13	64	11	12	70-79				
80 and upwards.	—	—	—	—	—	—	—	—	—	—	—	—	—	80 and upwards.				
	78	123	122	158	137	618	92	115	133	135	157	632	88	116	127	126	150	607

FEMALIS.

Table showing, in Decennial Age Groups, the actual Number of Deaths recorded in each of the First Five Years following Purchase and the corresponding Number that would have been expected to occur if the Rate of Mortality had been similar to that shown by (1) the Government Annuity Experience (1808-1875), (2) the British Offices Annuity Experience (1863-1893).

It will be seen, in the case of male annuitants, that the total number of deaths recorded in the five years was 618 as compared with 632 expected deaths according to the previous Government experience and 607 according to the British Offices experience. On the whole, therefore, there is little indication of any important change in the rate of mortality, although considerable divergencies exist in respect of individual age groups. If the ages below 60 are excluded the total deaths are practically identical in all three cases. Owing to the paucity of the data, the deaths occurring in successive years of duration exhibit various irregularities, but the light mortality in the year following purchase, year "0", is a noticeable feature of the new experience at the older ages.

In the case of females it will be observed that the results lead to more definite conclusions, the most significant feature being the comparatively light mortality exhibited by the lives included in the new experience. There are clear indications of a distinct improvement in the mortality of Government female annuitants during the period 1875-1904 as compared with the previous period, the total number of deaths recorded being less than the expected deaths for each year of duration—the aggregate difference for the five years being approximately 10 per-cent.

The Government annuity tables at present in force are based upon the assumption that the effect of selection may be neglected after four years have elapsed from date of purchase. A table based upon similar principles has been deduced from the present experience, and a comparison is given below between the actual deaths recorded at durations of four years and upwards and the corresponding number expected according to the experience of the period 1808-1875.

[See Table on p. 75.]

The table tends to confirm the conclusions based upon the relative rates of mortality in the first five years following purchase. In both cases the results indicate that the improved vitality of annuitants during the period under consideration, although to a certain extent common to nominees of either sex, was much more marked in the case of female lives.

The aggregate tables based upon the new experience, after excluding the facts relating to the early years of duration, are more in accord with the corresponding British Offices tables. This will be apparent from the comparative statement [given on page 76] representing the mortality according to the aggregate experience, after the exclusion in each case of the first five years following purchase.

*Comparison of the Actual Number of Deaths recorded amongst Nominees whose Annuities had been in force Four Years and upwards at Date of Death, with the Number that would have been expected to occur at corresponding Ages if the Rate of Mortality had been similar to that shown by the Government Annuity Experience, 1808-1875.*

Group of Ages at Death	MALES			FEMALES				
	Actual Deaths, (Government Annuity Experience, 1875-1904)	Expected Deaths, (Government Annuity Experience, 1808-1875)	Excess (+), or Deficit (-), in Expected as compared with Actual Deaths	Actual Deaths, (Government Annuity Experience, 1875-1904)	Expected Deaths, (Government Annuity Experience, 1808-1875)	Excess (+), or Deficit (-), in Expected as compared with Actual Deaths		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Under 30	2	3	+ 1	...	3	- 3	+	3
30-39	3	8	+ 5	9	11	+ 5	+	5
40-49	32	34	+ 2	59	57	- 2	-	2
50-59	120	133	+ 13	233	278	+ 45	+ 1931	
60-69	514	533	+ 19	1,112	1,219	+ 107	+ 962	
70-79	1,519	1,529	+ 10	3,268	3,528	+ 260	+ 796	
80-89	1,283	1,319	+ 36	3,276	3,464	+ 188	+ 574	
90 and upwards	215	211	- 4	608	674	+ 66	+ 1086	
3,688	3,770		+ 82		8,565	9,237	+ 672	+ 785

*Comparison of the Actual Number of Deaths recorded amongst Nominees whose Annuites had been in force Five Years and upwards at Date of Death, with the Number that would have been expected to occur at corresponding Ages if the Rate of Mortality had been similar to that shown by the British Offices Annuity Experience.*

		MALES				FEMALES			
Group of Ages at Death	Actual Deaths, (Government Annuity Experience, 1875-1904)	Expected Deaths, (British Offices Annuity Experience, 1863-1893)	Excess (+), or Defect (-), in Expected as compared with Actual Deaths	Actual Deaths, (Government Annuity Experience, 1875-1904)	Expected Deaths, (British Offices Annuity Experience, 1863-1893)	Excess (+), or Defect (-), as compared with Actual Deaths	Actual Deaths, (Government Annuity Experience, 1863-1893)	Per cent minus Col. (6)	Per cent minus Col. (6)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(9)
Under 30	1	"	- 1	"	"	- 1	"	1	25·00
30-39	2	6	+ .4	- 29·03	9	1·4	5	5	
40-49	28	16	- 12	51	60	9	216	6	2·86
50-59	106	136	+ 30	+ 28·30	210	1,037	1,058	21	2·03
60-69	482	488	+ 6	+ 1·24	52	3,158	3,163	5	.16
70-79	1,465	1,433	- 32	- 2·18	3,234	3,271	37	37	1·14
80-89	1,252	1,253	+ 1	+ .08	607	622	15	15	2·47
90 and upwards.	215	206	- 9	- 4·19					
	3,551	3,538	- 13	- .37	8,306	8,405	99	99	1·19

In deducing the expected deaths in the preceding tables the unadjusted rates of mortality were employed.

The extent to which the lives of annuitants entering at different ages were prolonged as a consequence of the lower rates of mortality to which they were apparently subject during the period 1875-1904 may be measured by comparing the select expectations of life according to the three different experiences already mentioned. This comparison is shown in the following table [p. 78], where the values appearing opposite each group of ages represent the mean expectations of life for the five consecutive ages at purchase comprised in the group. The values are deduced from the unadjusted rates of mortality, with the exception of those representing the Government Annuity Experience, 1808-1875, which are taken from the graduated tables given in Mr. Finlaison's report.

In the case of males it will be seen from column (5) that, although the expectations of life according to the new experience are throughout in excess of those based upon the experience of the period 1808-1875, there is a close agreement between the values for the important age groups 60-64 and 65-69. The corresponding differences for females are given in column (10), and it would appear from these figures that a considerable increase has taken place in the duration of life of female annuitants at all ages. The figures in column (11) show that the expectations of life for females, according to the new experience, are also at most ages in excess of those based upon the British Offices experience, but in this case the differences are smaller and vary in sign, superior vitality being exhibited by the British Offices annuitants entering at ages 55-59 and 70-74. The explanation of the comparatively low expectation of life for age group 55-59 according to the new experience is to be found in the low values of the expectations for ages at purchase 55 and 56. For some reason the female annuitants entering at those ages were inferior lives, and the heavy mortality amongst them had the effect of reducing the average expectation for the group to a lower level than would have ruled if these nominees had maintained the normal standard of vitality.

The rates of mortality deduced from the crude facts of the experience require to be subjected to a process of adjustment, in order to remove irregularities due to limitation of the data, and to obtain a smooth series of values which will serve as a basis for the construction of monetary tables. The graduated table must be in a convenient form for practical use, and produce results which do not materially differ from those given by the ungraduated table. When it may be assumed that the factor of selection is of no practical importance after a few years have elapsed from the date of purchase, such a table may be constructed on the basis of a combination of rates of mortality varying with the age and duration during a limited number of years, and subsequent rates varying only with the age. The present Government annuity tables are based on the assumption that the effect of selection is practically exhausted after the expiration of four years, and consequently after that time

*Comparison of Expectations of Life at Date of Purchase, based upon the Government Annuity Experience, 1875-1904, with the corresponding Values according to the previous Government Experience and the British Offices Experience.*

Group of Ages at Purchase	MALES					FEMALES					Group of Ages at Purchase	
	Mean of Five Values of Expectations of Life		Differences.			Mean of Five Values of Expectations of Life		Differences.				
	Government Annuity Experience	British Offices Annuity Experience, 1863-1893	Col. (3) minus Col. (2)	Col. (3) minus Col. (1)	Government Annuity Experience	British Offices Annuity Experience 1863-1893	Col. (8) minus Col. (7)	Col. (8) minus Col. (9)	Col. (10)	Col. (11)		
1868-1875 (2)	1875-1904 (3)	(4)	(5)	(6)	1868-1875 (7)	1875-1904 (8)	(9)	(10)	(11)	(12)		
50-54	1860	1889	1883	+.29	+.06	27.97	29.01	+1.04	+.46	40-41		
55-59	1566	1615	1609	+.43	+.06	24.51	25.17	+.66	+.46	45-49		
60-64	1277	1280	1346	+.03	-.36	21.07	22.21	1.14	+.43	50-54		
65-69	1022	1026	1032	+.04	-.06	17.72	18.13	+.43	-.33	55-59		
70-74	798	858	822	+.60	+.36	14.50	15.42	1.08	+.34	60-64		
						11.48	12.35	1.87	+.22	65-69		
						8.85	9.36	.51	-.13	70-74		
						6.74	6.98	.74	+.24	75-79		
						*5.64	5.73	.09	+.73	80-84		

\* This is the value for age 80, which is the limit of the table.

the select rates of mortality are merged into an aggregate table. In the case of the British Offices annuity experience the effect of selection was found to extend over a longer period, and the results of the present investigation lead to a similar conclusion. The new tables based upon the present experience have been constructed in such a manner that when rates of mortality varying with the age and duration during the first five years are merged into those of an aggregate table varying only with the age, the expectations of life deduced therefrom will approximate closely to the true values at the date of purchase and at the expiration of five years thereafter.

In the graduation of mortality tables considerable use has been made in recent years of methods involving the assumption of a law of mortality, more especially the law known as "Makeham's first Modification of Gompertz's Law." According to the law of human mortality, as originally propounded by Gompertz, the function known as the "force of mortality" increases with the age in geometrical progression. This theory was further developed by Makeham, who introduced an important modification resting on the hypothesis that the force of mortality is composed of two parts, one of which is constant and the other a geometrical series. The law thus modified has been found to represent with great accuracy the rates of mortality of adult lives, as shown by many important tables. Expressed algebraically Makeham's formula for the law of mortality is  $\mu_x = A + Bc^x$ , where  $\mu_x$  represents the force of mortality at age  $x$  and  $A$ ,  $B$ , and  $c$  are constants, the values of which are to be derived from the facts. Tables graduated by this formula present a perfectly smooth series of values, and, moreover, embody an important principle known as the "Law of Uniform Seniority", which enables the value of an annuity upon two or more joint lives of different ages to be deduced from that of an annuity upon the same number of joint lives of equal ages. Makeham's formula, therefore, possesses many advantages as an instrument of graduation if it can be applied without any material distortion of the facts. A graduation by this formula of the aggregate tables based upon the present experience was found to yield satisfactory results, and the method has accordingly been adopted.

The degree to which the features of the original experience are reproduced in the graduated tables constructed to represent the experience after the exclusion of the first five years following purchase may be judged from a comparison of the ungraduated and graduated expectations of life as shown in the two following tables.

## MALES.

*Comparison of Ungraduated and Graduated Expectations of Life.*

Group of Ages	MEAN OF FIVE VALUES OF EXPECTATIONS OF LIFE			Excess (+), or Defect (-), in the Graduated Expectations of Life as compared with the Values of	
	Ungraduated Select Tables, 5 Years after Purchase	Ungraduated Aggregate Table excluding First 5 Years	Graduated Table		
	$e_{[x-5]+5}$	$e_x^{(5)}$	$e_{[x-5]+5}$		
(1)	(2)	(3)	(4)	(5)	(6)
55-59	15.44	15.66	15.63	+.19	-.03
60-64	12.77	12.61	12.68	-.09	+.07
65-69	9.82	9.83	10.01	+.19	+.18
70-74	7.58	7.41	7.67	+.09	+.26
75-79	5.81	5.43	5.68	-.13	+.25
80-84	...	3.97	4.06	...	+.09
85-89	...	2.86	2.78	...	-.08

## FEMALES.

*Comparison of Ungraduated and Graduated Expectations of Life.*

Group of Ages	MEAN OF FIVE VALUES OF EXPECTATIONS OF LIFE			Excess (+), or Defect (-), in the Graduated Expectations of Life as compared with the Values of	
	Ungraduated Select Tables, 5 Years after Purchase	Ungraduated Aggregate Table excluding First 5 Years	Graduated Table		
	$e_{[x-5]+5}$	$e_x^{(5)}$	$e_{[x-5]+5}$		
(1)	(2)	(3)	(4)	(5)	(6)
45-49	25.36	24.99	25.08	-.28	+.09
50-54	21.20	21.60	21.57	+.37	-.03
55-59	18.23	18.04	18.14	-.09	+.10
60-64	14.42	14.61	14.86	+.44	+.25
65-69	11.84	11.58	11.80	-.04	+.22
70-74	9.00	8.73	9.04	+.04	+.31
75-79	6.60	6.38	6.66	+.06	+.28
80-84	4.79	4.48	4.68	-.11	+.20
85-89	...	3.12	3.12	...	...

The expectations of life for males commence with age group 55-59 owing to the paucity of the original data at the younger ages. It will be seen from the differences in columns (5) and (6) that for ages 70-79 in the case of males and ages 65-84 in the case of females, the graduated expectations of life are considerably in excess of those based upon the aggregate experience as shown in column (3), but agree approximately with the select expectations given in column (2). These ages comprise the sections of the tables where it was found necessary to modify the values based upon the aggregate experience as shown in column (3), which did not adequately represent the true expectations of life five years after purchase. The abnormally low vitality of the female entrants at ages 55-59, which is reflected in the reduced values of the select expectations of life both at date of purchase and five years afterwards, has already been referred to, but this feature has not been retained in the graduation, and it will be observed that the graduated value for age group 60-64, corresponding to ages 55-59 at date of purchase, is considerably higher than either of the ungraduated values for this group. The mean of the graduated expectations of life for each group of ages of both males and females lies either between the corresponding values of  $e_{[x-5]+5}$ , and  $e_x^{(5)}$ , or is in excess of both.

Makeham's formula, as originally expressed, is not suitable for the graduation of rates of mortality which vary both with the age and with the duration, but Mr. G. F. Hardy, recently President of the Institute of Actuaries, has shown that the formula may be adapted to this purpose, and demonstrated its practical application in the graduation of the British Offices tables. A similar course has been followed on the present occasion and advantage has been taken of the methods indicated by Mr. Hardy in his memorandum on the subject. It will not be necessary to reproduce here the various mathematical expressions employed to establish the requisite relations between the select and ultimate sections of the tables in order to preserve the law of uniform seniority.

The graduated rates of mortality for each of the first five years after purchase and for five years and upwards after purchase, together with the values of the functions  $p_x$ ,  $e_x$ , and  $a_x$ , at 3 per-cent interest, as deduced therefrom, are given in the tables appended to this report.

The following table gives a comparison between the values of annuities at 3 per-cent interest, as deduced from the graduated experience, and the corresponding values based upon the original data. For ages under 65 in the case of males, and under 60 in the case of females, where the select expectations of life five years after purchase were found on the average to be in close agreement with those based on the corresponding aggregate experience, the values appearing in columns (2) and (6) have been obtained by using the select rates of mortality for the first five years following purchase in combination with the five years and upwards aggregate table. For ages above 65 in the case of males, and above 60 in the case of females, the ungraduated values are based entirely upon the select tables.

*Comparison of Ungraduated and Graduated Annuity Values  
at 3 per-cent Interest.*

Group of Ages	MALES					FEMALES					Gro of Age	
	Mean of Five Values of $a_{[x]}$		Deviations Col. (3) minus Col. (2)			Mean of Five Values of $a_{[x]}$		Deviations Col. (7) minus Col. (6)				
	Un- graduated (1)	Graduated (2)		+	-	Un- graduated (6)	Graduated (7)	+	-			
(1)	(2)	(3)	(4)	(5)		(6)	(7)	(8)	(9)		0	
50-54	13.54	13.57	.03	...		16.69	16.71	.02	...		45-	
55-59	11.83	11.86	.03	...		15.13	15.11	...	.02		50-	
60-64	9.96	10.12	.16	...		13.22	13.38	.16	...		55-	
65-69	8.29	8.43	.14	...		11.60	11.55	...	.05		60-	
70-74	7.20	6.83	...	.37		9.70	9.69	...	.01		65-	
75-79	5.07	5.38	.31	...		7.67	7.88	.21	...		70-	
Average	9.31	9.36	.05	...		5.94	6.19	.25	...		75-	
						5.01	4.70	...	.31		80-	

The larger deviations at the older ages are mainly due to the irregularities which arise from the scarcity of the select data upon which the ungraduated values at those ages are based.

In the following table a comparison is shown of the values of annuities according to the new tables with the corresponding values of the annuity tables published in 1903, which were based upon the British Offices graduated experience.

*Comparison of Annuity Values at 3 per-cent Interest according to the new Graduated Tables and according to the British Offices Annuity Tables.*

Age	MALES				FEMALES				Col. (5) min Col. (6)	
	Value of an Annuity of 1		Col. (2) minus Col. (3)	(4)	Value of an Annuity of 1		(6)	(7)		
	New Government Table (1)	British Offices Annuity Table (2)			New Government Table (5)	British Offices Annuity Table (6)				
(1)	(2)	(3)	(4)	(5)	(6)	(7)				
40	17.34	17.60	-.26		18.69	18.26			+.43	
45	15.85	16.06	-.21		17.31	16.93			+.38	
50	14.24	14.40	-.16		15.78	15.51			+.27	
55	12.55	12.66	-.11		14.09	13.96			+.13	
60	10.82	10.88	-.06		12.29	12.23			+.06	
65	9.10	9.12	-.02		10.43	10.33			+.10	
70	7.45	7.44	+.01		8.59	8.41			+.18	
75	5.93	5.90	+.03		6.84	6.61			+.23	
80	4.59	4.54	+.05		5.27	5.05			+.22	

It will be seen from the differences in column (4) that the annuity values on male lives are in close agreement for ages above 55. At the younger ages the British Offices male life values were made to correspond approximately with those deduced from the experience of assured lives, and they are slightly higher at these ages than the values given by the new tables.

In the case of female lives the new values are throughout in excess of those based upon the British Offices graduated experience, the largest differences occurring at the beginning and near the end of the table.

The results of the present investigation point to the conclusion that the mortality tables on which the cost of Government life annuities is now based underestimate the vitality of annuitants, and, consequently, that these annuities are granted on too favourable terms to the persons who purchase them.

The following tables show the effect which the adoption of the new tables would have upon the cost of Government life annuities when the rate of interest is taken at 3 per-cent per annum.

*Cost of an Annuity of £100 payable in Quarterly Instalments with an additional Payment of one-fourth part of the Annuity after Death; the Interest of Money being 3 per-cent per annum.*

*Male Lives.*

Age	New Table of 1910	Present Table	VARIATION	
			In Amount	Per-cent
40	£ 1,783	£ 1,687	+ 96	+ 5·7
45	1,635	1,566	+ 69	+ 4·4
50	1,476	1,433	+ 43	+ 3·0
55	1,308	1,284	+ 24	+ 1·9
60	1,136	1,114	+ 22	+ 2·0
65	965	946	+ 19	+ 2·0
70	801	786	+ 15	+ 1·9
75	651	639	+ 12	+ 1·9
80	518	514	+ 4	+ .8

*Female Lives.*

Age	New Table of 1910	Present Table	VARIATION	
			In Amount	Per-cent
40	£ 1,917	£ 1,867	+ 50	+ 2·7
45	1,781	1,732	+ 49	+ 2·8
50	1,628	1,578	+ 50	+ 3·2
55	1,460	1,413	+ 47	+ 3·3
60	1,282	1,232	+ 50	+ 4·1
65	1,098	1,046	+ 52	+ 5·0
70	915	856	+ 59	+ 6·9
75	741	694	+ 47	+ 6·8
80	585	552	+ 33	+ 6·0

These tables show that the cost of annuities should be increased at all ages for both male and female lives. In the case of males the most noticeable feature is the large increase in the cost at the younger ages. Comparatively few annuities are purchased at ages under 50, and the present experience, in common with other annuity experiences, suffers from a scarcity of data at the younger ages. The new values at these ages, however, would be still somewhat lower than the corresponding values according to the British Offices annuity tables. In the case of females the new values, when compared with those at present in force, show a considerable increase at all ages, the additional cost being approximately equal to half a year's purchase of the annuities throughout the greater part of the table. The average increase in the cost would be 2·2 per-cent for male lives and 4·5 per-cent for female lives at the ages at which annuities are usually purchased, viz., 50 to 75.

I have to acknowledge the valuable assistance rendered by Mr. W. R. Jarman of this Department during the course of the investigation, especially as regards the graduation of the tables.

I am, Sir,

Your obedient servant,

J. BLAKEY,

Actuary to the National Debt Commissioners.

## MALE LIFE ANNUITANTS.

*The Value of an Annuity of 1, at 3 per-cent Interest, for the Year in which Life Annuities are purchased, for each of the Four subsequent Years, and for Five Years and upwards from Purchase.*

## FEMALE LIFE ANNUITANTS.

*The Value of an Annuity of 1, at 3 per-cent Interest, for the Year in which Life Annuities are purchased, for each of the Four subsequent Years, and for Five Years and upwards from Purchase.*

Age at Purchase [x]	YEARS ELAPSED SINCE PURCHASE						Age Attained x + 5
	0	1	2	3	4	5 and upwards	
	$a_{[x]}$	$a_{[x]+1}$	$a_{[x]+2}$	$a_{[x]+3}$	$a_{[x]+4}$	$a_{[x]+5}$	
40	18·689	18·311	17·951	17·616	17·304	17·008	45
41	18·428	18·042	17·676	17·333	17·015	16·712	46
42	18·159	17·767	17·394	17·045	16·720	16·410	47
43	17·885	17·486	17·106	16·750	16·418	16·102	48
44	17·604	17·198	16·810	16·449	16·110	15·787	49
45	17·315	16·902	16·509	16·140	15·795	15·465	50
46	17·021	16·600	16·200	15·824	15·473	15·137	51
47	16·719	16·292	15·884	15·503	15·145	14·803	52
48	16·411	15·977	15·563	15·174	14·810	14·463	53
49	16·097	15·656	15·234	14·840	14·471	14·117	54
50	15·776	15·327	14·900	14·500	14·125	13·766	55
51	15·451	14·994	14·560	14·154	13·774	13·410	56
52	15·118	14·654	14·215	13·803	13·418	13·048	57
53	14·781	14·310	13·864	13·448	13·057	12·684	58
54	14·438	13·960	13·509	13·087	12·692	12·315	59
55	14·090	13·606	13·148	12·722	12·323	11·942	60
56	13·738	13·247	12·781	12·353	11·950	11·567	61
57	13·381	12·884	12·416	11·981	11·575	11·189	62
58	13·021	12·517	12·044	11·606	11·197	10·809	63
59	12·657	12·147	11·670	11·229	10·818	10·429	64
60	12·291	11·775	11·294	10·850	10·438	10·048	65
61	11·922	11·401	10·916	10·470	10·057	9·666	66
62	11·551	11·025	10·537	10·089	9·675	9·285	67
63	11·179	10·648	10·157	9·708	9·295	8·906	68
64	10·807	10·271	9·777	9·328	8·916	8·528	69
65	10·434	9·894	9·399	8·950	8·538	8·154	70
66	10·062	9·518	9·022	8·573	8·164	7·783	71
67	9·691	9·144	8·646	8·199	7·793	7·416	72
68	9·322	8·772	8·274	7·829	7·426	7·054	73
69	8·955	8·403	7·905	7·463	7·064	6·697	74
70	8·592	8·038	7·541	7·102	6·708	6·346	75
71	8·232	7·677	7·182	6·746	6·358	6·002	76
72	7·877	7·320	6·828	6·397	6·014	5·666	77
73	7·526	6·970	6·480	6·054	5·678	5·338	78
74	7·182	6·626	6·139	5·719	5·351	5·019	79
75	6·844	6·288	5·806	5·392	5·031	4·708	80
76	6·512	5·958	5·480	5·074	4·721	4·408	81
77	6·188	5·636	5·163	4·764	4·421	4·117	82
78	5·872	5·323	4·856	4·464	4·130	3·836	83
79	5·564	5·018	4·557	4·175	3·850	3·567	84
80	5·265	4·723	4·269	3·895	3·581	3·308	85
...	...	...	...	...	...	3·060	86
...	...	...	...	...	...	2·824	87
...	...	...	...	...	...	2·599	88
...	...	...	...	...	...	2·385	89
...	...	...	...	...	...	2·182	90
...	...	...	...	...	...	1·991	91
...	...	...	...	...	...	1·811	92
...	...	...	...	...	...	1·641	93
...	...	...	...	...	...	1·482	94
...	...	...	...	...	...	1·334	95
...	...	...	...	...	...	1·195	96
...	...	...	...	...	...	1·066	97
...	...	...	...	...	...	·946	98
...	...	...	...	...	...	·834	99
...	...	...	...	...	...	·728	100

## LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., Barrister-at-Law.

Sale of equity  
of redemption  
of contingent  
reversionary  
interest. Right  
of vendor to  
indemnify in  
respect of  
mortgage debt.

THE recent case of *Mills v. United Counties Bank, Limited* [1912], 1 Ch. 231, is concerned with the question of the liability of a purchaser of the equity of redemption of certain contingent reversionary interests to indemnify the vendor against the mortgage debt. The material facts, for the purposes of these Notes, are as follows:—The plaintiff, Charles Frederick Mills, was entitled to one-eleventh share of his deceased father's estate in reversion, expectant upon the death or second marriage of his father's widow, Harriet Mills, and contingent upon his being alive at the date of such death or second marriage. The plaintiff, under two deeds dated respectively 24 June 1897 and 21 November 1898, mortgaged his reversionary interest to the defendant's bank to secure a sum of £10,000 and interest owing by his firm to the bank. The plaintiff, by a deed dated 14 March 1899, further mortgaged his interest to his father-in-law, John Mobberley, to secure an advance to himself individually. Upon the bank commencing an action against the plaintiff and others to recover the amount due to them, an arrangement was made by a deed dated 30 October 1902, under which, *inter alia*, the equity of redemption in the plaintiff's contingent reversionary interest was conveyed to the bank, subject to the second mortgage for £2,000 to John Mobberley, and subject to the payment by the bank of all duties payable to the Inland Revenue authorities in respect of the reversion. The deed provided that the mortgages to the bank should not merge in the equity of redemption but remain and be kept on foot as a charge so as to protect the bank against the moneys owing to John Mobberley, but for no other purpose, and it was declared that on the reversionary interest falling into possession, in the first place, the bank should retain the amount of principal and interest due under their charge, in the second place the charge in favour of John Mobberley should be paid off, and the surplus, if any, should be retained by the bank for their own absolute use and benefit.

John Mobberley died on 14 October 1907, and all mortgage debts and interest due to him became vested in Beatrice Maria Mills, the wife of the plaintiff. She called upon her husband to pay the debt under the covenant contained in the

mortgage deed of 14 March 1899, and he brought the present action on 13 July 1910, claiming a declaration that he was entitled to be discharged and exonerated by the defendant from all liability in respect of this debt of £2,000 and interest, and that the defendant might be ordered to pay the debt to his wife. The plaintiff's interest had not fallen into possession, but remained a contingent reversionary one.

The plaintiff alleged in his claim that the effect of the purchase by, and assignment to the bank of the equity of redemption in his reversionary interest was that the defendants became subject to an implied contract or obligation to the plaintiff to indemnify him against the mortgage of 14 March 1899, and the money thereby secured, but Eve, J., before whom the case came in the first instance, held, first, that the doctrine of *Waring v. Ward* (7 Ves. 332) did not apply to a mortgage of a contingent reversion unless the interest became vested in possession, and secondly, that on the construction of the deed of assignment of 30 October 1902, the existence in that deed of an express qualified covenant of indemnity negatived the implication of any fuller indemnity, and on these grounds he dismissed the action.

The plaintiff appealed against this decision, but on the case coming before the Court of Appeal, the judgment of Eve, J., was upheld, and the appeal dismissed. Cozens-Hardy, M.R., in delivering judgment to this effect, said: "The claim is based on this ground. It is said that according to the settled law of the Court a purchaser of an equity of redemption is bound under an implied obligation, or, as it is sometimes put, an obligation of conscience, to indemnify the vendor against the liability on the mortgage debts; and in an ordinary case that is, I think, obviously according to justice and common sense. If a property is worth £10,000, and is subject to a mortgage of £5,000, and the purchaser only pays the vendor £5,000 and gets the property, it would be almost shocking to say that in that case the vendor would be liable on the covenant to pay the full sum of £5,000 to the first mortgagee and that the purchaser was under no obligation to indemnify him. But that is a principle of law which must, of course, bend to the circumstances of the particular case. It is an implication and not an express obligation, and when you have to deal with an implication, you must, of course, have regard to all the circumstances of the case and to all the relations between

"the parties as expressed in the purchase deed; and if you  
"find in the purchase deed something to satisfy you that it  
"was not the real intention of the parties that the purchaser  
"should take upon himself the liability to indemnify the  
"vendor against the mortgage, there is nothing that binds  
"the Court to adopt the ordinary rule.

"Now there are two points made by the learned judge  
"which I should just like to refer to before I come to the  
"construction of the deed. The interest of the vendor here  
"was both contingent and reversionary. That interest has  
"not yet fallen into possession. The learned judge seems to  
"have thought that the doctrine of *Waring v. Ward* would  
"have no application to the purchase of a contingent re-  
"version which might or might not fall into possession, and  
"that the whole doctrine depended upon the purchaser  
"having gone into possession and enjoyment. If this case  
"had turned on that doctrine I should certainly have required  
"it to be more fully argued, because as at present advised I  
"cannot assent to that view of the law. But the learned  
"judge goes on to say 'that the implication of an unqualified  
"personal liability on the part of the purchaser is negatived  
"in this and in all cases where there is an express qualified  
"indemnity, or where the circumstances and nature of the  
"transaction are otherwise of a nature to rebut such  
"implication.' That is a proposition to which I give my  
"full assent, and it is on that ground that I think Eve, J.'s,  
"decision must be supported. . . . There is in this deed an  
"express covenant by the bank that they will indemnify the  
"vendors, and the extent to which they will indemnify the  
"vendors is stated. The covenant is by the bank with each  
"of the three vendors, including Charles, 'That the bank,  
"their successors and assigns will pay all duties payable to  
"the Inland Revenue authorities in respect of the shares,  
"interests and premises hereby assigned and will hold  
"harmless and indemnified the said parties hereto of the  
"first three parts from all claims and demands in respect of  
"or on account thereof.' If it were any part of the bargain  
"between the parties to this deed that the bank should in-  
"demnify the vendor against Mobberley's mortgage, where  
"would it be found? Undoubtedly in this last covenant.  
"That is the place where you would expect to find the  
"particular items against which an indemnity was to be  
"given, and in the face of the recitals in the deed, and in the

"face of the anxious endeavour to protect the bank against "Mobberley's mortgage, and in the face of this final covenant "in the deed, I cannot myself feel any doubt that the "doctrine of *Waring v. Ward* has no application to this case. "The circumstances of the parties and the terms of the deed "suffice to negative the implication, which of course must "give way to the express terms of the deed.

"In my opinion the judgment of Eve, J., was quite right, "and this appeal must be dismissed."

**Interest on  
arrears of  
annuity**

The case of *In re Salvin Worseley v. Marshall* [1912], 1 Ch. 332 raises the question as to whether

interest is payable on the arrears of an annuity in the following circumstances:—A testator, Bryan John Francis Salvin, died on 18 October 1902. By an indenture dated 24 February 1897, executed on the occasion of the marriage of his brother, the testator covenanted to pay to the trustees therein named an annuity of £400 during the joint lives of his brother and his then intended wife, and during the life of the survivor, by equal half-yearly instalments. The defendant, Edward Gleadows Marshall, was the surviving trustee of this deed. This annuity was duly paid down to and including the half-yearly instalment which accrued due on the 24 August preceding the testator's death. This action was commenced by originating summons on 25 April 1903, and on 31 July 1903, an order was made in the usual form for the administration of the testator's personal estate. At the date of that order the half-yearly instalment of the annuity due on 24 February 1903 remained unpaid, and the subsequent instalments were not paid. By an order dated 12 November 1907, enquiries were directed as to the amount of the arrears, and as to the capitalized value of the annuity on the day of the testator's death, and by the certificate filed in answer to such enquiries on 30 April 1908, the arrears—including £57. 5s. 3d. accrued during the testator's lifetime—were found to be £2,158. 6s. 6d. and the capitalized value of the annuity at the testator's death to have been £9,299. 18s. 6d. Further arrears accrued after the date of the last mentioned certificate, but ultimately on 24 August 1910, the arrears were discharged and the future payment of the annuity provided for, but no provision was made for the payment of interest on the arrears of the annuity.

On the case coming before Eve, J., he held that the

trustees were entitled to interest at 4 per-cent on the arrears. In delivering judgment to this effect he said : "The bulk of "the personal estate has been realized, and after discharging "the balance of the principal sums due to the creditors with "interest there will be a substantial surplus. Out of this "surplus the trustees of the annuity deed claim to be paid "interest at 4 per-cent per annum on the arrears of the "annuity, to be calculated on such part of them as had "accrued due at the date of the judgment, from that date, "and on such parts of them as accrued due subsequent to the "judgment, from the dates when they respectively accrued "due, and in all cases down to the actual date of payment. "This claim seems to be in strict accord with the provisions "of Order LV, rr. 63 and 63, but it has been argued on "behalf of the pecuniary legatees who are primarily "interested in resisting the claim, that there is a general rule "of practice under which interest is only allowed on arrears "of an annuity in very exceptional circumstances, that no "such circumstances exist here, and accordingly, that the "claim ought not to be allowed.

" I am inclined to agree with the contention that if there "be any such rule there are no sufficient grounds for not "applying it in the present case, but is there, in fact, any "such rule applied or applicable in the working out of the "judgment in an action of this nature ? Apart from authority, "the existence of such a rule would seem to be consonant "neither with good sense nor sound principle, for in what "attributes, it may well be asked, does a debt, arising out of "a covenant to make a series of periodic payments, differ "from one founded on any obligation to pay money where "there is no contract or statutory provision as to interest ; "and if, as would seem to be the case, there is no essential "difference between the quality of the two debts, why should "the one be treated as outside and the other as within the "Order I have referred to ?

" On behalf of the legatees the case of *Earl of Mansfield v. Ogle* (4 De G. & J. 38) is cited as authority for the "existence of such a rule. I do not think it has any "application to the present case . . . and if the case is to be "regarded as an authority for the proposition that in the "administration of the estate of the grantor of an annuity "the creditor in respect of arrears of such annuity is in a "different position from other creditors in that he is excluded "from recovering interest on his debt, it would seem to be

" quite inconsistent with *Lainson v. Lainson* (No. 2) (18 Beav. 7). " I do not, however, so regard it. The rule stated is a rule " applicable to different circumstances and in a wholly different " class of action, just as was the case in *In re Hiscoe* (71 L.J. " (Ch.) 374).

" The result is that, in my opinion, the claim to interest in " this case is well founded and must be allowed."

Voluntary settlement. Sale by voluntary donee after act of bankruptcy. Position of bona fide purchaser for value without notice.

The case of *In re Hart Ex parte Green* [1912] 3 K.B. 6, deals with the position of a *bonâ fide* purchaser for value without notice where the vendor holds under a voluntary settlement, and the

transaction takes place after an act of bankruptcy has been committed by the settlor. The facts of the case will be seen sufficiently from the extract from the judgment of Cozens-Hardy, M.R., which follows. In deciding in favour of the purchaser, he said : " The facts in this case are few and " are not in dispute, but they give rise to a question of law of " great importance and of very considerable difficulty. Hart " was the registered owner of fully-paid £1 shares in " a limited company of which he was director. On " October 14 1909 he transferred 250 of these shares " to his daughter, and the transfer was duly registered. " There was no consideration for the transfer. On " April 13 1910, the daughter transferred these shares " for value to Flora Lomas, and the transfer was " duly registered. Meanwhile a petition was presented on " 4 April against Hart, the act of bankruptcy alleged being " non-compliance with a bankruptcy notice which had been " served upon him on 15 March. A receiving order was made " on 22 April, and Hart was adjudicated a bankrupt on " 13 May. Flora Lomas had no notice of the act of bankruptcy " committed by Hart or of the presentation of the petition. " In these circumstances Phillimore, J., held that the transfer " of the shares to the daughter was and is void as against " the trustee under section 47 of the Bankruptcy Act, 1883, " and that the transfer to Flora Lomas was and is void " as against the trustee on the ground that the title of the " trustee had accrued prior to the date of such transfer, and " ordered Flora Lomas to deliver up to the trustee the " certificate for the shares and to execute a transfer of the " shares to the trustee. From this order Flora Lomas " appeals.

" Now there is no doubt that the title of the trustee relates " back to the act of bankruptcy on 31 March (section 43), " which was prior to the transfer; and it is contended that " the language of section 47 is clear, and that there is no such " saving of the rights of a purchaser for value without notice " as is found in section 48, sub-section 2, or in section 49.

" It is necessary to consider carefully the language of " section 47 and certain decisions upon that section. It was " held by Vaughan Williams, J., in the case of *In re Brall* " ([1893] 2 Q.B. 381) that when the statute says that a " voluntary settlement is 'void against the trustee in bank- " ruptcy' the word 'void' must be construed as meaning " 'voidable' and not absolutely void. He points out that in " all the earlier bankruptcy statutes dealing with this subject- " matter the settlement was made voidable by the Court " although neither the word 'void' nor 'voidable' was used, " and he concludes that the object of the Legislature had " remained unchanged and that the purpose of the alteration " was merely convenience of drafting. He added that the " moment it is assured that a settlement is not void *ab initio*, " those authorities apply which decide under the Statute 27 " Eliz. c. 4 that a purchase for valuable consideration from a " person making a title under a voluntary conveyance relates " back so as to prevent the original conveyance being a " voluntary conveyance within the meaning of the Act. So " far as I am aware, *In re Brall* has never been questioned, " and it was expressly approved of by the Court of Appeal in " *In re Carter and Kenderdine's Contract* ([1897] 1 Ch. 776). " It has also been held that section 47 does not vest the " property comprised in the voluntary settlement in the " trustee (*Sanguinetti v. Stuckey's Banking Company* [1895] " 1 Ch. 176). In the present case the shares are vested in " Flora Lomas, and Phillimore, J., has ordered her to transfer " them to the trustee. In my opinion the true view is that a " voluntary settlement is not void, but is only voidable by the " trustee, who must apply to the Court for a declaration to " that effect and for consequential relief.

" It seems to me to follow that the trustee is in the same " position as any other litigant who seeks a declaration that " a deed is voidable, whether on the ground of fraud or undue " influence or otherwise. It is settled law that, however good " a claim might be as against the grantee, the plaintiff cannot " succeed if there has been any subsequent transaction for " value and without notice.

"On 13 April who was the owner of the shares? Only "one answer is possible. Flora Lomas, who acquired them "from the registered owner. No question of title had arisen. "The property was of such a nature that no investigation of "the title of the transferor was possible, and it would be "almost shocking to deprive her of the property she acquired "in good faith. I think I am entitled to do what was done "by Lindley, L.J., in *In re Carter and Kenderdine's Contract*, "namely, 'to consider what the consequences would be if any "other conclusion were arrived at.' No purchaser of shares "or stock on the Stock Exchange would be safe if Flora "Lomas is to be deprived of her shares. If I may adopt the "words of Lindley, L.J., 'to my mind good sense is shocked "by such a startling construction as that.'"

Kennedy, L.J., in the course of his judgment, said: "I "have no difficulty in saying, and *In re Carter and Kenderdine's Contract*, I think, assists me in saying, that the whole effect "of section 47 is to render the transaction voidable as between "the trustee and the voluntary donee, but not as between "the trustee and the *bonâ fide* purchaser for value without "notice from the donee . . . For the decision of the present "case it is sufficient to say that the *bonâ fide* purchaser for "value without notice whose title was complete before there "came into existence a trustee who could seek to avoid the "voidable voluntary settlement has a good title."

#### ACTUARIAL NOTE.

*On the value of a Loan repayable by a Cumulative Sinking Fund.* By HERBERT PETTER.

A very simple formula may be deduced for the solution of the problem discussed in *J.I.A.*, xlvi, p. 401, and *Text-Book*, Part I, p. 94.

A loan of 1 bears interest at the nominal rate of  $g$  per annum, payable  $p$  times a year, and is repayable as to capital by a cumulative sinking fund of  $z$ , operating yearly\* and repaying the entire loan in  $n$  years. Find the present value of the loan to yield an effective rate of  $i$  per annum.

The successive capital repayments are

$$z, z(1+g), z(1+g)^2 \dots z(1+g)^{n-1},$$

\* By a suitable alteration of the formula, the interval for  $z$  may be varied.

and K, their present value at effective rate  $i$

$$\begin{aligned} &= vz[1 + v(1+g) + v^2(1+g)^2 + \dots + v^{n-1}(1+g)^{n-1}] \\ &= vz \cdot \frac{1 - v^n(1+g)^n}{1 - v(1+g)} \\ &= \frac{z - zv^n(1+g)^n}{i-g} \end{aligned}$$

But  $z[1 + (1+g) + (1+g)^2 + \dots + (1+g)^{n-1}] = 1$

whence

$$z = \frac{g}{(1+g)^n - 1}$$

and

$$z(1+g)^n = z + g$$

$$\therefore K = \frac{z - v^n(z + g)}{i - g}.$$

The expression for K on p. 403, *J.I.A.*, xlvi, may be readily reconciled with the above:—

In the notation there used

$$\begin{aligned} K &= \frac{j - i \left( j + \frac{z}{p} \right) \frac{1 - v^n}{i}}{j - i \left[ \frac{(1+i)^p - 1}{pi} \right]} \\ &= \frac{jv^n - \frac{z}{p} + \frac{zr^n}{p}}{j - \frac{(1+i)^p - 1}{p}} \\ &= \frac{pjv^n - z + zv^n}{1 + pj - (1+i)^p} \\ &= \frac{z - v^n(z + pj)}{(1+i)^p - (1 + pj)} \end{aligned}$$

In the notation of the present note  $1+i$  must be substituted for  $(1+i)^p$  and  $g$  for  $pj$ , so that the expression last given becomes

$$\frac{z - v^n(z + g)}{i - g}$$

which is the value for K previously deduced.

The value of the whole loan is :—

$$K + \frac{g}{i} (1-K) s_1^{(p)} \text{ in } Text-Book \text{ notation}$$

$$= K + \frac{\frac{1}{p} g}{\frac{(1+i)^{\frac{1}{p}} - 1}{(1+i)^{\frac{1}{p}} - 1}} (1-K).$$

As an example, taking that given on p. 404, *J.I.A.*, xlvi, and putting

$$n = 36, i = (1.0275)^2 - 1 = .055756, g = .05, p = 2,$$

$$v^{36} = (1.0275)^{-72} = .14181; z = \frac{1}{s_{36}} \text{ at rate } g = .01043446$$

$$\therefore K = \frac{z - v^{36}(z+g)}{i-g} = \frac{.01043446 - .14181 \times .06043446}{.055756 - .05}$$

$$= .32388,$$

and the value of the loan per-cent

$$= 100 \left[ K + \frac{\frac{1}{2} g}{\frac{(1+i)^{\frac{1}{2}} - 1}{(1+i)^{\frac{1}{2}} - 1}} (1-K) \right]$$

$$= 32.388 + \frac{.0250}{.0275} (100 - 32.388)$$

$$= 93.854.$$


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[In view of the attention which has been directed to the problem discussed in the foregoing Note it may be well to place on record the following neat and instructive solution by Mr. S. E. Macnaghten :—

In the usual notation the value of the loan

$$= K + g \frac{s_1^{(p)}}{i} (1-K) = K + \frac{g}{i} (1-K) + g \frac{s_1^{(p)} - 1}{i} (1-K) \dots (1)$$

Now, since the capital-repayments are independent of  $p$ , K has the same value whatever value be assigned

to  $p$ . But when  $p=1$  the value of the loan =  $\frac{a_i^{\frac{1}{n}}}{a_g^{\frac{1}{n}}}$ .

$$\therefore K + \frac{g}{i} (1 - K) = \frac{a_{\frac{n}{i}}^i}{a_{\frac{n}{i}}^g}$$

and  $\frac{1 - K}{i} = \frac{1}{i - g} \left( 1 - \frac{a_{\frac{n}{i}}^i}{a_{\frac{n}{i}}^g} \right)$

whence, by substitution in (1), the value of the loan

$$= \frac{a_{\frac{n}{i}}^i}{a_{\frac{n}{i}}^g} + g \frac{\frac{s(p)}{i} - 1}{i - g} \left( 1 - \frac{a_{\frac{n}{i}}^i}{a_{\frac{n}{i}}^g} \right)$$

The second term in this expression represents the increase in the value of the loan on account of the interest being paid  $p$  times a year instead of yearly, and the method of solution has the advantage of bringing out very clearly the connection between the general and particular cases.

For purposes of numerical calculation any one of the formulas which may be selected should, of course, be put into the form best adapted to the problem to be dealt with. Thus, in the usual case of the value being required, to yield  $j$  per annum convertible half-yearly, of a loan of 1 bearing interest at  $g$  per annum payable half-yearly and repayable in  $n$  years by an annual sinking fund of  $z$ , the formula in the Note will take the form

$$j + \left(1 - \frac{g}{j}\right) \frac{z - v^{2n}(z+g)}{\left(1 + \frac{j}{2}\right)^2 - (1+g)},$$

in which  $v^{2n}$  is to be calculated at rate  $\frac{j}{2}$  and  $z = \frac{1}{s_{\frac{n}{i}}^g}$  at rate  $g$ ;

and Mr. Macnaghten's formula becomes

$$\frac{1}{2\left(1 + \frac{j}{4}\right)} \frac{a_{2n}^{j/2}}{a_n^g} + \frac{\frac{1}{4}gj}{\left(1 + \frac{j}{2}\right)^2 - (1+g)} \left[ 1 - \frac{1}{2\left(1 + \frac{j}{4}\right)} \cdot \frac{a_{2n}^{j/2}}{a_n^g} \right]$$

—ED. J.I.A.]

## REVIEWS.

*Victorian Friendly Societies Experience, 1903-1907.* By A. M. LAUGHTON, F.I.A., F.F.A., *Government Statist and Actuary for Friendly Societies, Victoria.*

A REPORT with the above title, published by the authority of the Government of the State of Victoria, affords an extremely interesting view of the sickness and mortality experience prevalent among the more important of the friendly societies in that State. It appears that prior to 1909 the tables used in the valuations of the Victorian friendly societies were based on English experience. These being found unsuited to local conditions steps were taken to collect data from the experience of the Victorian friendly societies themselves with the view to the preparation of tables reflecting the actual conditions prevailing in the State. These tables are embodied in Mr. Laughton's report.

The experience includes about a quarter of a million years of life at risk, 2,451 deaths, and nearly 443,000 weeks of sickness. Mr. Laughton divides the data into two broad groups, "non-mining" and "mining", the latter accounting for about 37,500 years of life at risk or about 16 per-cent of the entire experience. This is a much larger proportion than obtained in the case of the Manchester Unity Experience, and the excessive risks of the mining group result in the exhibition of a substantially heavier sickness rate in the "Whole Experience" tabulation than appears in that of the "non-mining" group, although the latter appears to include every kind of risk with the exception of mining. At the age of 40, the respective rates of sickness (all durations) stated in weeks, are 1.0684 in the "Whole Experience", and .9949 in the "non-mining" group. The difference of 7 per-cent here shown and a corresponding difference of about 9 per-cent at the age of 50 seem to point to an excess of sickness liability in the case of the miners of fully 50 per-cent of the average of the "non-mining" group.

The outstanding features of the experience are the remarkably light rates of sickness and mortality shown throughout the working period of life. In respect of mortality, the "non-mining" experience shows distinctly lower death rates than that of the Manchester Unity, area 1, at nearly all ages under 60. The rate of sickness among the same group is substantially lower than that of the Manchester Unity, group A.H.J., and it is interesting to observe that in this respect it is in fairly close correspondence with the experience of friendly societies in other Australian States.

For instance, at the age of 40, the Victorian sickness rate is .995 of a week. The Manchester Unity rate, group A.H.J., is 1.313 weeks. The respective rates shown by the Friendly Society Experiences in New South Wales, 1900-1908, and South Australia, 1895-1904, are .896 weeks and .971 weeks. At the age of 50, the respective rates are: Victoria, 1.497 weeks; Manchester Unity,

group A.H.J., 2·177 weeks; New South Wales, 1·472 weeks; South Australia, 1·583 weeks. The low rate of sickness prevailing in the friendly societies in nearly all the daughter States as contrasted with that met with by the friendly societies in the Motherland is an interesting phenomenon, and is explainable probably by the differences between the conditions of life in a new and an old country.

The sickness rates were graduated by Mr. Spencer's 19-term formula, *J.I.A.*, xxxviii, p. 336. It is advisable in general to graduate sickness rates by a formula having a fairly restricted range, but where the data are limited and the ungraduated values tend to fluctuate widely it is advisable to adopt a formula of more extended range, and Mr. Laughton has exercised a judicious choice in employing a formula specially devised to meet such cases. It will be remembered that this particular formula was originally used to graduate the sickness rates of the E.F. group of the Manchester Unity Experience which, like Mr. Laughton's data, represented a very small experience. The mortality table was graduated by Mr. Spencer's 21-term formula, with entirely satisfactory results, as would indeed be expected in the ease of this powerful adjusting instrument. The "rate of sickness" is tabulated in the old form, the number of weeks of sickness at each age being divided by the number of lives entering on the age. At the ages between 65 and 75 the rates are gradually merged in those of the Manchester Unity, 1893-97 (group A.H.J.) experience, the latter being first adjusted to convert them from central rates to the older form of the function. From the age of 89 the rates are taken as constant, in both the whole society and non-mining experiences, at 33·488 weeks per annum. The point is not of any practical importance, but it may be suggested that Mr. Laughton would have satisfied the theoretical conditions more exactly had he completed his table with the Manchester Unity rates multiplied at each age by  $(1 - \frac{1}{2}q_x)$ . In this way he would have produced the same feature of a diminishing "sickness rate" at the older ages as is to be found in Sutton's Friendly Societies Experience. By the plan he has adopted he has really incorporated in his table a rate of sickness increasing to the limiting age, with the curious maximum at age 100 (assuming an even distribution of deaths throughout the year) of 67 weeks per annum.

The fact is that the old "rate of sickness" which Mr. Laughton has employed is not in any real sense of the term a rate of sickness at all, and it is not too much to suggest that it should be abandoned in favour of the central rate of sickness which is easier to work with in the preparation of Monetary Tables, and also expresses a definite statistical conception. In discussing social problems and in arranging actuarial methods of dealing with them, nobody wants to know the quotient of the number of weeks of sickness in a year divided by the number of persons beginning the year; but it may well be desired to know the true average quantity of incapacity during any year of age, or the average proportion of

incapacity prevalent among a given body of lives on any day in that year. The one is given by the central sickness rate at age  $x$  (say  $\bar{z}_{x+\frac{1}{2}}$ ), the other by  $\frac{\bar{z}_{x+\frac{1}{2}}}{52 \cdot 18}$  (say  $\sigma_{x+\frac{1}{2}}$ ). Again, the proportion of persons sick on entering age  $x$  may be required, and this is at once yielded by  $\frac{\bar{z}_{x-\frac{1}{2}} + \bar{z}_{x+\frac{1}{2}}}{2(52 \cdot 18)} = \text{say } \sigma_x$ .\*

As an example of the usefulness of the central rate of sickness the following method (suggested by Mr. G. F. Hardy) of obtaining mathematically the proportion of members becoming sick during a year will be of interest. This will readily be recognized as a problem with which actuaries have lately been called upon to deal, and since actual experience is not always easily obtainable, Mr. Hardy's very simple method of involving the central rate of sickness with one other tabulated function can hardly fail to appeal to the actuary who has need to estimate the value of a benefit payable in respect of the first few days of sickness.

By a standard table, such as the Manchester Unity Experience, the "proportion sick" in the course of a year of age is the ratio of the number of cases of sickness to the number exposed to risk in the middle of the year; and this proportion is compounded of the proportion sick at the beginning of the year ( $\sigma_x$  as above), and the proportion falling sick during the year.

Thus,

$$\begin{aligned} \text{Total number sick} &= \text{Full proportion sick} \times l_{x+\frac{1}{2}} \\ &= \text{New cases of sickness} + \sigma_x \times l_x \end{aligned}$$

Therefore,

$$\text{Number of new cases} = \text{Full proportion sick} \times l_{x+\frac{1}{2}} - \sigma_x \times l_x$$

and proportion of new cases,  $i.e.$ , probability of becoming sick during the year . . .

The full proportion sick in the case of the Manchester Unity Experience (whole society) is a tabulated function to be found in cols. (3) and (12), pages 210-13, of the Manchester Unity tables.

A. W. W.

\* This expression may be regarded as the "force of sickness" at age  $x$ .

### *Japanese Three Offices Life Tables (1910).*

THE two volumes comprising the Japanese Three Offices Life Tables (1910) furnish a welcome addition to the material at the disposal of the actuarial profession for the study of comparative mortality;

and although the data were drawn from Japanese assured lives, it may well be that the distinctive features of the tables will prove of use to British Actuaries in the investigation of extra risks.

Apart from the purely scientific utility of the volumes, they are of personal interest to the members of the Institute by reason of the fact that Mr. K. Ebihara—the only gentleman of Japanese nationality who has obtained the Fellowship of the Institute by examination—was chiefly responsible for the compilation of the tables. Mr. Ebihara had been sent to London by one of the contributory offices for the purposes of study, and after a brilliant course of little more than three years, in which he passed all the examinations of the Institute, he returned to Japan in 1906. With exemplary promptitude the work of constructing reliable endemic mortality tables, to serve as a scientific basis for the Life Assurance business he was called upon to administer, was commenced in May 1907. Little more than three years were taken in the collection of the data and the construction and graduation of the Life Tables, including the formation of a complete set of monetary functions, and the volumes embodying the results and describing, not only in the Japanese language but in English, the processes followed, were ready for issue early in 1912—a notable achievement considering the extent of the data.

Proceeding to a closer inspection of the volumes we note that the President of each of the three contributory offices has written a short preface. From these it appears that two of the offices previously used the English Seventeen Offices' Experience, and the third a native table based on more or less inaccurate statistics of the general population of Japan. We also learn that although life assurance was introduced into Japan not more than 30 years ago, it has made substantial progress, and that the 29 companies in existence (in 1910) had in force policies for sums assured exceeding 500 million yen (£50,000,000) and funds amounting to 60 million yen (£6,000,000). It is also pointed out that the three offices have taken special care in the medical selection of the lives proposing for assurance.

Following the preface is an account of the methods adopted in tabulating the data and in deducing the ungraduated rates of mortality. We need not here enter into detail, for in the main the methods of construction adopted closely resemble those followed in the British Offices' Life Tables. It must, however, be remarked, that the data used in the investigation are those of healthy assured lives, males and females, who were assured under whole-life and endowment assurance policies by the three oldest Japanese offices, namely, the "Meiji" (established 1881), the "Teikoku" (established 1888), and the "Nippon" (established 1888). The observations covered the whole history of these companies from their formation until the end of 1905, but all emigrants, and lives in military service who were assured in 1903 or after, were excluded, thus eliminating the special risks incurred during the Russo-Japanese War of 1904-1905.

The number of cards sent in indicates that endowment assurances are as popular in Japan as in the United Kingdom, the figures being as follows :

Whole-life with and without profits ... ... 229,844

Whole-life by limited payments with and without profits ... ... ... ... 52,728

Endowment Assurances with and without profits 203,143

The notation of the British Offices' Experience is adhered to even to the symbolic representation of the various descriptions of table, the letter J being substituted for O. Thus the  $J^{(M)}$  Table is the select experience deduced from the data relating to male lives. One new symbol  $B_{[x]+t}$  was used, however, to indicate a function designated "Brought down," which replaced the use of the function "net movement" in the construction of the British Offices' Experience. The nature of this function will be seen from the equations

$$B_{[x]+t} = E_{[x]+t-1} - [W_{[x]+t-1}^{(b)} + T_{[x]+t-1}^{(b)} + \epsilon_{[x]+t-1} + \theta_{[x]+t-1}]$$

and  $E_{[x]+t} = B_{[x]+t} - [W_{[x]+t}^{(a)} + T_{[x]+t}^{(a)}].$

Only two points here call for comment. The first is that the symbol  $\epsilon_{[x]+t}$  represents the existing at the end of the  $(t+1)$ th policy year, whereas in the British Offices' Experience it indicates the cases existing upon the  $t$ th policy anniversary; and the second is that in the absence of a column in the working sheets in which to record the total decrements, no very special advantage appears to accrue from the method of tabulation and the use of the new "Brought down" function, except that all the values involved in a single operation are on the same line.

Twelve tables in all were constructed for males, females, and males and females combined, on the basis of select, full aggregate, five years truncated aggregate, and ten years truncated aggregate data respectively.

Of these twelve tables the  $J^{(M)}$ ,  $J^M$  and  $J^{M(5)}$  are the most important, and they were the only ones that were graduated.

The following is a summary of the data of the tables in question.

*Summary of Data.*      *Japanese Three Offices Experience (1910).*

Table	$\sigma$	W	T	$\epsilon$	$\Theta$	E	Average Duration
$J^{(M)}$	350,302	157,511	5,393	166,117	21,281	1,631,459	4.66
$J^M$	336,073	152,075	5,079	158,784	20,135	1,532,110	4.69
$J^{M(5)}$	116,059	20,178	1,619	84,517	9,745	548,794	9.85

The aggregate and truncated tables ( $J^M$  and  $J^{M(5)}$ ) were graduated by means of Mr. J. Spencer's 21 term summation formula, supplemented at the early ages by a graphic process and at the older ages

by a Makeham curve, for which it was found that  $\log_{10}c = .038$  gave the best results.

In the graduation of the  $J^{(M)}$  Select Table, the extremely simple expedient was adopted of using the 5 year truncated table as the ultimate table. The  $J^{(M(5))}$  graduated rates of mortality were then multiplied into the exposed to risk for years of duration 0, 1, 2, 3, and 4, and it was found that the ratio of the total actual deaths to the total expected deaths in successive years of duration were .62; .87; .95; .97; and 1.00 respectively. These ratios were used to modify the rates for the first four years after entry.

Thus

$$q_{[x]} = .62 q_x^{(5)}$$

$$q_{[x]+1} = .87 q_{x+1}^{(5)}$$

$$q_{[x]+2} = .95 q_{x+2}^{(5)}$$

$$q_{[x]+3} = .97 q_{x+3}^{(5)}$$

and

$$q_{[x]+t} = Q_{x+t}^{(5)}$$

(when  $t = 4$  or more).

It necessarily follows from the way in which the ratios were obtained that the total expected and actual deaths in each year of duration are in substantial agreement. The deviations in each year of duration are also fairly evenly spread as positive and negative quantities, but an extended comparison over the first four years shows that the rate of mortality is too heavily weighted at the early entry ages 20–34, whilst for the group of entry ages 35–39 the expected deaths (1194.5) are in defect of the actual (1319) by no less than 124.5.

While, therefore the method was extremely simple to apply and doubtless gave sufficiently good results for practical use, it cannot be said to have satisfactorily met all the requirements of a good graduation.

Probably the greatest interest of the volumes to British Actuaries will be found in the comparison of the mortality rates with those corresponding thereto in the British Offices' Experience.

To facilitate reference the following comparisons are given between the graduated rates.

#### *Comparison of Rates of Mortality.*

$x$	$J^M$ $q_x$	$O^M$ $q_x$	$J^{(M(5))}$ $q_x$	$O^{M(5)}$ $q_x$	$J^{(M)}$ $q_{[x]}$	$O^{(M)}$ $q_{[x]}$	$J^{(M)}$ $q_{[x-4]+4}$	$O^{(M)}$ $q_{[x-10]+10}$
20	.00890	.00404	.01199	.00652	.00743	.00261	.01199	.00664
30	.00718	.00569	.00820	.00747	.00508	.00312	.00820	.00757
40	.00935	.00915	.00992	.00978	.00615	.00438	.00992	.00986
50	.02028	.01504	.02171	.01545	.01346	.00746	.02171	.01546
60	.04373	.02887	.04490	.02921	.02784	.01499	.04490	.02907
70	.08283	.06207	.08283	.06219	...	...	.08283	.06169
80	.19044	.13844	.19044	.13850	...	...	.19044	.13720

It will at once be noticed that the rates of mortality are much heavier than those of the British standard, except at age 40. It will also be seen that  $q_{30}$  is less than  $q_{29}$ .

In point of fact the  $q_x$  curve attains a secondary maximum at age 21 both in the full aggregate and 5 year truncated aggregate tables, the succeeding minima being located at ages 29 and 33 respectively.

In the  $J^{M(5)}$  Table the value of  $q_x$  from ages 34 to 39 inclusive are slightly below the corresponding rates by the  $O^{M(5)}$  Table, but this is the only section of the tables that at all closely approaches the British Standard.

The cumulative effect of the mortality is shown in the following comparisons of the curtate expectations of life.

### *Comparison of Curtate Expectations of Life.*

$x$	$J^M$ $e_x$	$O^M$ $e_x$	$J^{M(5)}$ $e_x$	$O^{M(5)}$ $e_x$	$J^{(M)}$ $e_{[x]}$	$O^{(M)}$ $e_{[x]}$
20	38·97	43·18	37·66	41·89	37·92	42·39
30	31·91	35·07	31·38	34·56	31·53	35·07
40	24·12	27·36	23·73	27·17	23·88	27·74
50	16·91	20·11	16·69	20·02	16·91	20·70
60	10·96	13·57	10·93	13·54	11·23	14·38
70	6·42	8·21	6·42	8·21	...	...
80	3·23	4·34	3·23	4·34	...	...

The monetary functions are tabulated at  $3\frac{1}{2}$ , 4,  $4\frac{1}{2}$ , and 5 per-cent interest on the basis of each of the graduated tables, and beside the usual commutation functions and their logarithms the values of  $a_x$ ,  $A_x$ ,  $P_x$ ,  $\bar{P}_x$ ,  $a_{xu}$ ,  $A_{xu}$  and  $P_{xu}$  are given.

In order to test the effect of the adoption of the new tables by the Japanese Offices the following comparisons with the Seventeen Offices Experience are given on the basis of interest at  $3\frac{1}{2}$  per-cent.

$x$	$J^M$ $P_x$	17 Offices $P_x$	Difference
20	.01460	.01379	.00081
25	.01636	.01563	.00073
30	.01910	.01796	.00114
35	.02299	.02094	.00205
40	.02834	.02485	.00351
50	.04461	.03703	.00758
60	.07273	.05888	.01385

This comparison shows that the Seventeen Offices' Table does not satisfactorily represent the mortality of Japanese assured lives, and the differences in net premiums seem to indicate that office premiums will have to be substantially modified at the older entry ages if the premiums at present in use have been based on the Seventeen Offices' Table.

The practical adoption of the Tables as a valuation standard would seem to involve the setting up of very considerably enhanced reserves, as indicated below. This increase of reserves combined with the reduction in premium loadings will probably make it difficult for the new tables to be adopted at once for valuation purposes.

$x$	$n$	100 $_n V_x(3\frac{1}{2} \text{ per-cent})$		$\frac{n V_x^{JM}}{n V_x^{17}} \text{ Offices}$
		JM	17 Offices	
20	5	3.51	3.72	.94
	10	8.50	8.05	1.05
	20	22.10	18.82	1.17
30	5	6.85	5.44	1.26
	10	14.86	11.71	1.27
	20	32.52	26.92	1.21
40	10	20.74	17.22	1.20
	20	41.66	36.73	1.13

C. W. K.

*The Proceedings of the Royal Society of Medicine—November 1911—  
Presidential address to the Epidemiological Section on the influence  
of Migration upon the Phthisis Death-rate.*

DR. W. H. HAMER in his recent Presidential Address to the Epidemiological Section of the Royal Society of Medicine suggests that after making allowance for altered nomenclature, better diagnostic methods, greater facilities for obtaining medical assistance in out of the way districts and the application of bacteriology to medicine, there still remains a sense of some influence operating on Phthisis Death-rates which is as yet undetected. This influence is he suggests "migration." The evidence he brings forward is based on the early work of Sir George Buchanan upon Phthisis in Kent, Surrey and Sussex, and on the general census figures of the United Kingdom with especial reference to Ireland. Dr. Hamer points out that in Ireland the Phthisis Death-rate has not declined as it has done in England and Wales and Scotland, and he explains this partly by "the fact that the deaths for the earlier decades . . . are underestimated" and partly by migration. He thinks that the latter cause altered the age constitution of the community, lowered the disease resisting power of the residual population owing

to the emigration of the healthier part of the community, and increased the death rate by the addition of a certain number of deaths among returned emigrants.

The statistical evidence is admittedly incomplete and unsatisfactory in many respects; so much so that it leaves us unconvinced, but Dr. Hamer's Address is very suggestive and should be read by those members of the Institute interested in such statistical problems.

W. P. E.

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*Some aspects of the National Insurance Act, 1911 [Part 1.—National Health Insurance], by REGINALD C. SIMMONDS, F.I.A., of the Alliance Assurance Company, Limited.*

[Read before the Institute, 27 January 1913.]

### INTRODUCTION.

THE following paper is intended to deal in an indicative rather than an exhaustive manner with certain aspects of the Health Insurance Legislation which has come into force recently in this country, and to draw attention also to the main points of similarity and of distinction which a review of the more important analogous foreign schemes brings to light. Needless to say, there is no attempt to fan the dying embers of politico-journalistic controversy, and wherever an opinion is expressed it is given with such reasons as appear to the author to support it.

The subject is so wide and so complex that it is imperative that somewhat strict lines of enquiry be laid down, and therefore it is thought that the following brief Syllabus will be deemed to be a necessary preliminary to the maintenance of that sharply-defined outline which is essential to a clear discussion of the points at issue. Accordingly the paper is divided into five main Sections, as follows :

- Syllabus.**
- (a) The Actuarial Basis.
  - (b) The practical working considered more particularly from an actuarial point of view.

- (c) The analogous schemes, either operative or projected, in foreign countries.
- (d) Amendments, complete or partial.
- (e) Social effects.

(a) THE ACTUARIAL BASIS.

In taking up this first section of the Syllabus it is desirable to follow in due order two main lines of enquiry. Firstly, the general outline of the scheme has to be considered, and secondly, the fundamental assumptions made in connection therewith claim mention.

Outline of the Scheme. As is well-known the National Health Insurance Scheme is designed to provide (usually by compulsion, but in some cases voluntarily) protection against sickness and certain other contingencies for a very large proportion of the industrial population of the United Kingdom, and so it embraces the five main benefits now known popularly as Sickness Benefit, Disablement Benefit, Medical Benefit, Sanatorium Benefit, and Maternity Benefit. These are the "Minimum Benefits" referred to by the Government Actuaries in their report, and it will be convenient just to set them out briefly here :

- (1) Sickness Benefit.—For men, 10s. per week for a period of 26 weeks, beginning from the fourth day of illness. For women, the rate is 7s. 6d. per week.
- (2) Disablement Benefit.—5s. per week for men and women alike, commencing upon the cessation of Sickness Benefit.
- (3) Medical Benefit.—Treatment by Doctor, including provision of medicines and certain surgical appliances.
- (4) Sanatorium Benefit.—Treatment in a Sanatorium or in other ways in case of consumption, and, possibly, of other diseases.
- (5) Maternity Benefit.—A sum of 30s. upon the confinement of the wife of an insured person or of an insured woman herself.

It must be understood, of course, that very many limitations, reservations and exceptions are made in each case, but it is neither necessary, nor, indeed, possible to specify them in this place.

The income of the scheme is raised by contributions from three distinct sources :

- (1) The community at large as represented by the tax-payer.
- (2) The employers of labour.
- (3) The workers affected.

A prominent feature of the scheme is that, with certain comparatively unimportant exceptions, all persons entering into insurance at the inception of the undertaking are allowed the benefit of a level rate of premium. In consequence a very large aggregate of initial reserve values has been set up which will be liquidated by the application during a term of about  $18\frac{1}{4}$  years of a portion of the contributions to be set aside for the purpose.

There are two main groups of insured persons, namely, the members of Approved Societies (corresponding largely to the Friendly Societies) and the Deposit Contributors who obtain their "insurance" through the Post Office.

Such in the very briefest, barest outline is the basis of the scheme, but brief and bare though the outline be it will be enough for present purposes, since it is only necessary at this stage to remember the plan of the Act sufficiently well to render the discussion of the fundamental actuarial assumptions intelligible. The multifarious points of interest which a detailed consideration presents will be reviewed in the next section—in the meantime the second line of enquiry under the present heading has to be taken up.

For information as to the assumptions made and the bases adopted in the fundamental calculations, it is necessary to have recourse to the several published reports of the Government Actuaries. As will be seen from the remarks which follow, there is revealed in the dispassionate statements of the Actuaries a condition of statistical unpreparedness in this country so serious as to make the very basic calculations of a great national scheme in many respects matters of conjecture instead of, as they should be, close approximations to the truth.

Its scope. The first estimate required was the number of contributors likely to come within the ambit of the scheme. In order to supply this information, the Actuaries were obliged to refer to the Census Returns of 1901, and to base their estimates upon the occupational statistics given therein. It was found that the age-groupings of the figures for

the respective countries included in the United Kingdom were not uniform in certain particulars, and therefore interpolations were needed to bring the results into line.

As regards the occupations all that could be attempted was to adopt broad assumptions as to the callings which would probably be embraced within the scheme—some vocations which seemed likely to furnish relatively few contributors were omitted, thus providing a certain counterbalancing influence to the inclusion of all the persons returned in the occupations actually selected. Thus an estimate was obtained of the probable total number of contributors in the various age groups.

These totals had to be divided into compulsory and voluntary contributors, and each of these into members of Approved Societies and Deposit Contributors.

In order to effect the former separation it was assumed that persons returned as working on their own account represented as well as possible the gross number of voluntary members.

In view of the option in this class to join or not at will, there was the further question of fixing the percentage of persons likely in fact to participate in the scheme, and this problem was complicated by the provision of a flat rate of contribution chargeable before age 45 (which feature of itself would probably afford a strong inducement to persons under that age) and of an increasing scale of premiums afterwards. In the circumstances it was thought sufficient to assume that the proportion of entrants in the voluntary section would be 35 per-cent of the possible number at the young ages, that it would rise to 60 per-cent at age 45, then fall abruptly to 30 per-cent, and decrease gradually thereafter to 15 per-cent.

The estimated number of deposit contributors was found by a somewhat similar process. The proportion of the gross totals in the age-groups was taken as 5 per-cent at age 16, and rose gradually to 15 per-cent at the superior limit.

These preliminary points being settled it was still necessary to bring the figures of 1901 for the various age groups down to 1912.

This was accomplished by making in the first instance broad assumptions as to the persistence during the decennium 1901–1911 of the rates of increase which obtained between 1891 and 1901. The method would be accurate enough if the rates of mortality and migration prevailing in the former term remained the same in the latter, and in the absence of any reliable in-

formation to the contrary, the rates of migration were treated as satisfying this condition. The death-returns of recent years, however, indicated a decided change in a downward direction in the rates of mortality, and therefore the ratios of increase already referred to were modified to allow for the decline. Thus the figures for each group were obtained as at 31 March 1911, and a further correction gave the results as at the date of commencement of the scheme.

Upon consideration of the nature of the data upon which these figures are based, and the large assumptions necessarily made on account of their incompleteness and obsolescence it must be admitted that the case for a thorough re-organization of the methods of compiling vital statistics in this country is enormously reinforced. Under present conditions it seems that it will be necessary, if a large scheme is to be launched, either to wait until the end of the current censal period or else to make large demands upon the assumed experience since the last investigation. Neither of these alternatives, it is suggested, is particularly creditable to a country of the resources which the United Kingdom possesses. Of course, as the Actuaries are careful to point out, the accuracy of the individual contributions is not necessarily affected by these statistical shortcomings, but the estimates of the magnitude of the scheme, and in particular of the periodical demands likely to be made upon the people of this country, whether as employers, workers or taxpayers, cannot be made without great difficulty.

*Bases of the calculations.*

Having thus noticed briefly the methods used in ascertaining the number and the age-distribution of the various classes of contributor, it is necessary to pass in review the bases upon which the mortality, sickness and other rates were fixed. While it is true that no benefits depending upon

*Mortality rates.*

life alone are included in the scheme, nevertheless the mortality basis is of great importance since the value of the various sickness and other allowances obviously depends upon the respective probabilities of survivance and of death. The published Experiences of the great Friendly Societies could not be expected to afford a suitable basis in this connection owing to the particularly select nature of the lives included therein as compared with those likely to be brought under the operation of a national scheme, and therefore a general population table was adopted. The particular experience selected, viz.: the English Life No. 6, is the latest

complete record, generally available, of the mortality prevailing among the people of this country, and probably no better basis could have been chosen, but at the same time it is important to notice that the progressive decline in the death-rates which the latest returns of the Registrar-General certainly indicate must have an effect in increasing the weight of the benefits. Especially is this so in the case of the sickness and disablement allowances which are rendered more onerous by the light death-rates, particularly as there is no set-off by reason of lessened funeral benefit such as is often found in an ordinary Friendly Society. Reference is made to this point by the Actuaries who state that some compensation may be found owing to the neglect of any profit from secessions, but, in view of the provisions of Section 10, it is rather doubtful whether the individual societies will derive any relief.

An interesting point regarding the relative mortality of married men and bachelors is that little difference occurs here at ages over 35, although in New Zealand marked divergencies were observed.

**Marriage rates.** The rates of marriage were deduced from the Returns of the Registrar-General for the ten years from 31 March 1896 to 31 March 1906. They were required in the case of females owing to the provision for suspension of ordinary benefits during coverture unless employment be continued. As a corollary it was necessary to determine rates of widowhood in order to allow for the contingency of subsequent re-entry into insurance.

**Issue rates.** The probabilities of issue which were needed in connection with the calculations for maternity benefit could not be obtained from British sources, and it was found necessary to go to the New Zealand figures for the requisite information. In passing, mention may be made of the fact that, in December 1892, Mr. George King, in his Paper entitled "On Family Annuities", made the following remark: "The absence of this information is a bar to the investigation of many problems important from both the social and the economic standpoints." That was more than twenty years ago—yet even now the defect remains unremedied!

**Sickness rates.** But important as the probabilities of death, of marriage and of issue are in connection with the National Insurance Scheme, it is unquestionable that the main feature of interest in these, the fundamental estimates, is that

of the rates of sickness, for, after all, the sickness and disablement benefits are the backbone of the scheme, as it were, and so are especially noteworthy.

The Actuaries adopted the Manchester Unity (1893–1897) Experience as the starting-point of their calculations. This very well-known series of tables allows for the effect of occupation upon the liability to sickness, and thus it was possible to adjust the tabular rates in order to obtain average results—as far as might be appropriate to what may be termed the vocational-distribution of the population included in the scheme. The contributors were divided into four classes, as follows :

Class.	Containing Occupations assumed to agree with—				
	1	..	..	M.U.	A.H.J. Risks.
2	..	..	..	„	B.C.D. „
3	..	..	..	„	E.F. „
4	..	..	..	„	G. „

Then the tabular values being weighted in each class according to the numbers at each age-group included, average rates of sickness were obtained which exceeded the Manchester Unity Whole Society Scale by about 10 per-cent.

From these figures a deduction had to be made in respect of sickness due to industrial accidents (included in the Manchester Unity results, but excluded by the terms of the National Insurance Scheme), and here the formidable barrier of inadequate statistics reared itself once more. The problem was solved by assuming the relief to be at least 10 per-cent of the net Manchester Unity rates.\*

Then further it must be remembered that the first three days of sickness are not paid for under the Act, and theoretically at any rate, some relief may still be expected on this account despite the modifications made during the passage of the Bill through Parliament.

Taking all these counteracting elements into consideration the Actuaries fell back upon the normal Manchester Unity Whole Society Rates as the final basis of computation. In connection with this, one or two points of interest arise. In the first place there is the question of a possible progressive rise in sickness rates. This feature has obtained in the past and may conceivably

\* In the original Paper this was stated to be the "weighted" rates—it has been amended subsequently to the Discussion, *q.v.*

persist in the future. Against this view there are, however, two weighty objections (*a*) that the Actuaries of the Manchester Unity Society stated quite recently that no material change in the experienced sickness rates had been observed since the compilation of the 1893–1897 tables, and (*b*) that the whole hope and social intention of the National Insurance Act are that it may “initiate a contrary tendency in sickness rates” (to adopt the words of the Government Actuaries).

Mention must be made in the second place of the assumption that female members will experience rates of sickness similar to those of male contributors. It is not by any means certain that this is the case, and in fact the German and the Austrian results suggest that upon the whole women do not show such high rates, although it is not altogether safe to dogmatize upon the point.

Secessions have been ignored entirely (except, of course, in the case of the marriage of female contributors), and in any event their effect, even if it were susceptible of accurate measurement would probably be relatively insignificant since at young ages withdrawal produces no material disturbance, and at higher ages comparatively few members will be likely to fall out of insurance. Nevertheless, some profit at least will arise here, and it will be remembered that the Actuaries bore this in mind when considering the probable future course of mortality rates.

**Unemployment rates.** The last basic element to be reviewed is the rate of unemployment. In this case the utter lack of suitable statistics prevented close calculation and eventually it was assumed that an all-round rate of unemployment of 5 percent per annum would meet the facts. This question only touches the Health Insurance Section of the Act because of the provision that contributions are not to be payable during a certain period of unemployment.

**The Contributions.** Gathering together the results of all these assumptions and presenting them in concrete form, the Actuaries reported that the total weekly contributions in pence required at age 16 to provide Minimum Benefits should be as follows :

Benefit	CONTRIBUTIONS (PENCE PER WEEK)		
	Men	Women	
Medical ...	1·51		1·51
Sanatorium ...	.32		.32
Sickness ...	2·39		1·74
Disablement ...	.78		.81
Maternity ...	.66		.17
Total Benefits ...	5·66		4·55
Cost of Administration ...	.92		.92
 Total	6·58		5·47

These contributions were computed upon the assumptions—

- (a) That they should cease at age 70.
- (b) That they should not be payable during sickness or unemployment.

The following points may be noted in connection with these figures—

- (1) The rate of interest assumed is 3 per-cent per annum.
- (2) The medical and sanatorium benefits (both of which extend throughout life) are treated upon the basis of a level cost of 6s. and 1s. 3d. per annum respectively.
- (3) The maternity benefit presents the reverse feature. Here the claims are all concentrated at the younger ages, but the premium payments extend up to age 70. This involves negative reserves at the higher ages for this particular benefit, but since all the allowances and risks stand together no danger is caused.
- (4) The cost of administration has been taken as 4s. per member per annum. It appears doubtful whether this sum will be adequate in the smaller Societies—in the larger organizations it will probably be ample.

**Basic Finance.** These considerations lead to the vitally-important question of the basic finance of the scheme. The normal rates charged are 7d. per week for men and 6d. for women, and in addition there is the Government subsidy which amounts

to  $\frac{2}{9}$  of the total cost of benefit and administration in the case of males, and to  $\frac{1}{4}$  in that of females.

In terms of Section 55 of the Act the Insurance Commissioners are empowered to retain out of each weekly contribution made the sum of  $1\frac{5}{9}d.$  in the case of men, and  $1\frac{1}{2}d.$  in that of women. Such deductions are to be used directly in the extinction of the initial reserves already very briefly referred to. It is apparent at once that  $1\frac{5}{9}d.$  is  $\frac{2}{9}$  of  $7d.$  and that  $1\frac{1}{2}d.$  is  $\frac{1}{4}$  of  $6d.$  Therefore, after these deductions are made there remain only  $\frac{7}{9}$  and  $\frac{3}{4}$  respectively of the original contributions to be passed on to the Societies, and since (ignoring margins) the full premiums are required to meet the minimum benefits and the expenses it is clear that there must be Government assistance to the extent of  $\frac{2}{9}$  and  $\frac{1}{4}$  respectively of the total disbursements if all charges are to be covered. As has been stated already, such subventions are actually provided, and in effect, therefore, the position is this : The Government becomes responsible upon an assessment basis for  $\frac{2}{9}$  or  $\frac{1}{4}$  of the whole cost throughout the whole term of insurance, and during the first  $18\frac{1}{4}$  years, or such other period as experience may show to be necessary, it retains a like proportion of the contributions in order to liquidate the initial liabilities incurred through treating the great majority of the contributors as if they entered into membership at age 16. After the reserves so created have been dealt with, there will be available the hitherto-deducted portions of the weekly contributions. These will then be applied in providing certain supplementary benefits as is explained at a later stage. Towards such additional allowances the Government will contribute its due quota.

(b) THE PRACTICAL WORKING CONSIDERED MORE PARTICULARLY  
FROM AN ACTUARIAL POINT OF VIEW.

In this section of the enquiry it will be more necessary than elsewhere to preserve sharpness of outline in view of the almost illimitable vista of discursive opportunity which is opened out.

Attention will be confined as strictly as possible to the following matters, and in cases where no especially interesting point occurs nought save a brief statement of fact will be given.

- (1) The Insured Persons.
- (2) The Contributions.

- (3) The Benefits.
- (4) Administration.
- (5) Approved Societies.
- (6) Deposit Insurance.
- (7) Valuations.
- (8) Special Classes of Persons.

These sub-headings are taken in order :

The insured persons. **(1) The Insured Persons** embrace practically all the industrial population between the ages of 16 and 70, British, Irish, and Alien alike, whose rate of remuneration does not exceed £160 per annum with the exception in the last-mentioned respect of manual workers, for whom there is no such restriction at all. Since most of the latter come within the Income-Tax limit it would not appear to be particularly necessary to extend its bounds in their case !

Where a worker is in receipt of a secured income (arising apart from personal exertions) of at least £26 per annum, or where there is dependence upon another for livelihood, exemption can be claimed, but the contribution of the employer is still payable. This provision is necessary in order to prevent persons already more favourably situated than the majority of their fellows from deriving a still further advantage in the daily conflict for that work which is essential to existence, and yet not available for all under the present system of social arrangements.

Very considerable controversy and difficulty have been caused by the question of what the term "rate of remuneration" really means, and such discussion would not appear to have been wholly set at rest by the plenary powers conferred upon the Insurance Commissioners under Sections 66 and 78 of the Act (entitled respectively "Determination of Questions" and "Power to remove Difficulties"). Fortunately, however, the point is not of special interest in this present enquiry.

The Insured Persons may be either employed contributors who must be insured or voluntary contributors who may be insured. The distinction between the two groups is important. The former as the name suggests are such as work under a master, while the latter are those who are engaged in occupations upon their own account.

The division is thus, broadly, between wage-earners and small employers such as blacksmiths, carpenters, &c. The employed contributors generally pay a fixed flat rate of premium,

but the others are denied this benefit except in a limited form, which it may be convenient to set out here—

- (a) For voluntary contributors entering the scheme within six months of its commencement and below age 45, the flat rate is charged, but those above 45 must pay the premiums necessary to provide at their respective ages,  $\frac{7}{9}$  for a man [or  $\frac{3}{4}$  for a woman] of the benefits.
- (b) For those who do not so enter, the contribution will be in accordance with the table prepared by the Commissioners under Section 5 (1) of the Act.

The two main classes of Insured Persons are further divided into two sections.

- (a) Members of Approved Societies.
- (b) Deposit Contributors.

These are dealt with in their due order.

**Contributions.** (2) **The Contributions** are collected by means of distinctive stamps purchased at the Post Office and affixed to special cards. For Employed Contributors they are normally 7d. per week for men, and 6d. for women, payable as follows—

By the Employer 3d. in each case.

By a Male Worker 4d.

By a Female Worker 3d.

Two main sources of modification may be mentioned here—

- (a) In Ireland there is no medical benefit, and accordingly the contribution is reduced by 1½d. per week.
- (b) Where very low wages are paid, the incidence of the contributions is varied in accordance with the first Part of Schedule 2 of the Act. In extreme cases of this kind Parliament bears part of the cost as a special matter.

**Benefits.** (3) **The Benefits** intended to be given at first have been mentioned already. It is necessary now that rather fuller consideration should be accorded to them.

*Medical Benefit* includes treatment and attendance as well as drugs and such surgical appliances as may be prescribed. This Benefit does not present any feature of immediate interest except, possibly, that in certain circumstances the Local

Authorities, &c., may become liable to assist in meeting the cost. This, however, does not affect the definite sums to be allotted out of the contributions of the insured persons.

Medical Benefit did not commence until January 1913.

*Sanatorium Benefit* is very similar to Medical Benefit from an actuarial standpoint. The fixed contribution included in the rates, supplemented by certain extraneous allowances, is available for the use of the Insurance Committees, and the rest is simply a matter of erecting Sanatoria, arranging for reception of approved cases and providing treatment. It must be noted that—

- (a) There is no guarantee that any particular case will be accepted for treatment.
- (b) The Benefit extends throughout life and may be granted to dependants of members.
- (c) Diseases other than consumption may be included.
- (d) Local Authorities, &c., may contribute towards the cost.
- and (e) This is the only Benefit which came into force immediately the Act was brought into operation.

*Sickness Benefit* naturally offers some very interesting points. Certain of these have been anticipated in an earlier part of this paper. The Benefit commences upon the expiration of the first three days of sickness, so that a fair amount of relief is obtained not merely from the intrinsic value of the numerous short illnesses which last for less than four days, but also, probably, because the exclusion of even three days is some deterrent to a malingerer, who is not able so easily to simulate a genuine illness of moderate or great duration as he is to pretend to those trivial little troubles which involve only a short absence from work and can be imitated fairly safely. The benefit is payable for twenty-six weeks only, after which it ceases absolutely as such—any further allowance coming in under the title of Disablement Benefit.

The other points to be noted regarding Sickness Benefit are as follows—

- (a) A waiting period of 26 weeks\* is imposed, and at least 26 contributions must have been paid.
- (b) After 26 weeks of benefit have been paid, the member must be off the funds for one year before he can become entitled to claim this particular allowance again.

\* Originally stated as "six months."—Altered after the Discussion, q.v.

(c) Sickness arising from an Industrial Accident or Disease which is or might be compensated is not allowed to count for benefit except as stated below. In such cases the member must claim compensation from his employer, and, if he do not proceed his Society may take the matter up and press it even to the length of litigation. Of course, if the weekly allowance under the Workmen's Compensation Act is not equal to the Sickness Benefit which would have been received otherwise, it is supplemented.

A point of interest arises here—Since certain maladies, such as anthrax and arsenical poisoning, &c., are already scheduled as Industrial Diseases, and other affections are liable to be so scheduled in future, it is possible that a progressively increasing relief from Sick Benefit might be given to the Societies at the cost of the employers (which means, of course, in most cases, at the expense of the Insurance Companies) until, in the limit, when every disease had been so treated, practically all the claims would be made to arise under the Compensation Acts. This last supposition is not at all likely to become actualized, but it shows the effect of enlarging the Industrial Disease List.

A further point of considerable importance is the effect of the provisions contained in Section 11, Subsection (c) of the Act. It is there laid down that notice of an agreement which provides for the payment of less than 10s. per week or of a lump-sum settlement of a Workmen's Compensation Claim shall be sent to the Insurance Commissioners, or to the Society or Committee concerned. There is no indication that these authorities have (or will claim to have) the right of preventing such of the settlements as appear likely to react unfavourably upon the Sickness Insurance Funds, but it is quite possible that, in time, such a problem will arise. If it do so arise, the companies which grant Employers' Liability Policies may be threatened by interference with the prompt settlement of claims and much trouble and expense may be caused.

- (d) Sickness benefit is not payable in connection with the confinement of a woman unless she be an employed contributor and married, or unless an illness not connected with the lying-in should occur.
- (e) In order to prevent malingering, it is provided that unmarried minors shall receive reduced benefits unless they have persons dependent upon them, and

for the same reason it is laid down that in the case of poorly-paid labour the sickness allowance itself may be restricted to  $\frac{2}{3}$  of the usual wages. The portion withheld from these last must be applied in other ways for their benefit.

- (f) Persons over 50 at entry receive reduced benefits as a rule. The reason is that the premium-curve becomes relatively so steep at these higher ages that the reserves required if full benefits were given would be prohibitive.

*Disablement Benefit* corresponds, of course, to the reduced pay for prolonged sickness which is given by the majority of Friendly Societies.

The main points to be noted are as follows—

- (a) A substantial waiting period is imposed before eligibility is secured—104 contributions must have been paid, and 104 weeks since entry must have elapsed. This is quite reasonable in view of the heavy liability involved, inasmuch as the benefit may be equivalent to the grant of an annuity continuing until age 70. The experience of the Friendly Societies in this direction has evidently been considered carefully, for it is safe to say that very much of the financial trouble which overtakes so many of these institutions is due to the excessive drain of the permanent sick pay, especially when this is aggravated by light mortality.
- (b) The benefit is defined as being an allowance while “incapable of work.” Upon a strict interpretation of this wording it would probably be found that many of the claims for the analogous benefit now presented to Friendly Societies would be refused. If the line be drawn strictly there will be much disappointment as well as much hardship in genuine cases. On the other hand any laxity will react most unfavourably upon the position of the Approved Societies whose officials will find in this case at any rate great difficulty in duly tempering justice with mercy. It is appropriate to mention at this point that the British Invalidity Benefit is very much restricted in comparison with the analogous allowances

given by the various foreign schemes. In these cases a limited earning capacity does not debar the member from assistance, whereas in this country disability must be complete. Reference must be made to the alternative scheme of National Insurance which was propounded by a body called the "National Insurance Inquiry Association." The proposals were published in *The Times* of 4 April last, and they are discussed in detail in the section relating to amendments. The reason for mentioning the matter at this point is that one of the chief criticisms which the Association made of the Government Scheme was the inadequacy of the Disablement Benefit.

*Maternity Benefit* provides a sum of 30s. payable either in cash or in kind upon the birth of a child to the wife of an insured person, to his widow within due time after his death, or to any woman who is herself an insured person. Only three points claim notice—

- (a) Where the mother is an employed contributor and is married she receives Sickness Allowance as well as Maternity Benefit.
- (b) The Maternity Allowance is paid from the husband's society except where the mother is herself insured and is not the wife of an insured person. In this last-mentioned case it is treated as a benefit on the mother's account.
- (c) By Section 20 the Insurance Commissioners may provide for the reinsurance with themselves of the benefit in respect of all Approved Societies. This would appear to be a very desirable feature since the application of a uniform contribution over the whole country for maternity allowances would be a source of great loss in districts with a high birth-rate and a source of profit in others differently situated. Obviously the Societies should not be called upon to run a risk which they have not the slightest means of controlling. Up to the present time the point has not been decided.

\* This wording has been modified from that in the original Paper, in accordance with the suggestion of Mr. Burn (*see Discussion*).

Medical and Sanatorium Benefits extend throughout life, but Sickness and Disablement Benefits cease at age 70, after which the separate provision of the Old Age Pensions Act becomes available in the majority of cases.

It is interesting to notice also that a large measure of discretionary power is left to the bodies charged with administration of benefits as to the precise form which the assistance should take. The question cannot be considered in full detail, but, for example, mention may be made of Section 12, which gives various options where an insured person is an inmate of a hospital, &c. This power is still further extended by Section 13, which allows Approved Societies to meet special circumstances where ordinary benefits are not particularly suitable, by submitting a scheme providing for alternative allowances. In the case of insurance clerks, for instance, owing to the liberal rules of the Companies the first six months (say) of illness can be treated as carrying full salary. Accordingly the value of the Sickness Benefit could be applied in some alternative manner which would take, in all probability, one of the following main forms—

- (a) A pension benefit.
- (b) An increased long-sickness or disablement allowance.

As regards (a) there is a comprehensive official report, issued by the Insurance Commissioners, which embodies the conclusions of a special Sub-Committee appointed to consider the matter and contains also many numerical results, including the actual life and other statistical tables upon the basis of which the calculations were made. The special provisions of the National Insurance Act make it necessary to lay down elaborate rules for the granting of alternative pensions, and a brief consideration of the main scheme recommended is of interest.

The report shows the yearly amounts of pension, commencing usually at age 65, and either continuing in full, or being reduced by 5s. per week after age 70 or then ceasing altogether, which can be granted in exchange for the relinquishment of either the whole of the sickness and disablement allowances or of certain portions thereof. It is proposed that a central Super-annuation Fund should be formed with separate sections for males and females, and that the actuarial value of the sickness and disablement benefits which would have been paid if the pension had not been substituted should be transferred periodically from the Societies to the Fund. This course is simple and also enables arrears to be allowed for automatically. In the case of women

provision is made for allowing a portion of the contributions to be returned with interest upon marriage. While, within limits, membership of the Superannuation Fund could be allowed at any age, obviously it would not be safe to permit members to elect to resume sick benefit at any time inasmuch as a serious option would be involved. A further question is that of the future course of the rate of mortality, any progressive decrease in which would cause a serious burden. The Committee recognized this, and proposed that certain accruing elements of profit, such as the gain from surrenders, surplus interest over the rate assumed (3 per-cent), &c., should be treated as an offset. It would appear that the Life Table published as an Appendix to the Report is based upon new data, the nature of which is not indicated, but the mortality, while much lighter than that shown by the English Life Table No. 6, approximates fairly well to the corresponding figures of the Manchester Unity (Whole Society) 1893–1897 Experience, as is seen by the following statement :

*Values of  $q_x$ .*

Age $x$	NEW TABLE		E.L. No. 6, J.I.A., Vol. 43		Manchester Unity (W.S.) 1893–1897
	Males	Females	Males	Females	
20	.0035	.0030	.0046	.0041	.0031
30	.0051	.0044	.0068	.0061	.0051
40	.0084	.0069	.0118	.0099	.0084
50	.0154	.0117	.0194	.0150	.0146
60	.0317	.0245	.0358	.0292	.0304
70	.0686	.0572	.0725	.0628	.0690
80	.1480	.1295	.1552	.1393	.1573

Perhaps the opinion may be hazarded that this new table is really an advance copy of the English Life No. 7. If that be so, and if the decline in mortality rates should continue, there is obvious need of caution in calculating pension rates.

The chief objection to the pension alternative is the comparative smallness of the future annuities receivable in return for the long-continued sacrifice of present benefits, and probably, therefore, unless past experience be falsified, there will be little disposition on the part of insured persons to adopt it, particularly in view of the following considerations—

- (1) The possibility of an extension of the present Old Age Pension Scheme which might be supposed (rightly

or wrongly) to render independent provision unnecessary.

- (2) The unfavourable position of (say) a domestic servant who, having chosen a pension in lieu of early sick benefit, subsequently becomes employed by a mistress who has elected to pay reduced contributions in terms of Section 47. In such a case the Approved Society would not be responsible for the sick benefit relinquished, and would receive, of course, no consideration in respect of it. Accordingly the equivalent amount could not be passed over to the Superannuation Fund. Thus, Section 47 might have a nullifying effect upon Section 13, and it is easy to see that the knowledge of this (which the Commissioners carefully point out) will exercise a deterrent effect upon prospective applicants. In this connection it is desirable to mention that Section 13 was not included in the original Bill, and being an addition it suffers somewhat from the usual defects of such excrescences.

The alternative (*b*) is extremely attractive from many points of view. It is possible, of course, to produce almost an infinite variety of schemes since the proportions of benefits, both Sickness and Disablement, to be given up, &c., may change endlessly, although probably there are but few really suitable bases for adoption particularly in view of Section 13 (4) of the Act which precludes disturbance of the reserves. One very important Approved Society has suggested what seems to be a distinctly promising arrangement and, as an illustration of what can be done upon a sound actuarial basis, the main outline may be reproduced. A domestic servant, aged 21, may have her Sickness Benefit for the first 6 weeks reduced from 7s. 6d. per week to 5s. 7d., and in return the allowance for the remaining 20 weeks is increased to 13s. The apparent advantage of this plan from the point of view of a servant is that present benefit is not surrendered for a far-off and problematical allowance, and in the particular case cited there is an adjustment to typical conditions of employment which surely is the criterion of suitability. It is understood, however, that certain difficulties of a legal nature have to be overcome ere the Insurance Commissioners can give their approval to the scheme.

These suggestions represent two of the best alternatives yet put forward—others may be evolved as time goes on, but whatever their nature it is safe to say that the Insured Persons with whom, ultimately, the choice rests will demand schemes which do, in fact, meet the special circumstances of their particular employment and will not be disposed to accept any other suggestions however soundly they may be based. Apart altogether from the question of the demand for the alternative benefits there is good reason for thinking that the supply will be restricted very greatly by the difficulties, both of arrangement and of administration which are bound to occur. It must be remembered that alternatives will become important mainly in what may be termed “class-societies” which, from the very fact of their being organized in the special interest of a particular type of person, will present abnormal experiences as regards rates of claim. Consequently in view of the present lack of data it is not surprising that many societies are either discouraging or at least deferring the subject. It is common knowledge, for example, that the Insurance Officials’ Society has decided to take no steps for at least three years towards providing alternative benefits.

The benefits conferred by the Act are subject to reduction or suspension when the insured person falls into arrear. Contributions are not payable during sickness however long-continued, nor during unemployment which does not amount to 4 weeks a year upon the average since entry into insurance. Further, no arrears are counted in the case of employed contributors during the first year of the operation of the Act. Thus, the rules upon this point are very liberal, and, in cases where the arrears have passed beyond the free limit, sickness benefit is reduced progressively until it reaches 5s. per week. In certain cases it might have fallen below this figure, but this has been prevented by adopting the alternative scheme of extending the waiting-period of three days.

The Extra Benefits specified in Part 2 of Schedule 4 are comprehensive and in many cases socially interesting, but they do not necessitate discussion here.

**Administration.** (4) **Administration.** The Central Authority charged with the control of the scheme as a whole is the Insurance Commission which, although intended originally to be one and indivisible, has now become separated into four distinct bodies known as the Insurance Commissioners for England,

Scotland, Ireland and Wales respectively with a Joint Committee representative of all of them as the supreme head. The local working is carried out by means of Approved Societies, Insurance Committees and the Post Office, and the purpose of this sub-section is to indicate the manner in which the administration is apportioned and to explain the relationships which subsist between the different bodies. The eligible lives will be, almost without exception, members of the Approved Societies (which are discussed in detail in the next sub-section), and in order to preserve as far as may be possible the usual organization of the Friendly Society, it is provided that the cash benefits, *i.e.*, the Maternity, Sickness and Disablement Allowances, shall be controlled by the Society, which will thus have a direct influence upon the claims. The other benefits, namely, Medical and Sanatorium Treatment, are more of the nature of National Health Services than Friendly Society Allowances, and therefore they are administered by the Local Insurance Committees who are able, presumably, to make arrangements in respect of the whole of the areas coming under their control, and who, by reason of their knowledge of peculiar circumstances and their connection with the other public bodies in the same neighbourhood, may be considered to have the best chance of dealing comprehensively with the questions arising. In addition the Insurance Committees are liable to arrange for *all* benefits in the case of the Deposit Contributors. The funds necessary for these purposes are derived partly from the amounts included in the contributions and paid over to the Committees, and partly also, in certain circumstances, from extraneous assistance by the Treasury and the Local Authorities.

Sufficient will have been said upon the subject of Administration if a further statement be added to the effect that the Insurance Commissioners receive from the Post Office the revenue arising from the sale of the special stamps together with the Government subventions, and periodically credit the Approved Societies with the proper share of the income. They also hand over to the Insurance Committees such sums as may be necessary to meet the expenses of conducting the Deposit Branch of the Scheme. It is no part of the present enquiry to deal with the question of book-keeping.

**(5) Approved Societies.** Inasmuch as very large numbers of the persons to be insured were known to belong to Friendly Societies and therefore to be well acquainted

with the principles of collective association for the common good, it was decided that a predominant share of the local work should devolve upon the existing organizations who would thereby be spared as much interference with their own operations as was possible. Yet, since a very considerable part of the total funds required would be derived from sources other than the contributions of members, it was essential that a large measure of control should be exerted from some authoritative centre in order to prevent mismanagement and secure reasonable uniformity. The problem was solved by granting recognition to any Society which satisfied certain conditions laid down in Section 23, and thereafter designating it an Approved Society. Briefly stated the requirements are : (a) that there shall be full self-government, and (b) that the business shall not be carried on for profit. The scope was made wide so that existing Societies with constitutions which did not accord with the conditions laid down, as for example, Industrial Insurance Companies with Shareholders, might be allowed to form a separate Section under the Act and so participate in the administration. At the same time a special provision was passed to enable Societies to overcome any particular legal difficulties which might otherwise have prevented them from sharing in the Scheme. Before approval is granted, the Rules of the Society, or Section, must be passed by the Insurance Commissioners, who insist upon the inclusion of certain very necessary provisions.

The Friendly Societies have reserved hitherto the right of selecting members, and so it was considered advisable to continue the practice in the case of the Approved Societies, but with the distinct rule that a person should not be refused admission merely on account of age. In this connection it may be stated that many Friendly Societies have a fairly low limit as regards age at entry, and since many of the new insured members would exceed such boundary it was desirable to prevent a wholesale refusal of facilities to these persons. Of course, applicants over age 65 and also aliens could be kept out, if desired, by the simple expedient of offering absolutely unattractive benefits.

Approval is only effective so long as the Society observes the provisions of the Act, and complies with the regulations of the Insurance Commissioners. It may be withdrawn if offences occur.

The question of removal from one Society to another involves the actuarial problem of settling the transfer value which should

be handed over by the late Society to the new one. Provision is made for this, but a wise restraint is placed upon frivolous changes by discontented members through the provision that the value is not passed on if the person leave without cause and the Society prove that it did not withhold consent unreasonably. There will be also a large amount of emigration, particularly among the younger members. This feature will be met in time by making reciprocal transfer arrangements between this country and other countries having similar schemes. When this has been done the transfer values of the retiring persons will be passed over to any Societies which they may join in the new lands to which they go, but, before doing so, the Insurance Commissioners will satisfy themselves that the Societies in question make proper provision for those who belong to them.

In certain cases where an emigrant still remains a member of his Society, the net transfer value may be carried from the State Section to the Private Section for disposal as may be arranged.

**(6) Deposit Insurance.** Mention has been made already of the fact that the Approved Societies have the right of selecting members, and therefore, since it was safe to assume that some at least of the applicants would be rejected, provision had to be made for giving an ultimate refuge which should be at hand if all else failed—a sort of Poorhouse and Ever-open Door combined. In addition there might be a number of otherwise eligible persons who, by reason of neglect, incitement to resistance, &c., would fail to enter the Approved Societies within the allotted period. All these persons are classed as Deposit Contributors, and it is necessary that the special features affecting them be reviewed. In the first place, there is no real insurance provision whatever, since the Scheme involves simply the crediting of the aggregate amounts paid by, or in respect of, the contributors to a special Post Office Fund, out of which benefits as required are drawn and expenses taken as is mentioned hereafter. As soon as the sum standing to the credit of a member is exhausted, the allowances cease. Provision is made for charging the individual accounts with the cost of administration, and of medical and sanatorium benefits. Secondly, upon death or permanent removal from the United Kingdom  $\frac{4}{7}$  of the credit balance of a male contributor may be returned—in the case of a woman the proportion is  $\frac{1}{2}$ . These ratios represent, of course, that part of the balance which arises

from the contributions paid in by the member personally. It is to be noted that if the whole of the amount were given there might be a distinct encouragement to specially healthy lives to pass by the Approved Societies in favour of the Deposit Scheme, and thus to secure, for example, a death-benefit very substantially in excess of the contributions actually paid in by them. Or, to take a very likely case, a man might join at a young age and emigrate within a few years. Obviously it would be distinctly to his advantage to prefer the Deposit Insurance if at the time of joining he had the intention of leaving this country after a while.

The only really satisfactory feature of this Deposit Scheme is its transience. The whole question must be reviewed before 31 December 1914, by which date it is possible that accumulating experience and more exact statistics as to the actual magnitude of the class to be dealt with will enable better provision to be made.

Undoubtedly the problem was and is a very difficult one, and yet, somehow, it might have been expected that more adequate arrangements would have been made even at the cost of a little extra outlay.

The transfer of members of Approved Societies to the Deposit Scheme may become necessary, and accordingly it is provided that the net value, if any, held in respect of such persons, shall be credited to the Post Office Fund. On the other hand, if a Deposit Contributor should join an Approved Society the amount standing to his credit in the Post Office Fund is transferred to the Society subject to the provision that no sum greater than the requisite reserve at date of transfer shall be passed over, and that, if there be not enough in his name to meet such reserve he shall be considered to be in arrear to the extent of the deficiency.

*valuations.* (7) **Valuations.** This question is of especial interest in the present circumstances, and certainly calls for brief notice. According to Section 36 of the Act, Valuations are to be made at triennial intervals by duly-appointed Valuers, and upon a prescribed basis. The Insurance Commissioners have power to modify the length of the periods elapsing between successive investigations as may be deemed advisable. Passing mention may be made of the definition of the word "Valuer" which occurs in Section 79. "A person possessing actuarial qualifications as may be approved by the Treasury." It appears, therefore, that there is a distinct opportunity for securing that

only really qualified men should be allowed to perform the duties involved, and doubtless steps will be taken to obtain regulations which will prevent in future the trouble and distress caused in the past by inefficient work in this direction.

If a surplus be found as a result of the investigation it is laid down that such part of it as is certified by the valuer to be disposable may be applied as follows—

- (a) In the case of a Society without branches the available surplus can be used to provide additional benefits according to a scheme to be approved by the Insurance Commissioners.
- (b) In the case of a Society with branches, one-third of the sum in question is to be passed to the central fund, and the remaining two-thirds may be distributed according to an approved scheme. Any surplus transferred to the central fund is to be utilized, firstly in making good deficiencies in any branches (with right of refusal to assist in any particular case where a charge of maladministration against the branch can be sustained before the Insurance Commissioners) and any balance remaining is to be divided amongst all the branches which showed a surplus in proportion to the actual amount thereof.

It is important to notice that no death benefits may be granted, and also that the choice of additional allowances is limited definitely to those specified in the second part of Schedule 4 of the Act.

If the Valuation should disclose a deficiency, remedial measures must be taken at once. This will mean, in the case of very many of the smaller Societies at any rate, that a much-needed check will be imposed upon the dangerous custom of ignoring shortages and letting matters take their course. The procedure to be adopted is as follows—

In the case of a Branch, three-quarters, or in some instances the whole, of the deficit is to be made good, if possible, out of the surplus fund in the hands of the central authority with the qualification (already referred to) that such assistance need not be given if maladministration be proved.

If, after these steps have been taken, the Branch be still in deficiency, or in case of a deficit in an independent Society (which has, of course, no central relief fund upon which to draw),

provision is made for the submission of a scheme calculated to make good the shortage within three years, *i.e.*, before the next valuation date laid down by the Act. The scheme will provide either for a levy or for curtailment, postponement or reduction of the Sick Benefit or for any other approved method.

An interesting feature is that levies are made very effective by the provision that the employer may be called upon to pay them in the first instance with the right, of course, of recovery from the worker affected. The wisdom of this provision may be doubted, for, although it enables the societies to enforce their demands, it does so by means of an entirely inconsequential method, *viz.* : by saddling with the burden a person in no way responsible for the deficiency and leaving him to reimburse himself as best he may. The probable effect of this will be that a very considerable amount of friction will be generated between master and man, and in addition the worker will feel that his employer is aware not only of the name, but also of the state of the Society to which he belongs. Certainly in view of the care taken when issuing the contribution cards to prevent the employer from ascertaining this information it is strange that not only these particulars but other much more vital ones should be thrust under his notice. There would seem to be no good reason whatever for bringing the employer into the matter in question, and, surely if the Societies cannot be trusted to secure from their members loyal adherence to the measures taken for the common good, then very much of the force and usefulness of these organizations vanishes altogether!

It is likely also that, from the workers' point of view, any factor which might lead to discrimination upon the part of employers against members of the Societies in deficiency would be deemed extremely unsatisfactory.

Provision is made for the strict enforcement of remedial measures since the Insurance Commissioners have power to coerce recalcitrant or negligent Societies, and ultimately, if all else fail, themselves to take over the administration for a time. Self-government must be re-established within three years, otherwise arrangements must be made for transferring the members either to another Approved Society or to the Post Office. It is easy to see that in a very bad case with a heavy deficiency and an unmistakeably adverse sickness expectation, the Insurance Commissioners might be unable to arrange with any Approved Society to take over the undesired members, with the result

that these persons would pass inevitably into the Deposit Scheme (assuming such to be still in operation). Protection against secessions of healthy lives from Societies in deficiency is secured by the rule that any reduction in benefit imposed shall follow the member to his new Society, and if no scheme have been completed, such adjustments of the transfer value shall be made as the circumstances demand.

In reviewing generally the provisions relating to Valuation, it is almost inevitable that queries should arise as to the probable results which these investigations—especially the first—will bring to light. It would seem obvious that very wide discrepancies will be observed between the ascertained figures of the different bodies—discrepancies due in many cases, of course, to accidental fluctuations, to relatively poorer investments, to slack or strict administration, and so on. All these are not susceptible of estimate, but there are yet other causes of divergence which may be considered more profitably. First in importance is the class of member which the particular Society may obtain. The freedom of choice paves the way for the segregation, into more or less homogeneous groups, of the various types and occupations to be found in the general body of members, with the obvious result that the more select classes (such for example, as the Insurance Officials or the Bank Clerks who are medically examined upon entering their respective professions) are cut off entirely from the worse groups. The effect is that, while the uniform contributions for the whole body may cover on the average the total benefit claims and expenses, yet in individual Societies the very widest fluctuations must occur, even although the management be extraordinarily economical and superlatively efficient.

The "Class-Societies" already mentioned will reap enormous advantages from the fact that nearly all their members have been strictly examined shortly before entry, and in consequence large surpluses may be expected which will allow of substantial extra benefits. But, on the other hand, the Class-Societies of a different kind, namely, those which are compelled either by locality, past connections or even by the competition for members, to accept the worst risks will certainly show very unsatisfactory results upon valuation, and, in fact, in many cases will have not only to forego all expectation of added benefits at the end of the first triennium, but also will be forced to submit to reductions in their allowances.

In this respect there seems to be a very grave problem ahead of the scheme. The compulsory membership combined with freedom of choice on the part of Societies suggests and encourages—it might almost be said enforces—this segregation of good and bad with results the magnitude of which time alone can reveal. It is the price of incorporating an existing system into a comprehensive new scheme, and nought but experience can tell how the difficulties thus caused will be overcome.

On the other hand it is essential to remember that the divergence between the sickness-experience of the various classes of person (as shown, for example, by the Manchester Unity Tables) is caused to a very large extent by the Industrial Accident Liability which is so severe in some cases and so light in others. The exclusion of this risk will tend undoubtedly to bring the various occupations more nearly together although there will still be scope for wide variations on account of the other factors already mentioned.

Fluctuation due to mere paucity of membership will be prevented to some extent by grouping together those Societies which have less than 5,000 members at the first valuation in such a way that no aggregate shall be below this standard. If a particular society do not join an association formed for the purpose, it will be grouped automatically according to locality. Provision is made for the establishment of a joint financial committee, and for preventing the destruction of the individual characteristics of the various societies. It would seem imperative for any organization which is likely to fail to secure 5,000 members to select its own associates, so to speak, since otherwise it might become forcibly related to very undesirable companions as, for example, in the case of a small agricultural society in the neighbourhood of a village industrial fund. The grouping of two such bodies would certainly not be to the pecuniary advantage of the agricultural members.

Three other points of interest in connection with the subject of valuations may be noted briefly—

- (a) That any branch of a Society may reinsure, with the parent body, all or some of its benefits.
- (b) That the male and female members of an Approved Society without branches may be separated for purposes of valuation.
- (c) That in terms of Section 72 provision has to be made

for adjusting the contributions and benefits of those members of existing registered Societies who will, henceforth, take a part of their insurance through the National Section. A proper scheme has to be prepared and submitted for approval to the Registrar of Friendly Societies, and it is laid down that such alterations as are proposed shall not affect prejudicially, the solvency of the Society. If, on subsequent valuation under the new conditions, a surplus is revealed, such amount must be applied for the benefit of members in general or of those only who join the State Section. Many interesting points arise which it would be necessary to consider somewhat closely at this juncture were it not for the fact that very detailed treatment has been given to them by Mr. A. W. Watson in the last of his recent Lectures. The completeness and breadth of the information presented by Mr. Watson render it, not merely permissible, but desirable, to omit the necessarily briefer and distinctly narrower treatment which the author of this Paper could hope, at best, to give.

Two other matters allied, more or less, with the question of Valuation may be cited. The subject of Investments is extremely important, and unless a decided extension of the field of Trustee Securities can be obtained, it appears likely that considerable disturbance in prices (and a probable diminution of yield) will occur. Possibly the funds of the Health Insurance Scheme may be employed in the financing of those large measures of social reconstruction which are anticipated in many quarters. Whether this will be so or not is, of course, quite uncertain, but the suggestion indicates one possible solution of the problem. The question of Excessive Sickness (in its social aspect) is dealt with in the last section of the Paper.

Special Classes. **(8) Special Classes of Persons.** In the opinion of the majority of students, the National Insurance Act would be sufficiently difficult were there no classes of members requiring special treatment. Inasmuch as there are ten such classes, there is an enormous increase in complexity.

It is neither necessary nor possible adequately to discuss all these groups—therefore only the more important will be referred to at any length and the others will receive cursory notice.

## (a) Women.

In Section 44 the special difficulties incidental to the insurance of women are dealt with. The main complication is due to marriage with its usual, and desirable, concomitant of exit from the ranks of the wage-earners. Owing to the practical impossibility of adequately supervising the sickness claims of unemployed married women, it became necessary to provide either for suspension of benefit upon marriage, or at least for a closely-restricted form of assistance. Thus, under the Act, upon the marriage of a female contributor who ceases to be employed, ordinary benefits are suspended, and one of two courses must be followed—

- (1) The woman may become a special voluntary contributor paying 3d. per week and receiving in return medical and reduced sickness and disablement benefits as stated in Table D of Schedule 4, Part I.
- (2) She may have two-thirds of her net transfer value applied towards the payment (so long as the amount will last) of special benefit upon confinement together with compassionate allowances at the discretion of the Society or Committee responsible.

In either case one-third of the transfer value passes automatically to what is known as the "Married Women's Suspense Account"—a fund whence will be met the sums required to enable widows to come back into insurance with full rights without paying arrears accruing during coverture.

If a woman remain employed after marriage the insurance is not affected.

Upon the death of her husband a female member of an Approved Society who is obliged to go to work, as so many of the widows of wage-earners are compelled to do, can simply resume ordinary benefits and be to all intents and purposes exactly as if she had remained a full member uninterruptedly. If she do not go to work she will have the option of becoming (if eligible otherwise) an ordinary voluntary contributor at the rate chargeable at her original age at entry, or she may, whether qualified by income-limit or not, become a special voluntary contributor as mentioned above.

Provision is made that a presently-married woman who was married when the Act came into operation shall be allowed, if and when she becomes an employed contributor, to pay the flat rate and to receive full benefits provided she enter into insurance either before or within one year after the death of her husband.

If a special voluntary contributor become employed the full contribution of 6d. per week is due, and may be applied firstly to meet the 3d. required to pay for the special benefits mentioned already, and secondly, for the advantage of the member, as the Society may decide (or of course the woman can be an ordinary contributor—but, if so, she will be regarded as a new member, subject however to certain regulations, as yet unissued).

A few points arise in connection with this intricate network of provisions.

In the first place the Married Women's Suspense Account must be considered. This fund will be built up from the prescribed proportion (one-third) of the transfer values of women who become married and cease to be employed, and it will be drawn upon later when widowhood or other cause necessitates re-entry into insurance. Now the probabilities of marriage, of not remaining employed, of widowhood, of then becoming an employed, voluntary or special voluntary contributor are all seen to be involved directly an effort is made to solve the problem of the adequacy or otherwise of the proportion assigned. Since of these four elements, the first and third are not known accurately, while the second and last are largely matters of conjecture, it is obvious that at present no decided opinion can be pronounced. In this, as in many other points of the Act, experience alone can teach. It is provided that any deficiency in the account in question shall be made good by a draft upon the money in hand for the extinction of initial Reserve Values. The effect of this is that, practically speaking, the general body of insured persons is made to guarantee certain privileges to the female members at the risk of postponement of the promised additional benefits. There is hardly likely to be a very significant call upon the "Reserves Fund", and consequently the matter is not of importance. In view of the generally tentative statistical basis of the Act, perhaps it is quite reasonable that all fluctuations should be distributed over the whole of the membership.

Secondly, the case of women who marry and who do not wish to pay the special Voluntary Contribution claims brief mention. The effect of Section 44 in this instance is to put such persons into the position of mere deposit holders without the right of withdrawal at will.

(b) Aliens.

In Section 45 Aliens are considered. Insurance is compulsory, partly in order to avoid giving a stimulus

to employing such persons in preference to English people, but except in special circumstances reserves are not credited and no Government Subsidy is allowed. Consequently full benefits cannot be given, and each Society has to determine what rates of sickness, disablement and maternity allowances it can afford, regard being had, of course, to the actual age of the member at entry. It is important to notice that the Government assistance is refused as regards *Benefits* but not as regards *Contributions*. This means that the State Subsidy will be given in the case of low-paid labour and of persons over 65 at entry. To produce a sort of correspondence Alien Deposit Contributors receive only  $\frac{7}{9}$  (males) or  $\frac{3}{4}$  (females) of the usual amount of each money benefit. The Insurance Commissioners have prepared for the use of Societies, at their option, a table of allowances applicable to the special circumstances of aliens. A few persons, namely, those who on 4 May 1911 had been in this country for five years and belonged to a Society now approved, are subject to no restrictions and get full benefits.

*(See Soldiers and Sailors.)* The position of Soldiers and Sailors is a distinctive one. As drafted, the Bill provided that no actual benefits should be given during service, but that certain reduced contributions should be payable (both by the men and the War Departments) in order to secure (a) Reserve Values sufficient to enable healthy men to join Approved Societies upon discharge from the Service, and (b) some poor provision for those who by reason of ill-health were unable to do so. The Government promised a contribution equal to  $\frac{2}{9}$  of the amount which would have been payable had the men involved been ordinary civilians. The Clause produced very strong opposition and was drastically amended, though not quite in the direction desired by many persons. As the Act now stands there is a deduction of  $1\frac{1}{2}d.$  per week from the pay of each soldier and sailor (with the exception of certain minor classes), and the following main provisions apply—

- (1) For those men who belong to (or who join within a prescribed period) an Approved Society, the Authorities contribute  $1\frac{1}{2}d.$  per week, and, until discharged, the member receives no benefit other than maternity benefit, which, however, is payable whether his wife be outside the United Kingdom or not. The Insurance Commissioners deduct only 1d. of the sum allocated for extinction of Reserve Values from

the 3*d.* received on behalf of the Approved Societies —the balance of  $\frac{2}{3}$ *d.* coming from the special Navy and Army Insurance Fund referred to below. The normal Reserve Values are credited.

- (2) In the case of other men a special Navy and Army Insurance Fund is constituted. This Fund is really built up from (a) the Contributions of 1*½d.* per week made by the men in question, (b) the Government Subsidy of  $\frac{2}{3}$  of the benefits which would be payable, but for the special circumstances to *all* the men, whether members of Approved Societies or of this particular fund, (c) Reserve Values in respect of men serving at the commencement of the Act, (d) the contributions of the Naval and Military Authorities, which, in this case, are fixed at the sum required to keep the fund solvent.

During service the men in the Special Fund get nothing but Maternity Benefit.

Upon discharge, provision is made for passing as many men as possible from the Special Fund into Approved Societies, but, where there is inability to join one of these organizations through ill-health, the person affected remains in the Fund and receives the normal benefits subject to special conditions which it is not necessary to mention here. But while protection is thus afforded to the invalid men there is no right of staying in the Fund at will.

It appears from this that the broad principle is to induce as large a proportion as possible of the men to belong to Approved Societies, and therefore to avoid the question of arranging for transfer values upon discharge. Thus, so far as these men are concerned, the main point of actuarial interest is whether the reduced contributions receivable during service set against the freedom from all benefits (except maternity allowances) will suffice to provide, as and when required, not merely the normal reserves, but the greater reserves which naturally require to be made in view of the heavier sickness expectation arising in consequence of service abroad. As there is, in such cases, no option of selecting only the healthy discharged men, the Approved Societies will require to be satisfied as to their position ere they accept soldiers and sailors at the outset. The Government Actuaries consider that they will be perfectly safe in so doing. It will be very interesting to watch the effect of a

considerable number of such members in any Society and, incidentally, of course, the problem of assessing their future sickness expectation will have to be dealt with. In this connection, a possible relief owing to heavier mortality must be borne in mind.

The Navy and Army Insurance Fund is guaranteed by the Admiralty and the Army Council, and therefore its experience, while it may be watched with interest, does not present scope for much discussion.

A comprehensive view of the whole question of the application of the Act to the Naval and Military Forces of the Crown suggests that it might have been much more desirable if the proposal of certain experienced gentlemen had been adopted, and if, accordingly, an entirely distinctive Society had been formed. Space prevents a discussion of the merits of this scheme, but one especial benefit likely to be secured by the members of such a special society may be mentioned, namely, the provision of assistance in order to tide men over the transitional period extending from the date of discharge up to the time of finding suitable and permanent employment. As things stand at present, there is likely to be great hardship for a considerable number of the men so circumstanced.

(d) *Persons partly provided for.* In order to meet the case of persons who, by custom or otherwise, are entitled to receive full remuneration during short illnesses, provision is made in Section 47 for reducing the contributions by a fixed amount assumed to be equivalent to the value of the benefit actually surrendered. This Section is designed to cover in a rough sort of way the needs of those who do not require a duplicated provision of benefit. It is not of particular actuarial interest, except as regards its reaction upon the subject of substituted benefits which has been referred to already. Individual employers, however, have a somewhat delicate task if affected by the Section, for upon them lies the onus of deciding whether the relief from contributions is desirable having regard to the rather serious obligations involved. It is possible that a distinct selection may be set up in future by employers of, say, domestic servants (one of the classes most affected) with the idea of taking the reduction in premiums and effecting a saving in benefit charges by engaging only very healthy persons. This, however, is a domestic rather than an actuarial problem.

(e) *Seamen* The normal provisions of the Act are obviously inapplicable in some particulars to persons in the

Mercantile Marine, and therefore they receive special treatment. Members of Approved Societies receive no sickness, disablement or medical benefits so long as such are covered by the provisions of the Merchant Shipping Acts. It is found that on the average persons engaged in the foreign trade are employed only for four-fifths of a year, and in order to protect them from falling into arrear owing to these special circumstances, it is laid down that four contributions shall count as five. The employer pays for every week of employment, but he is relieved of one penny in consideration of the definite liability laid upon him by the Merchant Shipping Acts. Seamen in the home trade are not so circumstanced as regards unemployment, and therefore the provision as to alteration of contributions does not apply to them.

A special Seamen's Society is set up, and this organization, besides receiving the prescribed contributions of its members will be credited also with the employers' payments in respect of the considerable number of foreign sailors who are not domiciled in this country and who are not insured. These sums represent pure profit, and it is intended that they be utilized in providing additional benefits, not merely for the members of the special Seamen's Society, but for the general body of mariners. It is laid down that pensions for long sea-service shall be granted, preference being given, if the Society desire, to those who have served in the foreign trade. The reason is that the accretions of benefit arise from the contributions paid in respect of aliens who compete with British seamen almost exclusively in this branch of the industry.

*(*Other Special Classes.*)* The remaining sections relating to special classes of insured persons refer to workers over 65 at the commencement of the Act, to persons in seasonal trades, to inmates of charitable homes, to elementary school teachers, and to certain grades of the Service of the Crown.

Regarding these, there is little to be said. Persons over 65 receive such benefits as the societies care to give in return for the contributions paid by the workers and their employers together with a direct Government subsidy of 2d. per week. Reserve Values, &c., are not credited, and, of course, the State does not bear  $\frac{2}{3}$ ths of the cost of benefits and administration.

No points arise in the case of the other special classes.

This completes a brief survey of the chief points of practical and actuarial interest arising out of Part I of the Act, and

accordingly the enquiry passes into its next phase which involves a comparison of the British scheme with the analogous arrangements, operative or projected, in certain foreign countries.

(c) THE ANALOGOUS SCHEMES, EITHER OPERATIVE OR PROJECTED IN FOREIGN COUNTRIES.

At the very outset of this part of the Paper it is necessary to emphasize once again the fact that the treatment is indicative and not exhaustive. Considerations of space and opportunity preclude any detailed comparison of the schemes to be discussed.

As the result of somewhat extensive enquiries (referred to in the acknowledgments at the end of the Paper) it appears that only in Germany, Austria-Hungary, Luxembourg, Norway, Russia, and Roumania, are there schemes of Social Insurance at all comparable with the English Act, and in the case of the three countries at the end of the list the legislative measures are very recent, so that no experience has been accumulated. Other nations have taken no definite steps as yet, although in France and Belgium there are schemes in favour of one or two occupations; and at the present time action is foreshadowed in Denmark, Holland, Sweden, and Switzerland. In many instances provision is made for protection against Industrial Accidents and Old Age (with a certain measure of invalidity cover), but with these two branches of Social Insurance the present enquiry has no immediate connection.

It is thought that the best method of treatment will be to take each country in turn, to outline its scheme and to point out the practical issues involved. Obviously this plan causes a certain amount of overlapping between the present Section and that dealing with Social Effects, but there is a distinct gain to be secured by adopting it.

*Germany.* Germany affords the outstanding example of National

Insurance upon an enormous scale, and it is fitting that attention be accorded to this great Empire first of all. The chief sources of information in this case are the well-known Paper by Mr. T. E. Young, in Vol. xxix of the *Journal of the Institute*, the Official Memorandum on "Sickness and Invalidity Insurance in Germany", and the recent publication of the United States Labour Department. While, however, the brief account which follows is drawn to a considerable extent from these documents, it has been amplified and modified by reference to numerous other books and papers of which the most important

is the volume by Mr. W. Harbutt Dawson, entitled "Social Insurance in Germany, 1883-1911."

German Social Insurance is organized in three distinct branches—

- (a) Sickness Insurance.
- (b) Accident Insurance.
- (c) Invalidity and Old-Age Insurance.

The corresponding British arrangement, adopting German nomenclature for comparison, is

- (a) Sickness and Invalidity Insurance.
- (b) Accident "Insurance."
- (c) Old Age "Insurance."

Thus, while in England the friendly society tradition of regarding invalidity allowances as continuations of early sick pay has been adhered to officially, in Germany these benefits are considered as being part of the provision for old-age—a view which is quite easily justified if old age be defined in terms of practical incapacity to earn wages. It may be noted also in passing that the Accident and the Old Age Insurances in the two countries present very distinctive features, but they do not fall to be discussed here.

The German Schemes have recently undergone a process of codification and extension. The various branches have not been united, but a central control has been established, and the official element is now much more evident. A considerable amount of modification has been introduced, and some important new features are added, chief of which is the provision for Widows and Orphans.

It will be convenient to consider the Sickness and Invalidity Schemes separately, and in each case only the following outstanding points will be treated—

- (a) Scope.
- (b) Organization.
- (c) Contributions.
- (d) Benefits.

After these have been discussed a few general matters of interest will be mentioned.

#### *German Sickness Insurance.*

##### *(a) Scope.*

The new Law has enlarged the bounds of the Scheme very considerably. Agricultural labourers, domestic servants, casual workers, &c., are now included, besides the more

usual occupations, and if all those who are eligible, do, in fact, insure, the percentage of occupied persons protected will be at least 65.

There is no income limit for workpeople. Voluntary Insurance is permitted within limits, but in this instance the sickness funds have power to call for evidence of health which they cannot do in the case of the compulsorily-insured persons.

(b) **Organization.** The organization of the Sickness Protection is carried out through various types of fund, corresponding roughly to the previously-existent voluntary associations, thus agreeing with the British method of working as far as possible through the Friendly Societies and similar bodies. Inasmuch, however, as no health test is required on entry (see above) no question of rejection is likely to arise, so that Deposit Contributors are not created. Employment is considered to be a reasonable guarantee of fitness, and it is laid down that no illness actually existing when the insurance commences can rank for benefit.

(c) **Contributions.** The Contributions are assessed as a percentage of the basal rates of wages, which are fixed periodically by the Authorities. Usually 5s. is the maximum day-wage considered, but occasionally the official limit may be raised to 6s. As a rule  $4\frac{1}{2}$  per-cent of the wages represents the utmost which may be levied, but this is not absolutely uniform. Of the total amount charged, the employer pays  $\frac{1}{3}$ , the worker  $\frac{2}{3}$ , and the State nothing. It is interesting to note that a Government proposal to make the proportions  $\frac{1}{2}$  and  $\frac{1}{2}$  respectively was opposed strenuously by the Socialists on the ground, chiefly, that it would involve giving the employers equal representation in the management instead of one-third as at present! In certain unhealthy trades the employers may be charged more than the normal amount.

Contributions are not payable during sickness, and there is a certain amount of latitude as regards unemployment. There are reduced contributions in some cases, as in England.

(d) **Benefits.** The Benefits are—

- (1) Sick Pay for 26 weeks from the fourth day of illness and Medical Treatment for not more than six months.
- (2) Maternity Benefit.
- (3) Funeral Benefit.
- (4) Family Benefit.

The chief points of interest to be noted are as follows—

*Medical Treatment* is not continued throughout life.

*Sickness Benefit* is not uniform in amount in all cases since it is related directly to the officially-determined wages. Usually 50 per-cent is allowed, but in some cases as much as 75 per-cent may be given.

It may be granted alternatively in the form of Hospital Treatment with a subvention to the family of the patient. At the expiration of the period for which sick pay is allowed the member, if still disabled, must claim a new benefit from a different branch of the Insurance Organization, *i.e.*, from the Invalidity Section. This would seem to be cumbersome and likely to lead in some cases to difficulty in obtaining the assistance which is really required.

*Maternity Benefit* takes the form of eight weeks of sick pay, and is limited to women who are themselves insured, but may be extended by the Societies as a voluntary matter if they choose. It is not taken from the Husband's Fund.

*Funeral Benefit* is normally a sum equal to twenty times a day's wage.

*Family Benefit* is a voluntary allowance which may be given to the dependents of insured persons if the Sick Fund adopts a rule to that effect. It may include all the foregoing Benefits, but as it is an additional assistance, extra contributions may be levied upon those who enjoy it.

### *German Invalidity Insurance.*

(a) *Scope.* This does not differ much from that of the Sickness Insurance. Mention may be made, however, of the fact that persons whose earning capacity is less than  $\frac{1}{3}$  of the normal amount for their class, are exempted from insurance. This amounts, in a way, to exclusion of bad lives.

(b) *Organization.* This is upon a territorial basis, and there is no Friendly Society control as in this country.

(c) *Contributions.* The Contributions vary according to the particular wage-class to which the insured person belongs. There are five classes, and in each case the Authorities fix the premiums for a period of 10 years. The employer and the employed pay equal proportions, and the State subsidy consists of a fixed addition of about £2. 10s. per annum to the pension which these contributions provide.

(d) Benefits. There are now five Benefits—

- (1) Old Age Pensions (claimable at 70).
- (2) Invalidity Pensions.
- (3) Survivor Pensions.
- (4) Gratuities to Widows and Orphans.
- (5) Sanatorium Benefit.

Only (2) and (5) are of eminent interest in the present enquiry.

Invalidity Pensions are either—

- (a) The Sickness Pensions which are intended to be continuations of the Sick Benefit ; or
- (b) The Incapacity Pensions allowed to members who are unable to earn  $\frac{1}{3}$  of the normal wage for persons of their status. (This would seem in some respects to be a better basis than the English one, since the insistence upon complete invalidity in the latter case is likely to afford a direct discouragement to persons who might be able to undertake light work, but who will not do so because of the attendant loss of the insurance provision. Against this consideration, however, must be set the undoubted fact that in England a large part of the financial trouble in Friendly Societies has been caused by allowing persons to draw reduced sick pay when not completely incapacitated.)

Ere any right to Invalidity Allowances accrues a waiting-period of 200 weeks must be passed and at least 100 contributions must be paid.

The Pensions consist of the following items—

- (a) A fixed Imperial Subsidy of £2. 10s. per annum.
- (b) A basic amount, differing according to wage-class, but independent of the number of contributions made.
- (c) An accretion varying with the number of weekly premiums paid.

There is a special supplement of 10 per-cent for each child under age 15, the maximum addition on this account being limited to 50 per-cent.

It is laid down that Pensions arising under the Accident Insurance Law must not be supplemented by Invalidity Allowances except in certain circumstances.

Members are allowed to make voluntary additional payments in order to obtain increased Pensions.

Sanatorium Benefit does not claim special notice here.

Such, in the briefest compass, is a description of the German schemes from which the National Insurance Act was largely adapted.

A few points of difference, &c., may be mentioned here conveniently—

- (1) The varying contributions and varying benefits in Germany as compared with general fixity in England.
- (2) The larger number of benefits given in Germany, but their distinctly more limited character.
- (3) The much smaller measure of State assistance in Germany combined with more stringent official control in some ways though not in others—notably as regards the latitude allowed in the matter of benefits.

[The addition to the Invalidity Pensions and the maintenance of the Imperial Insurance Office (corresponding, of course, to the Insurance Commissions in the British Scheme) constitute practically the entire assistance which the State gives in Germany.]

- (4) In Germany the first 13 weeks of accident pay falls on the Sickness Funds—in England the Approved Societies do not have to provide benefit in case of compensated disablement.
- (5) In Germany the user of “cheap labour” is relieved of contributions in comparison with the employer who pays better wages—in England the reverse is the case.
- (6) In Germany the Invalidity Insurance is administered territorially in contrast with the Friendly Society control permitted in this country. It might appear, at first sight, that in many respects the British plan is better, since an effective check upon long-sickness claims is most easy when members of the same Society as that to which the beneficiary belongs have a direct personal knowledge of the case, and a clear interest in preventing fraud, but this is not

found to apply universally in Germany. It has to be remembered, also, that the apparent advantage of the English practice may be neutralized by the fact that many of the workers will obtain insurance, not through the local Friendly Societies, but through the great Organizations whose administration is centralized. Of course, much depends upon the means employed to distribute the benefits, and it is quite possible that the use of agents who have an intimate acquaintance with their clients may prove, in practice, to be a very effective check.

In the view of competent authorities in Germany, the more centralized the administration becomes, the more effective is the safeguard against malingering, and the better is the provision afforded. A large Insurance Organization is found, in practice, to be able to take a broader view of the questions at issue and to use its greater resources more effectively. To cite one instance—it is found that the employment of special inspectors is most useful in preventing simulation, but the local societies cannot afford to take such measures owing to the expense involved. This is especially important in view of the apparent tendency, already manifest in England, towards concentration, and, in fact, the point has received special attention in the new German code which encourages the creation of societies larger than those which have existed hitherto.

- (7) In Germany the Sick Funds are deemed to be solvent if, each year, they put aside 10 per-cent of the contributions as a reserve. In 1910 rather less than one-half of them attained this standard !
- (8) Although there are at present six types of Fund in the administration of Sickness Insurance, there is evidence of a distinct tendency towards reducing the number and, consequently, obtaining more uniformity.

Quite recently Office Employees above the income-limit for Invalidity Insurance have been brought within its scope by a separate law. There are nine wage-classes—the incapacity requisite to secure a pension is  $\frac{1}{2}$  (and not  $\frac{2}{3}$  as in the case of the workmen) and no Imperial Subsidy is given.

The next country to be considered is Austria. In the main the Social Insurance Scheme in question

is a copy of the German model, and consequently it will be necessary merely to indicate its distinctive features. The actuarial basis was treated very fully in a recent Paper by Mr. G. W. Richmond, but the present enquiry is concerned merely with the general outlines of the scheme.

As regards Sickness Insurance the chief differences between the German and the Austrian methods lie in the conditions of benefit, the assessment of contributions (the maximum official day-wage being lower in Austria), and so on. Upon the whole the allowances are less full in the latter country. Until recently Invalidity Insurance in Austria has been compulsory only for miners (since 1889), and employees in offices (since 1906). The German feature of wage-classes and varying pensions is followed, but the State subvention is not fixed irrespectively of group, but depends upon the class to which the man belongs. Within the last few years a movement in favour of extending the Social Insurance Schemes, and in particular of making Invalidity Protection general, has been noticeable. The Author is informed by Dr. Ernest Blaschke that the question has been discussed, but that action has not resulted, at present, although it is likely to do so very soon.

**Luxembourg.** Luxembourg is mentioned in passing. The scheme there is modelled upon the German Law.

**Norway.** Norway, Russia, and Roumania, are new entrants into the field of Social Insurance. In Norway a territorial organization is created to provide Medical and Invalidity Benefits. The members are classified under four income-limits, and sickness is not covered by the scheme.

Russia and Roumania have passed their laws quite recently and it is only through the great courtesy of the Labour Department of the British Board of Trade, whose officials gave access to certain hitherto-unpublished documents, that the following particulars have been collected.

**Russia.** The Russian Law would seem to contemplate the formation of Factory or Works Funds for the purpose of Sickness Insurance with the provision that a definite minimum membership (usually 20, but sometimes 30) be obtained. The benefits to be given are—Medical Aid, Sick Allowances, and Funeral Money. Medical Aid includes Maternity Benefits and is chargeable upon the employer. Sick pay is proportionate to wages, and varies also according to the existence or absence of dependents. If the workman have dependents it is from  $\frac{1}{2}$ — $\frac{2}{3}$

of the wages—if he have none it lies between  $\frac{1}{4}$  and  $\frac{1}{2}$ . The allowance commences on the fourth day of illness and persists for 26 weeks. The maximum period of benefit in any one year is 30 weeks. Maternity Allowances also vary according to wages between the limits of 50 and 100 per-cent. Funeral Benefit consists of a lump sum equal to 20 or 30 days' wages, as in Germany, Austria, &c.

The Law allows a certain elasticity with regard to the conditions and limits of benefit.

The Contributions may be either a direct percentage of the wages with a restriction as to the figure to be charged, or in some cases there may be definite wage-classes, as in Germany. The employer pays two-thirds of the amount contributed by the workers, and although the latter may be called upon for increased subscriptions if a deficiency arise in the funds, the former is definitely exempted from any surcharge. Apparently there is no intention of creating strict actuarial reserves, but, instead, a small percentage of the contributions (from 5 to 10 per-cent) is to be set aside as a margin.

The administration of the funds is quite representative—in some cases one fund may be established for several different factories.

*Roumania.* The Roumanian Law provides firstly for Sickness Insurance, and secondly for Invalidity Protection. The former includes Medical and Maternity Benefits as well as the usual sick pay, and the main cost falls upon the workers—the State contributing a certain subsidy. Five wage-classes are adopted and the benefit varies not merely according to the existence or otherwise of dependents, but is governed also by the question of treatment at home or in hospital. Not more than 50 per-cent of wages is to be allowed. Contributions are based upon the mean salary of the wage-groups, and have been assumed to remain in force for 45 weeks in each year upon the average.

The Invalidity Insurance provides for a pension commencing upon certification as a disabled person after at least 200 weekly contributions have been paid. The allowance increases from fcs. 150 per annum (after the minimum number has been credited) to fcs. 384 after 49 years' payments have been made. Apparently the pension is to commence not later than age 65 in any event. There is a triple contribution by the State, the employer and the worker, aggregating 45 centimes per week.

These two cases, the Russian and the Roumanian, represent the latest examples of Social Legislation upon the lines which are being considered in this paper. It is safe to say that, ere long, the example will be followed by other nations who have not yet dealt with the problems which these Insurances are designed to meet.

Brief mention may be made of the papers, &c., which appear in the *Transactions* of the various International Actuarial Congresses—more particularly the second and the sixth.

In concluding this Section it will be well to point out that the tendency in all these foreign schemes is towards a more and more comprehensive system of provision. The first step is usually the substitution for sporadic voluntary protection of compulsory sickness and perhaps invalidity insurance. As time goes on it is found that partial protection causes friction and waste since the Industrial Accident Legislation overlaps the Sickness Provision, and the Old Age Pension Scheme is more than complementary to the Invalidity Allowances. These considerations and the pressure of public opinion cause inevitably a demand not merely for extension but for codification. This phenomenon has been witnessed in Germany, it is being seen in Austria, it is undoubtedly to be in evidence in Great Britain before many years have passed away.

The problems which such a tendency involves are of very great sociological interest, particularly at the inception of a scheme such as the present British Act unfolds. There is a certain amount of evidence that overlapping has been considered in the framing of the Act, but it is suggested that as time goes on a more deliberate effort at what may be termed a synthetic combination will have to be made. In this connection reference is given to the proposals of the National Insurance Inquiry Association which are dealt with in the next part of the Paper.

#### (d) AMENDMENTS—COMPLETE OR PARTIAL.

The next Section of the enquiry to be taken up is entitled, "Amendments—complete or partial." At the outset it is necessary to say, that although a very considerable amount has been written in the Press and elsewhere as to the importance of speedily amending the Act, yet concrete suggestion has been distinctly lacking. The question may be approached in two ways—either it may be held that the Scheme should be remodelled as a whole, or it may be contended that sufficient

improvement will be made if some particular features be altered.

The two points of view must be considered in turn. One possible root-and-branch alternative would be to make the Scheme non-contributory. This plan is reserved for discussion in the last Section of the Paper since the chief points which arise are distinctly more sociological than actuarial. Coming now to practicable and at the same time far-reaching suggestions, it will be found that perhaps the most important arguments in favour of the reconstruction of the Scheme as a whole are to be found in the assertion that it duplicates very largely provision which already exists—that in consequence it forces persons with little to spare to pay for what they do not need—and that such facilities as are available for obtaining alternative benefits are cumbrous and unsatisfactory. This line of argument leads to the question of an alternative measure, and it is stated accordingly that what is required is a scheme of benefits which will be contiguous, as it were, with those already secured voluntarily and which will not—at any rate appreciably—overlap them. This is a view which has received wide support, and it is worth while to discuss it briefly but not dogmatically, for in such a matter as the question of amending a largely untried measure the best course to take is distinctly that which avoids unreserved assertion. As regards the contention that much of the provision made by the Act is a mere duplication of that already secured voluntarily, there is room for considerable variation in opinion. From one point of view the assertion appears to be justified when it is remembered that very many of the compulsorily insured persons are members of the well-established, sound and progressive Friendly Society Orders which do afford a really good protection to their members. But on the other hand it may be asserted with much force and truth that a considerable proportion of the persons who belong to the Friendly Societies, especially, of course, those connected with the smaller bodies, is not really provided for at all owing to the admittedly unsatisfactory financial position of these organizations. Moreover, it is known that the existing Friendly Societies do not in point of fact reach by any means all the workers in the country. The balance of these conflicting arguments is certainly a matter of individual judgment, and consequently no decision is definitely laid down here, but it is suggested that the latter point of view is more in consonance with the whole of the facts.

At this juncture it will be advisable to refer to an alternative Scheme which was put forward by an influential committee, termed "The National Insurance Inquiry Association." In *The Times* of 4 April last there appeared a letter and a memorandum sent by the Association, and inasmuch as the proposals were put forward as the result of careful deliberation and were based upon sound actuarial foundations (merits not common to many of the alternatives suggested), they demand attention.

It was proposed that the total income of the Fund and the proportionate share of the contributions made by the employed, the employer and the taxpayer, should remain as at present, and that the provision whereby generally all entrants are rated as at age 16 should be retained. The modifications proposed referred to the nature of the benefits to be given and to the simplification of the administrative machinery, and they were submitted in order to secure advantages such as are here specified—

- (1) To prevent the risk of the cessation of the widely prevalent custom of paying full wages during temporary sickness.
- (2) To deal satisfactorily with the Deposit Contributors by including them directly in the Insurance Scheme.
- (3) To avoid interfering unduly with the existing Friendly Societies, &c., while nevertheless giving them (subject to proper safeguards) an important share in managing those parts of the Scheme with which they are especially fitted to deal—and to include the other benefits in a central Scheme.
- (4) To guarantee the benefits offered.
- (5) To widen the scope of the Invalidity Insurance by including Partial Disablement.
- (6) To make the Insurance Provision more elastic by enacting that only Invalidity, Maternity and Sanatorium allowances need be taken universally, the other part of the benefits (quite  $\frac{2}{3}$ rds of the whole in value) being selected at will from a comprehensive list.
- (7) To provide retiring pensions at an age earlier than 70 and to make provision also for widows and orphans.
- (8) To secure by centralized control much cheaper administration.

Proceeding upon these lines the Committee recommended that there should be two distinct schemes—

- (1) Central Insurance with definitely guaranteed benefits open to all.
- (2) Subsidized Society Insurance administered largely upon the present lines adopted by Friendly Societies &c., but with safeguards to secure solvency.

The benefits in the Central Scheme would include—

- (a) Invalidity Allowances as in the present Act, but seeing that twice as much provision as is made now would be available there would be the definite intention of including Partial Disablement when experience rendered such a course safe.
- (b) Pensions of 5s. a week commencing at age 60 (with increase of 8d. a week for each year of deferment of claim and continuance of contribution up to age 65). These pensions would be coupled with those available under the existing "Old Age Pensions Act" so that at age 70 the Insurance Pension would continue only so far as it exceeded any old age allowance which the beneficiary might be able to claim.
- (c) Maternity and Sanatorium Benefits largely as in the present Act.
- (d) Allowances to Widows during invalidity and after age 60, and maintenance of Orphans until age 16. These benefits would be such as might be secured by the allocation of a definite proportion of the income (the amount so available is stated to exceed considerably the average sum provided for a similar purpose in Germany).

It was suggested also that the scale of reductions in benefit necessitated by non-payment of contributions should be graduated so as to run smoothly from the maximum point when no arrears at all occur, thus preventing the present anomaly that persons who pay 52 weeks' contributions in a year secure no tangible advantage over those who pay only 49. It must be pointed out, however, that since contributions are compulsory for every week of employment, and since there is no appreciable option of non-payment (the expedient of throwing one's self out of work for three weeks annually in order to escape contributions amounting to 1s. not being sufficiently attractive to warrant its extensive adoption) there is really no more anomaly

in making the allowance of, say, three weeks for possible unemployment than exists in the similar relief from payment granted on account of sickness.

Persons now over 16 would be liable to deferment of pension age according to the actual age at entry. Evidently the calculations showed this to be necessary.

Women would be treated in all respects similarly to men.

It will be noticed that no reference has been made hitherto to the question of a medical test upon entry into insurance. This point is vitally important in regard to the Invalidity Allowances, inasmuch as the inclusion or exclusion of a considerable number of impaired lives would naturally be reflected strongly in the rates of claim. The recommendation of the Committee was to the effect that where evidence of health was not produced at entry, there should be a postponement of the right to benefit for such time as might be necessary beyond the uniform period of two years. This suggestion is extremely vague, and probably it would mean in practice that the Invalidity Fund (already credited with twice the contributions provided in the present Act) would commence by granting to healthy persons Disablement Benefit upon much the same conditions as are now laid down, and would impose in the case of the unhealthy some arbitrarily-fixed scale of postponement which would be reviewed from time to time as experience accumulated. Thus, the effect might be that the present bad lives would be assisted, partly, out of the extra contributions of the healthy—the other portion of the penalty due to inferior eligibility being met, of course, by the deferment of benefit already mentioned. The comparative advantage or otherwise of such a method depends entirely upon the treatment to be meted out to the Deposit Contributors when their case comes up for reconsideration at the end of the year 1914. In the meantime, owing to the waiting period, no appreciable difference could arise in the matter of Invalidity Allowance. Since Sickness Insurance would be left practically as it was before the present Act was passed, it is evident that the invalid lives who could not get into the Friendly Societies would not receive even the meagre assistance at present afforded by the compulsory creation of a Subsidized Deposit Account. After a time, of course, the Invalidity Allowances would be extended generally in the manner already mentioned.

An important difference of principle regarding the form of the State assistance was suggested. It will be remembered that

under the present Act the State provides a definite proportion of the emerging benefits and expenses. Partly in order to secure a more equal distribution of State costs, present and future, and partly also on account of the pension insurance included in their Scheme, the Committee suggested that the assistance should take the form of a proportion of contributions. In view of the very heavy ultimate liability which the pensions would cause, the recommendation is certainly timely.

The Subsidized Society Insurance would give any benefit which the members required, provided always that proper Invalidity, Maternity and Sanatorium Allowances were included. As has been stated these are calculated to absorb less than  $\frac{1}{3}$  of the full contributions available. There would be regulations requiring separation of private from subsidized business, proper audit and valuation, immediate rectification of deficiencies, and so forth. Reserve values would be credited as in the present Act, but the Sinking Fund deduction would be only 1d. per week, the period being correspondingly extended to between 40/45 years.

This is the main Scheme.—Certain supplementary suggestions included, firstly, the provision that for all persons now over age 21 insurance should be optional. Employers would pay their ordinary contributions in such cases, but the sum so received would be carried to the credit of an Accident Insurance Fund, which by its operation would tend to relieve them generally of other payments. Secondly, it was proposed that the present Workmen's Compensation legislation should be linked up with the National Insurance Scheme, so that the combined provision might secure :

- (a) Very much more economical administration. (It was remarked that a very large proportion of the amount now expended by employers is absorbed in litigation charges and general expenses.)
- (b) More comprehensive benefit for the workers.

When disablement arose there would be no question of repudiating liability, for complete protection against invalidity, however caused, would be available, and the substitution of weekly allowances for lump-sum settlements would be distinctly advantageous on the whole.

The chief difference between the present Act and the suggested Scheme is, of course, the absence in the latter of any

definite provision for Sickness Benefit. In view of the fact that so many persons have already insured voluntarily, in order to obtain this allowance through the ordinary Friendly Societies, it must be assumed that in their case there is a very real need for it. It is suggested that since the provision so made has proved, in many cases unfortunately, to be insecure, there is good ground for attempting, as the National Insurance Act undoubtedly does, to consolidate and extend it, and further, it is quite obvious that if a part of the contributions be diverted towards providing pensions, the cost of sickness protection will have to be met separately. It is a matter of opinion whether there would not be in practice a tendency to neglect to secure sickness cover either on account of the difficulty of meeting the extra demand or merely through inertia. Further, if it be the case that insurance against sickness would be neglected, a situation would arise which might very easily react most unfavourably upon the invalidity experience. There is also the question as to the extent to which the working classes of the country have really an effective (and not merely a vicariously-imputed) guarantee of wages during short periods of sickness—to say nothing of any interval between the expiration of such guarantee, and the commencement of the invalidity allowance. On the other hand, it is equally obvious that the duplication by the present Act of what is in very many cases quite a satisfactory voluntary sickness provision is undesirable, and is not rendered appreciably better by the machinery for obtaining alternative benefits.

It will be noted, also, that no medical benefit is provided in the suggested Scheme.

Bearing in mind the premise that the primary aim of National Insurance is to conserve and improve the general health of the population rather than to become a huge benefit-paying organization, it would seem that the first consideration to be secured is a thoroughly sound provision against sickness. If this be not obtained the whole fabric of Social Insurances is imperilled at the foundations, and that which otherwise might have become a protection for the worker and a shelter for the poor fails largely of its purpose.

The idea of co-ordinating Industrial Accident Compensation with Invalidity and Pension Insurance is extremely good, because such a union would lead to economy of management, comprehensiveness of provision and general efficiency. Perhaps

these advantages may be expressed metaphorically by reference to the actuarial axiom, that when two or three benefits are combined in one policy it is practicable to offer a better aggregate of protection at a lower total cost than if separate contracts covering individual risks were issued.

Passing now to the question of amendments in points of detail the enquiry becomes rather difficult, chiefly because most of the defects will become apparent in the course of actual working, of which there is as yet no available experience. Presumably the Insurance Commissioners have encountered various points which will require modification at some later date, but in any case their results are naturally not public. In the course of this present Paper certain defects have been noted, and it is thought that a brief reference to them at this point will suffice as a treatment of the second portion of the Section.

In the first place, it is submitted that the position of the Deposit Contributors is thoroughly unsatisfactory, and must be altered radically, since the weakest who need the most get the least. It is a question for consideration whether it might not have been possible to form a central fund for these persons with the right of modifying the normal conditions of benefit in such a manner as might prove advisable—the maintenance of the fund being charged partly upon the National Insurance Scheme and partly upon the community in general. Stringent precautions in administration would be required, and at best the plan would be a compromise, but the chief point to remember is that these rejected lives need insurance protection—such cover can only be obtained upon onerous terms whatever be the course adopted, and therefore all that has to be decided in reality is the apportionment of the burden.

Secondly, it is suggested that the separation for Insurance purposes of the four countries at present composing the United Kingdom is likely to produce not merely increased charges, and a certain duplication of official machinery, but (and this is more serious) to cause also considerable difficulties in many of the Societies owing to transfers from one country to another.

Thirdly, it is thought that there is likely to be great dissatisfaction when, upon valuation, certain Societies with an abnormally high proportion of heavy risks (and there must be many such) show unsatisfactory results.

This problem is extremely difficult and probably incapable of completely satisfactory solution except as the result of long-continued and carefully-analyzed experience, and it is not suggested in this Paper that a solution has been found, but at the same time it may be well to indicate a few possible lines of approach. If compulsory grouping for valuation purposes had been widely extended, it would have been feasible to secure average results, while preserving the practical autonomy of each individual Society. The disadvantage is, of course, that Societies with very favourable sickness experiences would object to the curtailment of their profit from this source, and it is to be feared that the spirit of co-operation for a common good is not yet strong enough to induce them to agree. Another plan would be to have one central fund combining all the risks, but this has the very serious disadvantage of alienating the interest of the existing Friendly Societies, and thus of making the restriction of unjust claims a matter of extreme difficulty. Neither of these ways of approach is completely satisfactory, and it is almost, if not quite, certain that no other plan will ever be free from difficulty. In this connection it is well to remember the unifying effect which the exclusion of the Industrial Accident Risk is certain to produce.

It is desirable to emphasize the fact (ignored almost entirely by the majority of the critics of the Insurance Act) that the problem involved was not the construction of an ideal system, operating among an ideal people under ideal conditions, but rather the setting-up of a fresh scheme in a country where there was already much provision of a miscellaneous nature, and where in spite of relatively high civilization there is certainly not a completely-developed social consciousness. This simple and obvious fact alone must appeal, it is suggested, to all who consider the issues involved in a national scheme, such as has been brought into being in this country.

#### (e) SOCIAL EFFECTS.

The last Section of the Paper touches what is perhaps the most interesting feature of all these State Schemes of Social Insurance, and raises immediately that age-long question which is summed up in a single word "Whither?" For, while very much may be undetermined in regard to these Schemes there is one fact which is supremely, absolutely unquestionable, and that is the certainty that the details of

administration, the collection of contributions, the payment of benefits, even the control of huge reserve funds, are insignificant in comparison with the effect which will be produced upon the character of those nations who organize such insurances. The actuary can assess the charges at least with a fair approach to certainty, the financier can invest the funds with a moderate prospect at any rate of safety, but it needs someone more than either of these—someone greater, even, than both of them with a modern eugenist added—to say what are the reactions of those social forces which begin to have play at the first mention of such a scheme. Therefore it might seem hopeless to essay the almost impossible task, but if definite assertion cannot be made, it is surely at least practicable—even profitable—to consider the issues at stake. That is all which this Section of the enquiry is intended to accomplish, and it is proposed that the problem be approached from two different directions. In the first place, there will be the appeal to experience ; that is to say, the German evidence will be examined, and the conclusions to be drawn will receive attention, and in the second place general, and it is hoped not altogether unhelpful, suggestions will be given.

With regard to the test of experience it has to be remembered that the German Imperial Insurance Laws have not been in operation yet for so long as 30 years, and that, therefore, the really vital social effects—those which become as it were part of the heredity of the nation—have certainly not had opportunity of revealing themselves.

Such a consideration imposes necessarily a severe restriction upon the usefulness of the appeal to the past, since it limits the comparison to a period insufficient for the observation of any but the more rapid and possibly the transient changes. This, however, would not appear to be an obstacle to some enquirers since in a book published nearly 10 years ago (when parts of the German System had been in operation for less than 15 years) the following conclusions were reached—

- (1) That the German Scheme while originally intended to promote thrift and self-reliance had largely failed of its object.
- (2) That there was a growing demand for larger and larger measures of State assistance as shown in the decline of voluntary provision.
- (3) That gradually the Scheme was becoming more and more a matter really of State relief, and that control of claims for benefit was slackening.

- (4) That such success as it had achieved was due entirely to mere compulsion (whereas, admittedly, willing co-operation is essential to real progress).
- (5) That so far from discouraging pauperism the Scheme had, in fact, augmented it.

This condensed statement represents very fairly the main arguments and conclusions which are set forth in the book. It is suggested that these views are extreme, and although doubtless in many respects legitimate deductions from the limited evidence available, they cannot be said to take a sufficiently long and complete view of all the circumstances.

The following brief digest of the opinions of various authorities in Germany given in response to a request from the British Government is of great interest, and very considerable value in the question now under discussion.

The verdict of representative employers is almost unanimously to the effect

- (1) That the productive capacity and general comfort of the workers have been materially improved through the sense of security which the Insurance Schemes give and the prompt treatment of incapacity when it arises.
- (2) That the employers recognize the necessity for the compulsory provision, that upon the whole they regard it as the cheapest method, but that any great extension of its scope would not be acceptable.
- (3) That there is distinct evidence in many quarters of the awakening of a social consciousness among the employers which has induced them largely to supplement the minimum legal provision.
- (4) That the anticipated improvement in the cordiality of the relations between master and man has not in fact occurred. [One referee says that this is the fault of human nature and not of the Insurance Laws!]

Some employers suggest that malingering has been very greatly increased, and one declares that persons out-of-work resort to sickness benefit as if it were an unemployment allowance, but generally these opinions are not advanced.

With regard to malingering it is definitely stated by a former official of the Imperial Insurance Department, that there is no evidence to support the assertions of fraudulent claiming, and

that the idea is obtained by concentrating attention on the minority of unscrupulous persons who occur not in one stratum, but in all strata, of society.

The view of the President of the same central authority is that new moral influences have been awakened in the nation, and that generally (except upon the ground of increased discontent) all anticipations have been fully realized.

The statements of various Poor-Law Authorities suggest a striking explanation of the apparent increase in Poor-Law Relief, noted by the writer already referred to.

They state emphatically that the Insurance Laws have produced very real and very marked relief to their obligations, but that, nevertheless, there is no pecuniary saving because the progressive improvement in the standard of living and the direct encouragement which the Laws have given to comprehensive and costly schemes of treatment have necessitated largely-increased Poor-Law subventions.

Quantitatively there is an augmentation—qualitatively there is a real decrease—savings have been devoted to ever wider assistance and more thorough help, as, for example, in the provision of Forest Schools for Invalid Children, Dental Clinics, Sanatoria, &c.

Mention may be made in this place of the Berlin Pension Board, which expended, in 1911, no less than 28 per-cent of its revenue upon prophylactic measures, and is acknowledged to have achieved more good for society at large by this work than by all its invalidity and old-age pensions.

Since this is so, it would appear, indeed, that the idea of all-round social schemes has taken deep root !

Such is the latest representative opinion upon the record of the German effort—the only comment which will be made is to raise the issue as to whether any or all of these observed effects will persist or whether time will prove their transience. That this is not merely an academic suggestion is clear, it is submitted, from the consideration that the introduction of a great social change must cause early disturbances, just as the violent agitation of a volume of water produces temporary waves which, though important and impressive at first, subside in a longer or a shorter interval. Thus, in order truly to assess effects, allowance must be made for initial maladjustments which have no lasting significance. May it not be possible, for example, that the tendency to malinger will disappear as the idea of the aim of the insurance becomes

more and more clear to the persons affected, and as, accordingly, a greater development of the sense of social responsibility emerges ?

Inasmuch as no other country has any experience comparable in point of length or magnitude with that of Germany, the appeal to the witness of the past is necessarily concluded, and it remains only to consider a few general indications.

The first of these has been elicited already—it is that the social effect of any scheme has to be viewed not quantitatively, but qualitatively, for immediately the plan is put into operation it commences to modify, fundamentally, the very conditions which evoked it. This is abundantly clear from the simple illustration of sickness insurance. A new scheme may be based upon the latest available data, but directly it comes into operation it not merely provides protection against the expected incidence of disability—it changes probably the whole character of the future sickness experience.

It is urged, further, that the very widest possible view needs to be taken if a true conception of the results of any scheme is to be formed. A social change, for example, may relieve immediate distress, and be thereby considered desirable ; it may also encourage thriftlessness, and on that account be condemned. It is recognized that to look at either of these aspects singly would lead to fallacious results, but is it always so clearly perceived that a true result is not obtained even if all present factors be included ? It is submitted that no analysis of social consequences can be of any ultimate avail unless it take account of the fact that any present step projected, debated or carried out, has its roots in the past and its fruition in the future. The life of a community is being recognized more and more as an indivisible entity from which short periods cannot be safely taken out as blocks of stone from a quarry. Therefore the only valid view is that which looks both backward and forward. For a similar reason it is essential also to make as broad a survey as is possible. Objection can be raised to the effect that these conditions are ideal, impossible, and practically productive of no settled conviction. It may be—but surely it were better to take no decision than to adopt a false one, and surely the knowledge of the greatness of the problem should but act as an incentive to the patient investigation and unhurried effort by which alone success is to be achieved ? In this direction the members of the Institute have a very special opportunity of rendering useful

service to the community to which they belong, and there is more than enough scope for all who are willing to assist.

It is clear also that the National Insurance Scheme will have a profound effect upon the voluntary thrift organizations of this country. Indications are not wanting already that in future there will be a great and ever greater concentration of the work so that many of the smaller societies will cease to exist. This tendency towards centralization, which has been observed in Germany, makes for greater administrative efficiency and more secure control, but while it achieves these ends it also produces, or at least encourages, other results of a less desirable nature. Of these last the chief is the decay of the spirit of voluntary association with all that it brings, in the way of mutual knowledge and helpfulness. On the one hand centralization offers great but mechanical efficiency, on the other the old system of purely local organization produces good social effects, but hopeless financial results. It is a task of the future to extract from each system its best possibilities and to blend them in the finest and most useful way.

With regard to the general question of the tendency towards what is known as pauperization, which term is used here to include malingering, there is one phase of the subject which seems to have received but little notice hitherto. It is suggested that there is good reason for believing that the supposed increase in readiness to seek State aid upon the slightest pretext or even upon none, is not a permanent characteristic developed by the operation of the schemes of social assistance now under consideration, but is either

- (a) The bringing to the surface of a latent but none the less real disposition. After a time the bosom will have been cleansed of much perilous stuff, and the supposed effect will vanish.
- or (b) A merely temporary feature due to the lack of proper acquaintance with the provisions and the spirit of the Schemes of which it is safe to say that the majority, have at first no clear conception. [This has been touched upon previously with reference to Germany.]

If these suggestions be well founded it would seem to follow that the remedy is merely one of waiting patiently and guiding carefully. It is not contended, of course, that there will be a cessation of malingering in every case but rather that the passing

of the years will reveal the true and ultimately small dimensions of the problem to be attacked. Many ideas have been made public with regard to the best means of doing this—most of them offer purely mechanical devices, consisting mainly of a detective organization. It is suggested that the only valid and persistent check is to be found in the development of the spirit of co-operation and the encouragement of a growing social consciousness.

In a previous section mention was made of the fundamental question as to the relative merits of Contributory or Non-contributory Schemes, and it was stated that the points arising in that connection were chiefly of social rather than of actuarial interest. It is desirable that brief reference to the matter be made at this point. The main criticism levelled against a non-contributory scheme, is that it offers the greatest possible encouragement to malingering, and where the provision made extends over the working years of life during which every capable citizen is supposed to contribute his or her quota of assistance, there is no doubt that the system is impracticable. The case of Old-Age Pensions is distinctly different, for the possible scope for malingering is very much more restricted. It must be admitted that this objection is thoroughly well-founded if the matter be regarded from a purely practical standpoint, but since a rather wider outlook is being adopted in the present section of the enquiry, it is necessary to mention also that there are at least two reasons which may be adduced upon the opposite side. The former of these is that illness and disablement are not direct functions of industry, and therefore that the cost of insuring against them should no more be charged specifically against employers and employed than are the expenses of the Public Health Services. The latter is that neither the employers nor the employed can properly afford the outlay involved.

These considerations open up a most interesting prospect of further enquiry, but limits of space absolutely prevent any excursions in such a direction.

This Section will close with a short reference to the marked connection established by various portions of the National Insurance Act with matters involving a wider social outlook. A good example of this feature is found in the treatment of Excessive Sickness outlined in Section 63. It is laid down that any excess greater than 10 per-cent according to prescribed tables may be laid to the charge of the employer, local authority,

property owner, or water company, found to be responsible. Provision is made for the holding of official enquiries in cases where the parties concerned repudiate liability. While it is likely that the working of this Section may cause considerable friction, it is submitted that there is a distinctly interesting social consideration involved, and that is the reason for its being referred to in this place. Not only in this particular but throughout the Act there is clear evidence that very much more than mere benefit payment was contemplated. (See, for example, Section 60, which provides for periodical reports by the Insurance Committees as to the health of their districts and for the dissemination of information upon questions relating to physical well-being.) It is this fact which makes the National Insurance Scheme so interesting from a social as well as an actuarial, standpoint, and justifies the adoption of a distinctly broader method of treatment than the strictly professional aspects of the Act demand. And yet, may it not be said that the constantly enlarging domain of actuarial interests will include, sooner or later, the whole range of Social Insurance Schemes and their developments? It is submitted that even now this is true, and it must become yet more so for in the age-long battle against misery, and wretchedness, and poverty and disease, there is pre-eminent need for men of specialized training, of wide experience and of warm sympathy—a combination of qualities which it is surely the aim of all the members of this Institute to possess.

Thus the enquiry closes, and but one task remains. In conclusion the author would acknowledge with sincere appreciation the assistance received from the many persons and Societies, from whom information (mainly concerning foreign schemes of Social Insurance) has been sought. Amongst them he would mention Dr. Ernest Blaschke of Vienna, the various British Consuls abroad, certain of the foreign Ambassadors, Ministers, and Chambers of Commerce in London, the British Board of Trade, the Royal Statistical Society, the British Institute of Social Service, the Permanent Committee on Social Insurances of Paris, &c. Special thanks are also due to Mr. Joseph Burn for supplying information relative to alternative benefits under Section 13. To all these and to the various other gentlemen who have made suggestions, granted introductions, and given help, grateful acknowledgment is offered. Without such response very much of interest would have been omitted inevitably.

## ABSTRACT OF THE DISCUSSION.

Mr. R. C. FIPPARD said it would be admitted that, under a modest title, Mr. Simmonds had submitted to the Institute a fairly comprehensive account of the principles and methods of National Health Insurance, not only in this country but throughout the world.

Dealing first with the technical side of the subject he supposed the most important question for an actuary was that of valuation. He thought that this could only properly be considered in conjunction with the methods of calculating the contributions required and the initial reserve values. In making these calculations due account was taken of the varying conditions as to employment of the population entering into insurance, but the values thus calculated were made generally applicable and no account was, or had been, taken of the fact that persons engaged in the same or similar occupations would exhibit an inclination to become members of the same society. The effect must be that the same differences in the experience of Approved Societies would be found in the future as had been found in the old Friendly Societies' experience in the past, and the sums that were adequate or more than adequate as rates of premium and initial reserves for one society would prove insufficient for the maintenance of the same benefits in a society not so favourably constituted.

It seemed highly probable that the first triennial investigation into the affairs of the Approved Societies would result practically in a modification of the provisions of the Insurance Act, and that in place of the proposed scheme giving the same benefits to members of all societies, in exchange for a level and equal contribution, it would be found that—apart from the effects of differences in management—while all insured persons might continue to pay the same contributions, and would receive on the whole the same State subsidy, their benefits would vary to a greater or less degree. In effect, the result would be similar to the German scheme, under which, as Mr. Simmonds reminded them, certain trades contributed more than the normal contribution, the difference being that, in this country the extra cost would be borne by the insured person in the form of a reduced benefit, while in Germany the employer was required to pay an additional contribution in order that the workman might enjoy the normal benefit. Mr. Simmonds pointed out that the triennial valuation was to be made on a "prescribed basis." Reading Section 36 in conjunction with sub-section 4 of Section 63, it appeared at first sight that one basis would be prescribed for all societies. A wider interpretation could, however, probably be put upon the wording of those sections and they might be read as meaning that a basis of value would be prescribed, not for all societies, but for each society under consideration. Such a reading was certainly called for if it were agreed that Approved Societies would be found to have differing constitutions.

Closely related to this subject was the question of transfer

values. An insured person was permitted, within certain limits, to move freely from one Approved Society to another and, generally speaking, when he had become a member of the second society, a sum, called his transfer value, was credited to that society at the expense of the first society. Transfer value was defined in Section 81 as a sum representing the liability of the society in respect of a member. At a first glance this would appear to indicate that the sum transferred on account of any person would be his share in the funds of the society, but Sections 37 and 38 did not seem to uphold this contention. Section 38 provided for an adjustment of transfer value in respect of a member leaving a society in deficiency, indicating that some standard value was contemplated; and Section 37 provided for the division of an ascertained surplus in the form of additional benefits among insured persons who were members of the fortunate society. He suggested that this must be held to mean members of the society when the benefits became payable. This was by no means a small point, as the number of transfers between societies seemed likely to be very considerable. The question whether a removal from one society to another was frivolous or not, to which Mr. Simmonds looked for a check on this movement, would probably be very difficult to decide. In this connection it was interesting to notice the exact wording of the proviso to Section 31 which dealt with the power of a society to refuse the transfer values. It stated that the first society must prove that the insured person voluntarily ceased to be a member of that society without consent, and that that consent was not unreasonably withheld. This gave the Insurance Commissioners considerable discretionary power, and it remained to be seen how wide an interpretation would be given to the meaning of reasonable refusal on the part of a society. He thought the chief value of this section lay in the power it gave to a decadent society to prevent an accentuation of the decay by the removal of healthy members. A refusal, based on this ground would, he believed, be upheld. It might be remarked that such refusal did not of itself prevent a man withdrawing and joining a second society, but the chance of this happening was very small, as the new society, in such circumstances, would not be credited with the transfer value and would thus incur a direct loss.

He wished to make one other remark, before leaving this subject, in reference to the transfer of a Deposit Contributor to an Approved Society. He understood that it had very generally been believed that in such a case no reserve value was credited to the society. This was not correct, however. Where a reserve value was required in respect of a person entering into insurance before 15 July next, it would be provided whether he joined an Approved Society or the Post Office Fund, but if he adopted the latter course it would not be placed to his credit, unless he subsequently joined an Approved Society. He thought, therefore, that Mr. Simmonds' remarks on this subject required supplementing by the additional information that when such a person transferred to a society that society would be credited not only with the whole or part of the sum standing

to his personal credit in the Post Office Fund, but also with his initial reserve value.

The remaining point of a purely actuarial nature with which he proposed to deal was the provision made in Section 63 for reimbursing a Society or Insurance Committee for excessive expenditure incurred through negligence or default of a public authority or private person. The meaning of this section was quite clear and the standard to which reference was to be made was provided for, but he imagined it would be very difficult for a society to take full advantage of it. It would evidently not be sufficient to take action because heavy claims had been made. The incidence of those claims and the benefits paid to the unaffected members would have to be considered, so that before a claim that was to be successful was preferred, an investigation into the sickness experience would have to be made, and the time and expense involved would in many cases be prohibitive. He thought the most successful way of meeting the difficulty would probably be found in the adoption of a system of continuous investigation of each area in which the society carried on business. It had occurred to him that it might be possible to calculate a series of standard values, depending on the age distribution of the membership of a society in different areas, representing the total anticipated expenditure by the society in those areas or the expected rate of sickness; and a periodical comparison with this standard of the actual experience would immediately show if an investigation was likely to be practically useful. Such an investigation would then take the form of a mere examination of the claim papers and a claim could be made, with all confidence, without the necessity of the ordinary sickness experience investigation. This plan could clearly be modified to give as close or as rough a check as might be considered desirable, and it would probably be found that one standard could be adopted for all areas, unless minute sub-division were required. This system would also serve as a useful check on local administration of the claims on the society, in cases where local secretaries were entrusted with discretionary powers in this direction.

Dealing in somewhat greater detail with the Paper, the first point to which he wished to call attention, admittedly a trivial one, was the statement that a waiting period of six months was imposed in respect of sickness benefit. The actual period was 26 weeks. In the same paragraph Mr. Simmonds referred to the relief afforded to societies from compensated Industrial Accident or Disease. This relief was also enjoyed in cases of accident or disease for which damages might be claimed at Common Law. This was of importance, and in some cases a judicious use of this provision might obviate the necessity of an application under Section 63 to the financial advantage of the society.

The author expressed a fear that Section 11 might lead to a great multiplication of diseases scheduled under the Workmen's Compensation Act, and also that the later provisions of that section might prevent the prompt settlement of compensation claims.

He did not think this fear was well founded. Some extension of the scheduled diseases was quite possible, but this would probably have been effected without the intervention of the Insurance Act ; and, as regards the settlement of compensation claims, it might be pointed out that no power was given to the Commissioners, the Insurance Committees or the Societies to prevent such settlements, but they would be informed that an arrangement affecting the insurance funds was contemplated, and could place the Registrar in possession of any knowledge they might have of the case and thus assist him in coming to a decision.

Reference was made in the Paper to the definition of "Sickness" for claim purposes, and he quite agreed that this was a difficult problem. It must happen in many cases that a man might be injured sufficiently to prevent him from following his normal occupation and yet be perfectly capable, physically, of taking up some other occupation. The obvious instance was that of a mechanic who injured one hand, and was thus unable to continue his ordinary work for a time, but might very well take up clerical or outdoor work. It would be most unfair to disentitle such a man to sickness benefit, and a very good working rule seemed to be to pay this benefit in all cases where a man was rendered unfit to follow his usual occupation, and to give the case further consideration if and when Disablement Benefit were applied for. In most cases, after six months' illness one should be able to tell whether the man was likely to be able to resume his work, and if there was no immediate prospect of this taking place it was clearly time to consider whether he could take up some lighter occupation. At the same time, one might say that if a man was so severely injured as to be compelled to give up his ordinary work entirely he would most likely require at least six months to recover from his injuries, so that this length of time before re-consideration would hardly be too long. Such a rule was naturally liable to considerable modification, and must be considered only as a working basis.

Referring to the treatment of deficiencies, Mr. Simmonds mentioned the peculiarity of the method of enforcing payment of a levy by recourse to the employer. He quite agreed that this was at variance with the method adopted in the issue of contribution cards, but would point out that Section 38 did not seem to imply that this should be the usual method of collecting such levies, but indicated that it was a cheap and easy means of enforcing payment from reluctant members.

Coming now to the final section of the Paper dealing with social effects, two suggestions occurred to him, and he offered them for Mr. Simmonds' consideration. The first referred to the possibility of social legislation stimulating the growth of a social consciousness, which it was suggested would have a greater effect in checking malingering than a purely detective organization. Would not such a condition, however, be the best detective organization that could be secured ? A man with a well-developed social conscience would, he thought, be very perturbed at seeing his neighbour

partaking unduly of the benefits of the Act. The other suggestion had reference to the claims for consideration of a non-contributory system of insurance. Such a system would have to be financed by the proceeds of taxation, and, unless the leisured members of the community were directly charged with the burden, this taxation would ultimately fall on the shoulders of the industrial population. He believed it was accepted as an elementary truth of Economics that such indirect taxation fell more heavily on the persons ultimately paying than if levied on them directly, so that it was at least doubtful whether any relief would be derived by the beneficiaries if the alternative financial arrangements were adopted.

In conclusion he wished to refer to the intimate connection established between the National Insurance Act and other legislative enactments. In addition to the Workmen's Compensation Acts already mentioned it afforded opportunities, under the provisions of section 63, for increasing the usefulness of the Factory, Public Health and Housing Acts, and other measures directed to the improvement of social conditions. This was a good illustration of the far-reaching effects of the Act—the extent of which was perhaps hardly conceived by those responsible for placing it on the Statute Book.

Mr. F. J. CAMERON said he quite agreed with the author's remark as to the paucity of reliable data available on which to base the rates of contribution and benefit. It was obvious, however, that in a few years' time a vast quantity of most valuable statistics would be available.

The Act presented some problems which did not admit of solution. For example, they were accustomed to seeing every day in the public Press authoritative statements of exactly what part of the contributions would fall on the taxpayer, but this was quite incalculable. Some people said that all beneath the Income Tax limit contributed practically nothing to the State portion of the benefit, whereas other people stated that they contributed the greater portion. Personally he was inclined to take the latter view. The question of the amendment of the Act was an extremely interesting one, but, as the author had said, few concrete suggestions had yet been put forward. Everybody agreed that the Act must be amended, but very few people said how it was to be done. Among the fundamental amendments Mr. Simmonds mentioned the scheme put forward by the National Insurance Enquiry Association; but in regard to this it seemed that a National Health Insurance scheme which omitted direct specific provision for both medical and sickness benefits somewhat resembled "Hamlet" without the Prince of Denmark. In regard to all the fundamental amendments it seemed to him, looking at the question from the practical point of view, that, as both the great parties of the State were agreed on the main principles of the Act, it would be an academic question to consider the fundamental amendment of the Act at the present time, and that, as the author pointed out, it was more practical to discuss subsidiary amendments.

In regard to the Post Office deposit contributors, it appeared to him that the provision that three-sevenths of the amount at the credit of a depositor would be forfeited at his death, taken in conjunction with the provision whereby Insurance Committees had power to continue certain benefits to depositors even after their credits were exhausted, seemed to point to a slight insurance element in the scheme.\* He was very glad, however, to see that on 1 January 1915, the whole question of the Post Office contributors would be re-considered, and he was sure everyone hoped that some real insurance scheme for this class would then be introduced. The National Insurance Act provided for investigation being made in the case of excessive sickness, which was defined as being sickness 10 per-cent over the normal expectation of sickness. It seemed to him that that was a very valuable provision indeed, and he hoped it would be strictly carried out. The same remarks applied to the periodical reports to be made in regard to the health of the insured population. In this way the National Insurance Act might really be the means of improving the health of the nation.

Mr. J. BACON desired to point out that the whole scheme of the National Insurance Act was erected on a most unsatisfactory basis. In his opinion it was a huge financial pyramid resting on a very small apex of scientifically ascertained fact, and this fact emphasized very strongly the plea that the Institute should at once set on foot a Research Bureau, in order that actuaries of the future might possess proper equipment for dealing with the social and insurance problems that would come before them ; and that they might thus enlarge the somewhat narrow sphere to which they were at present confined. One of the most valuable features of the National Insurance Act was that it would accumulate a vast body of facts on which to base a solution of such future problems.

Turning to one or two of the points in connection with the Paper, he did not think they need be greatly alarmed at the fall in the rate of mortality which the author had said was likely to occur. He found, on comparing the cost of sickness, that taking the Manchester Unity experience based on Area 1 as against the Manchester Unity experience based on Area 2, the difference in the premium where benefits and contributions ceased at age 70 was only 4 per-cent for all ages from 16 to 40. That arose from the fact that the better mortality took effect mostly after the age of 70, whereas the contributions and benefits ceased at age 70. It must be borne in mind that the areas he had quoted were the best and worst in the Manchester Unity experience, representing respectively the agricultural and the manufacturing and mining areas. At age 16 the complete expectation for Area 1 was 48.921, and, for Area 2, 45.154, a difference of nearly four years ; but the difference in the

\* Mr. Cameron informs us that since the meeting he has ascertained, by enquiry of the Commissioners, that the three-sevenths of the amount standing to the credit of a deceased deposit contributor will remain in the Post Office Fund pending further legislation as to the Deposit Insurance scheme.—ED. J.I.A.

contributions at 3 per-cent interest was only 4 per-cent. Comparing the values of  $q_x$  given on page 124 with those of the Manchester Unity experience, they fell between Area 1 and Area 2 with a large margin on either side, so that there was room for a great deal of improvement in the death rate before the contributions required for the sickness benefits would be greatly affected. Far more important, he thought, was the question of the future rate of sickness. There seemed to be a general agreement that the rate of sickness was going to rise, and there were one or two considerations that had not been referred to in the Paper in connection with that point. On the one hand there was, as Mr. Simmonds had pointed out, the belief that the preventive measures that were being taken and the more adequate care of the national health would cause an ultimate decline in sickness, and there was the business consideration also that the doctors would now have every reason to effect a cure as soon as possible. On the other hand, there was no doubt that the offices administering the sickness benefit would feel that somehow or other if there was an excessive drain on their society, a great many other societies would be similarly affected, and that in the long run it was the Government that had to pay. Again, the claimant might be insured for the State section with one society and he might also be further insured in another society ; and he could imagine there would be a great deal of competition between those two societies that the one should not be less generous than the other in the treatment of the insured person. That particularly applied where the insured person was insured for State purposes with an Approved Society attached to a Collecting Friendly Society or an Industrial Insurance Company. Where an industrial agent was administering the sickness benefit under the Act, the very greatest pressure would be brought to bear upon his superiors to grant the most generous treatment possible within the terms of the Act in order that the industrial premium debit might not be upset. Further, he desired to point out that an unforeseen element had been introduced by the practical failure of Section 72. It was anticipated when the Act passed through the House that the vast majority of the members of friendly societies would take the State benefits in lieu of their existing voluntary benefits. As a matter of fact the great majority of friendly society members were taking both benefits. There was, therefore, a much higher scale of sick pay, and judging from past experience that meant a much longer duration of sickness. When a man in ordinary health who was earning perhaps 25s. or 30s. a week was receiving 10s. a week sick benefit, he was not likely to keep on his Club so long as when he was drawing 10s. a week from the State and an additional 10s. a week from his Club.

He wished to refer to the point raised by the author as to payment during partial disablement. Judging from experience it was the most dangerous thing possible to provide sick pay whilst a man was allowed to do any kind of work whatever. When a man must stay within doors, or at any rate refrain entirely from work,

the society knew at once whether he was ill or whether he was not ill ; but when he was allowed to do a little light employment it was by no means easy to determine whether he could fully resume work or not. With regard to the question of female sickness, all the experience available was totally opposed to the suggestion put forward by the author that female sickness was lighter than male sickness. Comparing the rates given in Sutton's Friendly Society Experience for Males and Females, the female sickness was greater up to age 53 than the male for both the periods under two years and over two years ; and as a matter of fact the difference up to age 45 for the first two years was equivalent to about one-third of a week's sick pay. That was supplemented by certain experience he had recently had before him, in which in districts where the benefits were in all respects similar, except that in most cases the women received a lower rate of sick pay than the men, the female experience had almost invariably been heavier than the male experience. It must also be remembered that such sickness experience as was available was mostly drawn from agricultural districts, whereas under the Act the women who would be mostly insured would be taken from industrial and textile areas. So that he thought a much greater difference was to be expected in the future between the male and female rate even than had been experienced in the past.

Considering that the Paper was presented to a body of actuaries, he was somewhat surprised that the author had practically omitted all reference to the most important section of the Act from the actuarial point of view : he referred to Section 72. A great body of work had been thrust upon actuaries in connection with that section in preparing valuations and schemes for submission to the Registrar under the Act. It was well known that while the Bill was before the House it was stated that many millions of reserves would be released and would be available for placing societies in a state of solvency, or for extra benefits. He had recently had through his hands the experience of something like two thousand lodges of different Orders, and he found as a matter of fact in a great many districts, particularly in colliery districts and in districts where there was a great number of iron works, where wages were high, that in lodge after lodge there were no reduced members ; and, taking societies in industrial and mining centres as a whole, probably the number of reduced members was not on the average more than 10 per-cent. In agricultural lodges he found it was rather different, but he had hardly come across a case where the number of members reducing had been more than half of the whole. So that as far as he could see only a small proportion of the amounts anticipated would be released under the Act. He desired to say at that point that, considering the work which had been thrust upon actuaries and the important part they would play under the Act, he was rather surprised, in view of the many concessions that had been obtained by another professional body, that the Institute was unable to obtain a more satisfactory definition of the term "Actuary" than appeared in the Act. He thought it was too early to appreciate the

social effects of the Act, but one thing was quite clear, namely, that it had sounded the death knell of friendly societies as they had known them. Already some hundreds of small separate societies had either dissolved or had joined the affiliated Orders, and it was pretty clear, he thought, that in future the administration of sickness benefits would be in the hands either of large centralized or affiliated societies, or of great approved societies in connection with collecting friendly societies or industrial insurance companies. Whether that was a step on the way to complete State control or not he could not say, but it was quite clear that the voluntary spirit would go out of the administration of friendly society benefits. There was a strong probability that, considering the amount of work that was to be thrust upon the secretaries of approved societies and the complex records they had to keep, either their capacity or their time would fail, and, whether they liked it or not, they would be driven to amalgamate with the big societies that could provide skilled and adequately-paid organization.

There was one other very small point which had been raised by Mr. Simmonds to which he desired to refer. The author spoke of the grouping of agricultural and industrial lodges in the same area, and thought that might be bad for the agricultural lodges. Personally he was not at all sure that that was so. He had found in many cases that, where a very light rate of sickness was expected, a very heavy rate of sickness was obtained in agricultural districts, mainly through lax administration. With the better administration which would probably arise from the more competent work that could be given by skilled artisans in industrial areas, the rate of sickness was likely to go down in such amalgamated lodges. One quite unforeseen social effect of the legislation had been, he believed, the offer of one large approved society to provide free actuarial advice. There was no Disciplinary Committee attached to the Institute, but he desired to point out to the President that that was a matter the Council might very well take under their consideration, because, although no doubt it was good for the societies concerned, it was exceedingly bad for those members who practised as consulting actuaries and was detrimental to the interests of the profession.

Mr. A. W. TARN said that the author had alluded to a considerable extent to the German system of insurance, upon which the British Act had been mainly based. In the last few days a Parliamentary Paper had been issued by the Government, entitled "Medical Benefit under the German Sickness Insurance Legislation," which contained a considerable amount of information as to the working of the existing Acts and also as to the new consolidating Act which was coming into force in 1914. The Paper to which he referred dealt with the organization of sickness insurance and the scope of medical benefit. The duration of the medical benefit was not the same in Germany as in England: originally it was for thirteen weeks and now it was for twenty-six weeks. The Paper also dealt with the methods of remuneration, the cost of medical

benefit, the German pharmacy system, and the expenditure of sickness funds on medical benefit. The medical benefits differed somewhat from those in this country. Although they were not of so long duration they apparently included a great deal more than was given in England. For instance, medical treatment often comprised X-ray applications, electrical treatment, mechanical exercises and the like. The treatment of teeth was usually regarded as falling within the scope of medical benefit, but artificial teeth were seldom provided, though some funds made a contribution towards their cost. The liability of the sickness funds in regard to appliances, on the other hand, was defined with sufficient clearness. The appliances which must be supplied were spectacles, trusses and similar appliances, while a number of additional benefits could be obtained if they were paid for. Hospital treatment in Germany was also different from what it was in this country. The Paper to which he was referring gave rather fuller details of the family insurance to which the author had alluded very briefly. The insurance of dependants was amongst the things which the law allowed, and a definition of dependants was also given. Apparently societies in various parts of Germany had different rules as to who might be regarded as dependants. For instance, in Bremen dependants were defined as "Wife or husband, and children, including adopted children"; in Dresden as "Wife or husband, and children and step-children under 15 years"; in Leipzig, "Wife or husband, parents, grandparents, parents-in-law and children under 16 years"; in Stuttgart, "Wife and children under 14 years"; in Remscheid, "Wife or husband or relative in charge of household, children and foster children, parents and parents-in-law"; and in the case of unmarried members having their own households, the mother or sister in charge"; and in Essen (Krupp), "Wife, children and other relatives of member by blood or marriage resident with him and wholly or mainly dependent upon his earnings, but not a house-keeper or a servant."

Mr. R. G. MAUDLING said that Section 72 of the Act had not received sufficient attention in the paper. Mr. Bacon had pointed out that the idea that a large amount of reserves would be released was erroneous. He might add to Mr. Bacon's remarks by saying that it was becoming rather difficult to allow the full fourpenny deduction which many members had been led to expect. The question of released reserves was very much bound up with the age distribution of the members, because if a society was valued on a normal experience and the value of the liability removed was estimated, it would be found—where the members were young—impossible to give a deduction of  $2\frac{3}{4}d.$  or  $2\frac{1}{2}d.$  in respect of the sickness benefits; and as the average cost of management of a society was something like  $\frac{1}{2}d.$ , and the cost of medical benefit generally  $1d.$ , it became difficult to provide for a full reduction. That had a great effect on the released reserves, as the members who were released were generally young. His own experience fully confirmed Mr. Bacon's remarks as to released reserves. On the

other hand it was important to note that a fair number of members who had previously declined to reduce now wished to do so. That raised another difficult point for the settlement of the Registrar. The word "continuing" which had produced all the trouble, was inserted subsequently to the introduction of the Bill in the Commons, and it was to be regretted that the alteration was made.

He desired to make a few remarks on the author's statement, with regard to the question of the three days' sickness, that theoretically, some relief might be obtained therefrom. Subsequently, however, he said : "The benefit commences upon the expiration of the first three days of sickness, so that a fair amount of relief is obtained not merely from the intrinsic value of the numerous short illnesses which last for less than four days, but also, probably, because the exclusion of even three days is some deterrent to a malingerer, who is not able so easily to simulate a genuine illness of moderate or great duration as he is to pretend to those trivial little troubles which involve only a short absence from work and can be imitated fairly safely." He could not reconcile those two statements. It seemed to him, with regard to women's sickness in particular, that probably the first three days would provide a substantial margin. Mr. Bacon had referred to the question of sickness of women generally ; and in this connection the Author, in referring to Mr. Dawson's book, might also have extracted a reference to the explanation given by the Imperial Insurance Office in regard to the peculiar experience which had apparently been noticed, namely, that the sickness was greater amongst men than amongst women. The Imperial Insurance Office pointed out that that could be explained by the fact that the women were generally younger, and were not engaged in such heavy occupations as men. As the experience was totally opposed to any experience he had seen of women's sickness, he was afraid it was a slender case to rest a conclusion of any kind upon. Another point was that sickness insurance in Germany covered the first thirteen weeks of compensated accident claims, and that would fully account for the heavy proportion of claims. Further, the duration of sickness in women was longer than that in men, a fact which also pointed to the inclusion of a large proportion of compensated accidents amongst the claims.

Mr. Bacon had stated that if people were allowed to work while they received sick benefit it had a remarkable effect on the experience. That, he thought, was to be expected. He had in mind a large society which allowed under certain sections excess sickness, and the experience in respect of the reduced pay amounted to about 170 per-cent of the expected. That was a fair example of the effect it might have upon societies and the restriction of sickness benefit to total incapacity seemed to be the only way in which a National Insurance Act could work successfully. Mr. Fippard made some remarks regarding the basis of the scheme, and seemed to take rather a gloomy view of it. The first three years might produce rather bad results, but as remedial measures were to be forced down people's throats by a Government Department it would probably

lead to a considerable improvement in the future sickness experience. It would also add considerably to the recognition of the value of actuarial advice, by the members of friendly societies generally. The Author had suggested one or two amendments of the Act. He did not think at the present early stage it was possible to make any reasonable suggestions for amendment. He did not consider that actuaries need concern themselves with amendments of a minor nature, and radical amendments were a question for the future. In any event Section 72 would come up again, because once a scheme had been provided under Section 72 it was necessary to be very careful in the amendment of the Act that they did not again upset the finances of friendly societies. It seemed to him that actuaries should see how the Act worked, and when they had an experience of its working then probably they would have some idea of the proper way to amend it.

Mr. V. A. BURROWS said that there were one or two points to which he would like to refer. In dealing with the compensated accidents margin, Mr. Simmonds said—"The problem was solved by assuming the relief to be at least 10 per-cent of the weighted rates deduced above," i.e., the Manchester Unity whole society rates. That was, however, not so; the relief was assumed to be 10 per-cent of the pure whole society rates. He also wished to refer to the question of female sickness. The author had not definitely stated that female sickness might be expected to be lighter than male sickness, but his reference to the German and Austrian results seemed to imply that that was his opinion. He (Mr. Burrows) had extracted a few particulars relating to the question. He took some figures relating to a very large affiliated Order doing friendly society work in England, Scotland, Ireland and Wales. He first selected the female lodges and then selected contiguous male lodges granting a similar rate of benefit, i.e., benefit reducing at the end of the same period. He compared the experience of these male and female lodges on the basis of the Manchester Unity experience; one interesting fact that emerged at once from the selection of those lodges was that the female lodges were nearly all (adopting the Manchester Unity notation) in Area 1. That threw interesting light on Sutton's experience, because evidently the districts in which female lodges had succeeded had been in Area 1, and consequently, in the main, Sutton's experience did not deal with the same class of lives as those which the Insurance Act would cover. It was rural rather than industrial.

Another point was that the female lodges were, on the whole, very much younger than the male lodges, and consequently light rates of sickness would be anticipated. When he looked into the figures, however, some very remarkable results were shown. He took, first of all, some lodges granting full pay during the first twelve months and half-pay thereafter; and he found that in ten lodges, chosen because of the number of members in them—the largest lodges being selected for comparison—there was an excess on full pay for sickness over the expected of 29 per-cent. The half-pay

was practically normal, there being a small saving of 2 per-cent. On the other hand, when he looked at the male experience in contiguous lodges, he found that on the full pay there was a gain of 13 per-cent as against a loss of 29 per-cent; and in half-pay there was a gain of 17 per-cent as against a small gain of 2 per-cent. When it was remembered that female lodges were, on the whole, very much younger and included one or two very young lodges, and also that the lodges were situated in Area 1, those facts were really remarkable. He then looked into some further results taken from the same affiliated order in lodges where full pay was granted for six months only and half-pay thereafter; and he found that the results were quite confirmed; in fact, the excess in the cost of full pay was even more aggravated. Then he took an affiliated order consisting entirely of females. The order included 16 lodges, one of which he excluded for reasons he would give in a moment. In full pay there was an excess shown of 32 per-cent, although against this there was a saving on half-pay of 40 per-cent. The 32 per-cent excess in the full pay confirmed what he had said in regard to other experience. One lodge was excluded because the benefits had been altered during the quinquennium he was investigating. The alteration consisted of a reduction in benefits, because the experience was so exceptionally bad. In the previous quinquennium, this lodge, which was the largest of them all, showed an excess of 47 per-cent. He thought it could be taken for granted that female sickness was not by any means lighter than male sickness, and in regard to full pay it was very much heavier, although in subsequent pay it might not be very different from male sickness.

In answer to a question put by a member, Mr. Burrows stated that the sick pay in all the cases to which he referred commenced from the first day, as was almost invariably the case.

Another point to which he wished to refer was the probable trend of sickness experience. As had already been mentioned, Section 72 was altered during the passage of the Bill through Parliament by the introduction of the word "continuing." As a result members of friendly societies had taken the option of continuing their existing contracts and contributing in addition for the State benefits. He was recently dealing with a mining district of an affiliated order, in which the figures were very remarkable. It consisted of 20 lodges, the total membership of which was 1,734: and the number of members reducing out of the 1,734 was 2. That result was all the more important when it was borne in mind that the expected sick pay during the quinquennium was £6,012 and the actual sick pay £10,512, *i.e.*, an excess of 75 per cent. Hazardous occupations naturally meant excessive sickness, quite apart from the question of compensated accidents. Hazardous occupations might also be taken to mean higher wages, which meant that the men could afford to pay for the double benefit. Consequently the result was obtained that just where the members ought to have reduced their existing contracts they had elected to take

double benefits, so that excessive insurance was obtained where it would be most disastrous. The district to which he was referring was certainly an exceptional one, but he had taken out the results in a great number of districts, covering over a hundred thousand lives, and found the average number of members reducing was 18 per-cent only.

He wished to comment on the omission of any reference, apart from an incidental one, to Section 72. The author had stated as his reason for the omission that Mr. Watson had said the last word on the subject. He (the speaker) thought that a better reason would have been that to deal with Section 72 would require a separate Paper, inasmuch as Section 72 opened up (to borrow one of the author's phrases) an "illimitable vista of discursive opportunity." He would like to have known what Mr. Simmonds would do as regards special classes in preparing schemes under Section 72 —such classes as women, aliens, soldiers and sailors, seamen, mercantile marines, or members over 65.

Mr JOSEPH BURN in concluding the discussion desired to offer his heartiest congratulations to the author on his very useful Paper. He thought Mr. Simmonds was very bold in undertaking to deal with such a big subject, and the modesty with which he had attacked it was admirable. The author had, he thought, seen the dangers and the difficulties which many of them had failed to see when the Act was passed, and he thought the author was to be commended, and not in any way found fault with, for having refrained in the majority of cases from giving any opinion on the more doubtful matters which arose under the Act.

He quite agreed with Mr. Bacon that the future sickness rates were of very much more importance than the future death rates. With regard to the rates of sickness which were likely to be experienced among females, he had listened with very great attention to the remarks of the different speakers; and the statistics which were given by Mr. Burrows were, in his opinion, most interesting. It had been his somewhat painful duty to have to give an opinion as to the probable rates of sickness amongst the female members of a very large approved society, and he had come to the conclusion that the available information was absolutely useless for the purpose. On examination he had found that in different societies the attendant circumstances were widely different. For instance, it was found in many of the female societies that a very large number of the members were unemployed, and there was good reason to believe that the rate of sickness amongst unemployed female members would be entirely different from that experienced amongst the majority of the employed persons under the Act. Then, again, the experience with regard to the length of sickness was entirely different from what he was in the habit of regarding as the normal experience of males. He entirely agreed with what seemed to be the general opinion of the different members who had discussed the Paper, that the probable sickness experience of females in the future was an unknown quantity, and that in this

connection actuaries would have to do as they had been told to do with so many things lately—Wait and see.

He was particularly sorry to hear one speaker remark that it was his candid opinion that the National Insurance Act sounded the death-knell of the majority of friendly societies, as they now knew them. He certainly hoped that that would not be the case. So far as his experience had gone, he believed friendly societies had served a very useful purpose. They had had their troubles, and he certainly would not be right in saying that in his own experience he had found very many of them solvent. He did not say that in any flippant way, because he had been pleased to notice that most of them were getting, if he might use the expression, more nearly solvent, and there was a growing tendency amongst them to take proper means of putting their finances in order. He thought it would be a very sad thing if the National Insurance Act resulted in retarding the general friendly society movement. He quite sympathized with what had been said as to the difficulties of local secretaries, and so on. It certainly seemed that the work which was necessitated under the Act was of such a complicated and difficult nature that it was almost impossible for small societies to undertake it, but he still hoped that this difficulty would be overcome, as he thought it would be conceded that, generally speaking, such societies served a very useful purpose.

Turning more particularly to the Paper, the author said in dealing with medical benefit: "The medical and sanatorium benefits (both of which extend throughout life) are treated upon the basis of a level cost of 6s. and 1s. 3d. per annum respectively." He wished to point out that this was on the assumption that the contribution ceased at age 70. The same thing would be found in the case of the cost of administration which the author had referred to as 4s. Mr. Simmonds also referred to the maternity benefit. On reading the Paper he thought, at first, that the manner in which the author had expressed the paragraphs (a) and (b) of the maternity benefit was incorrect, but he found on looking at it again that such was not the case. But although, in his opinion, it was quite correct he thought the wording might be made a little plainer. He would not attempt at the present time to suggest the form that the alteration should take, but desired to point out a few things in reference to the maternity benefit. In the first place, the maternity benefit, as he understood it, would be granted to a woman who was an employed person and whose husband was not an employed person. If her husband was not an employed person under the Act, then on the birth of a child the wife would receive maternity benefit, and sickness benefit in addition. If, on the other hand, her husband was an insured person, then the maternity benefit would be paid from his society, and the woman would receive no maternity benefit, but would receive sickness benefit from her own society. It was true from one point of view, as the author had pointed out that she did in both cases receive maternity benefit and sickness benefit, but it was particularly

to be noticed that where the husband was an employed person the maternity benefit was paid on his account. A case likely to result in some hardship was that of a husband, in a very small way of business on his own account, who decided to become a voluntary contributor. In that case, the wife, if she were an employed contributor, would not be entitled to the maternity benefit, as this would come through her husband's society. The Act, however, provided that a voluntary contributor was not entitled to claim for maternity benefit until he had paid 52 weeks' contributions, and this provision was inserted in the Act evidently as a necessary precaution; but, in such a case as this, a child might be born only a few weeks subsequently to the husband becoming a voluntary contributor, and the result would be that the husband, having paid only two or three contributions for his own insurance, by that means deprived his wife of the maternity benefit to which she would otherwise have been entitled. In the case of illegitimate children, the mother received only the maternity benefit.

The author referred to deposit insurance, and some of the speakers had spoken on the difficulty of transfers from the Post Office to an Approved Society. The difficulty seemed to arise principally from the curious wording of the Act—see Section 43 (2) (a) and (b).

He desired to touch on one further point, namely, the author's statement with regard to alternative schemes. As he, personally, was to a considerable extent responsible for the particular scheme referred to he would like to be allowed to make a few remarks in reference to it. The scheme which he proposed was one intended to be most suitable for domestic servants. The argument was that in most cases a domestic servant would be provided for by her mistress for about a fortnight, at the very least. If at the end of that time the mistress thought the servant should return home, the latter would be entitled, it might be supposed, to a month's salary. From that point of view it seemed that a larger benefit would be most advantageous to a domestic servant at the end of about six weeks. Accordingly, in his scheme, which was submitted to the Government actuaries and passed as being the exact equivalent, he had proposed that a domestic servant could, if she so wished, give up a certain portion of the 7s. 6d. benefit to which she was entitled for the first six weeks, and in return receive, if she were ill for so long, 13s. a week instead of 7s. 6d. a week for the next 20 weeks, resuming at the end of that period the normal rate of benefit. This scheme had certain characteristics which were the outcome of very careful consideration. For instance, he had been careful not to interfere with the normal reserves, and to provide for the payment of some sickness benefit from the first day on which sickness benefit became due, as by that means the evident difficulty was avoided which would be experienced if the servant gave up the whole of her sickness payment for a certain time but remained ill for a longer period. The result would be that in certain claims for sickness it would be necessary to make

investigations into past records as to whether she had or had not been ill. The scheme, as the author had stated, was objected to mainly, as he understood, on certain legal grounds. Probably most of those present were aware that the Government had since issued a scheme of their own, to be obtained in Form A.S. 26, which, as it appeared to him, was a distinct copy of the scheme he had proposed, but without its evident advantages. The plan proposed that all benefits should be given up until the eleventh day after sickness. As a result, the difficulty was introduced which he had already mentioned. Having given up all benefits until the eleventh day, the servant was then to be entitled to 10s. instead of 7s. 6d. a week for some period dating from the eleventh day, varying according to her age from eleven weeks to four weeks, and then after that she would receive 7s. 6d. a week for the remainder of the twenty-six weeks, and after that return to the 5s. So far, he had been unable to find any advantage which would be derived from the adoption of such a scheme, and he failed to find the legal difficulties which were present in his own scheme and not present in the Government's. Moreover, he was still anxious to discover some society bold enough to adopt it, because, having considered it very carefully, it would, so far as he could judge, be absolutely impracticable for any large society.

The PRESIDENT, in proposing a hearty vote of thanks to the author for his able, useful and interesting Paper, said it was a compliment to the author that such a large audience should have attended the meeting at which his Paper was read. As he (the President) had recently had an opportunity of expressing his own views on health insurance he would not say much on the present occasion. The National Insurance Act was a forcible example of the trend of modern legislation ; but whether that class of legislation was for the uplifting of the people at large, time alone would show. Some maintained that such legislation partook of the nature of doles out of the public purse, which undermined self-reliance and induced indolence, and thus was a sign of decay and of effete legislation ; on the other hand, some contended that such legislation was for the amelioration of the lot of the people at large ; and it remained to be proved which of those contentions was right. Actuaries, however, could at any rate, do one thing : they could, as opportunity served, do their utmost to help administer the Act on solvent and honest lines, and aid in steering it through the very difficult waters through which it undoubtedly had to pass.

The resolution of thanks was then put, and carried with acclamation.

Mr. R. C. SIMMONDS, in reply, first of all thanked the members very heartily for the kind reception which they had accorded to the Paper. In answer to Mr. Fippard's remarks he quite agreed that there was a very distinct probability of a certain modification of benefits as time went on, and in that case one naturally thought of the German example where a definite effort had been made to

differentiate according to occupational risk, etc. With regard to the question of excessive sickness, he thought the chief difficulties which would arise would be the trouble in getting societies to proceed and the expenses which would follow such action. If the old feature, for example, of honorary members should be preserved to any great extent he thought it conceivable that employers, landlords, and so on, might join from purely benevolent reasons, and, accordingly, the societies might hesitate to take proceedings for the prevention of excessive sickness owing to the risk of offending them. He thought that Mr. Fippard rather misunderstood his allusion to the questions of industrial diseases and of workmen's compensation settlements. He did not suggest that he had any fear of the progressive increase of the schedule of industrial disease. If reference were made to the Paper it would be found that he merely gave what was rather an interesting speculation, without expressing any opinion upon the point. As a matter of fact, he said, later on, that he saw no reason why any extreme tendency of that sort should become manifest. With regard to workmen's compensation claims, he desired to mention that he had the opportunity recently of discussing the question with the accident insurance manager of a very large Company, and that this gentleman was careful to point out that the Commissioners, the Societies and the other National Insurance Authorities certainly had no power to prevent a settlement. The point which he wished to make in the Paper had relation to the fact that the Commissioners were given a certain amount of information about these claims, and that feature suggested that the thin end of the wedge was being inserted. In fact, from information which came before him quite recently, he felt justified in suggesting that as time went on a certain tendency towards dealing with those settlements might become apparent. He referred to the question of aliens and persons over 65. As the Act stood, he believed that it was clear that societies might offer any benefits whatsoever. They could offer, on the one hand, absolutely unattractive benefits; or, on the contrary, they might offer absolutely impossible benefits. He understood, however, that the Commissioners, in issuing their own suggested table for the use of societies in such cases, had stated that unless that particular table were adopted, it would be necessary for the alternative to be submitted and approved. If he were rightly informed, that certainly indicated that the Commissioners were attempting to control a matter on which they had no absolutely clear direction according to the Act, and, that being so, he wished to make the same point regarding workmen's compensation settlements. He expressed no opinion on the question of the non-contributory schemes of social insurance—they were mentioned merely because a large body of opinion in this country supported them—but he thought that the actuarial profession generally would not consider them desirable. With regard to Mr. Cameron's remarks, he wished to point out that in the case of deposit contributors there was now no right of sharing in the remaining balance left by deceased

members. The original Bill did provide for such a benefit of survivorship, but that was subsequently altered.

Referring to what Mr. Bacon had said with respect to the statistical basis of the Act, he quite agreed that the national records were very unsatisfactory, but at the same time he hardly thought that it was the place of the Institute to undertake the collection of such figures. It seemed to him that a national concern like that under consideration should be undertaken nationally, with, of course, the expert advice and assistance which the Institute could give, and which it would, no doubt, be willing to render. The question of mortality and sickness rates had been dealt with quite fully, and, therefore, he did not propose to say much about it, except that, undoubtedly, from a social point of view, there was a distinct tendency to keep up the apparent rates by extending the scope of treatment. He had referred to that aspect in discussing the case of Germany, where certainly there had not been the anticipated fall of rates, largely owing to the adoption of wider schemes of relief. With regard to the question of female sickness, which had given rise to much interesting discussion, he wished to say, in the first place, that there was no intentional suggestion in his remarks. He had an absolutely open mind on the question, and he did not quite see how the paragraph in the Paper could be held to mean that he thought that females would have lighter sickness. He put in the small reference to Germany and Austria simply because he felt that it might possibly have some slight indicative bearing on the point. As the question had been raised so very fully, he desired to say that one important point which speakers seemed to have omitted was that maternity sickness did not have full operation under the Act, and that should have a decided influence upon the future rates in the case of females. Not only so, but he thought with Mr. Burn that in the past the female membership of friendly societies had been largely composed of unemployed persons; and as now only employed females, as a rule, would be included there was reason to think that an entirely different experience would be obtained.

With regard to the question of Section 72 he confessed that he expected the criticisms which had been raised. Nevertheless, he did not think that any useful purpose would have been served by his attempting to deal with the section more fully; partly because Mr. Watson had dealt with it at great length, and partly, also, because he thought that it was a question in which experience would have very much to teach. At the same time, he thought that it would have been distinctly interesting if those speakers who had raised the question had given the Institute the benefit of their views upon the points at issue. With respect to the question of grouping, Mr. Bacon referred to a little statement which he (the author) had made regarding an agricultural and an industrial society. His point there was merely illustrative. He simply used those terms at random to indicate the possibility that one society with a light experience would find its association with another society having a heavy experience rather inconvenient.

In conclusion, he desired to refer to two or three points raised by Mr. Burn, the first of which dealt with the medical and sanatorium benefits. He quite appreciated that there had been a commutation of those allowances, but being obliged to cut out very much, in order to reduce the Paper to some practicable proportions, he omitted any reference to the point. He agreed, on re-reading his remarks in the matter of the maternity benefit, that the wording was not entirely clear, and he would endeavour to modify it. The question of alternative benefits had hardly received the full consideration which might have been expected : in fact, Mr. Burn was the only speaker who had referred to the subject. Personally, he had intended to bring up himself the question of the new circular which the Government had just issued. It came into his hands too recently for him to mention it in the Paper. As Mr. Burn had dealt with the matter so fully, the only thing he would say was that he quite agreed with Mr. Burn's remarks. In conclusion, he again thanked the members for the courteous hearing which they had given to him.

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*"House Purchase" Companies : The "Bond Investment" sections of the 1909 Act and some actuarial features of the business returned thereunder. By CHARLES HUGH MALTBY, F.I.A., of the North British and Mercantile Insurance Company.*

[Read before the Institute, 24 February 1913.]

In putting forward the present Paper I am somewhat dubious as to whether it is not almost necessary to apologise for bringing before the notice of the Institute a subject which forms but one of the many minor branches of our Science, and which, moreover, has had a certain amount of odium—much of which is undeserved—thrown upon it from time to time. So far, however, as the Institute is concerned, it is practically untrodden ground, and the consideration, therefore, of some of the new problems suggested by it may not be without interest.

The valuations of the Companies carrying on the business in question have also now to be made by a qualified Actuary, and I hope, therefore, that the present Paper may save a certain amount of the preliminary investigation which takes up so much valuable time when a new field of inquiry is first approached. The consideration of the subject by the Institute, too, cannot, I think, but have a healthy effect upon the business generally.

It must be remembered that the class of business investigated is of comparatively recent growth, and that many of its features have been adopted with little or no reference to the actuarial principles involved. Many of the Companies now engaged therein have as yet hardly shaken down into their permanent form, so that the whole business is at the present moment to some extent in a state of flux, from which it is very difficult to foresee either the lines upon, or the extent to, which it will develop. As time goes on, however, it is only reasonable to expect that the publicity enforced by recent legislation, and the necessity for seeking skilled advice, will bear fruit in its growth on sound and healthy lines.

The Paper falls naturally into three sections, which, notwithstanding the title I have chosen, have been arranged in the following order—

- I. General description of the business covered by the sections of the Act in question, and the lines on which it is at present conducted.
- II. Some of the actuarial features of the business in question.
- III. The "Bond Investment" sections of the 1909 Act.

#### I.—GENERAL DESCRIPTION OF THE BUSINESS AND THE LINES UPON WHICH IT IS CONDUCTED.

The Companies in question were first brought prominently under notice by the appointment in 1905 of a Departmental Committee, which included one of our most eminent members, "to enquire as to the operations of Companies (not being Life Assurance Companies) which collect periodical payments from "the industrial classes in return for benefits promised in the "future." The Report of this Committee was reprinted in Vol. xl., p. 99, of the *Journal*, and on reference to this, it will be seen that, to quote the words of the Committee, "The "Companies to which this enquiry has been directed may be "described as Companies which issue bonds or certificates to "members of the public by which the Company, in return for "fixed monthly payments payable over a fixed period of years, "contracts to pay the subscriber a lump sum at the end of the "period, and to give him certain incidental advantages, including, "in many cases, the right to an advance on certain conditions."

At the date of the investigation the Companies in question fell into two distinct classes, which, for the sake of convenience,

are here placed in the reverse order to that in which they were treated in the Committee's report.

#### A. *Bond Investment Companies properly so-called.*

"Companies which in return for the monthly subscription "contract to pay the subscriber at the end of a fixed period, "usually 10 or 12 years, a lump sum, which is in excess, and "sometimes considerably in excess, of the amount of the sub- "scriptions, together with the compound interest calculated at "3 or even at 4 or 5 per-cent."

The house purchase element was usually absent from the contracts of these Companies, with which the Report, and the evidence submitted to the Committee, mainly dealt.

These Companies have now practically, if not entirely, disappeared; their temporary success being due partly to the high rate of interest promised, and partly to the ballot system whereby drawn bonds or coupons were redeemed in cash, the latter feature introducing a speculative element—highly attractive to some natures—into their transactions.

#### B. *"House Purchase," or "Property Investment," Companies.*

"Companies which in return for the monthly subscription "contract to pay the subscriber at the end of the period—usually "30 years—a lump sum equivalent to the amount of the sub- "scriptions, together with compound interest at a low rate, "varying from  $1\frac{3}{4}$  per-cent to  $2\frac{1}{2}$  per-cent, together with, in some "cases, bonuses periodically allotted out of the annual profits of "the Company, as ascertained by an actuarial valuation.

"These Companies put forward as a prominent feature of "their business the advancement to bondholders of money at "interest for the purpose of house purchase, the loan being made "on the security of the property purchased, with the partly paid "bond or certificate as collateral security."

#### A. BOND INVESTMENT COMPANIES.

From the evidence placed before the Departmental Committee, it appears that the first Bond Investment Company was floated in Liverpool about 1899. This was followed in quick succession by many others between 1900 and 1905, but no means exists of tracing the exact number founded, or projected. Even if they were all registered under the Companies Act—which is somewhat doubtful—neither the name of the Company, nor

the memorandum of association, always revealed the true nature of the concern ; as powers were commonly taken to transact many classes of business besides Bond Investment, which was often only incidentally mentioned. Moreover, even if a concern is identified in the Registers as a Bond Investment Company, there is no way of finding out whether or no any business was actually transacted.

As already indicated, it is now unnecessary to say very much about this class of Company, for although they undoubtedly attracted a relatively large amount of business from the date of their foundation up to 1905, the publicity given by the Press to their methods and the Report of the Departmental Committee, dealt them such a severe blow that at the present time their operations are only a matter of history. No doubt, too, their disappearance was hastened by the fear of legislation, which culminated in the 1909 Act.

Anyone interested in the subject will find a very full account of their methods, and of the working of the ballot system, in the Report of the Departmental Committee, the evidence laid before this body, and also in a Paper read by Mr. Harold Dougherty, A.I.A., before the Insurance Institute of Newcastle-on-Tyne, in April, 1911, published in Vol. xiv of the Journal of the *Chartered Insurance Institute*. From the evidence laid before the Committee it is evident that the whole system was hopelessly rotten, and founded on an inherently unsound basis. This was also borne out by the results of two Valuations made by Mr. Dougherty, in consultation with Mr. T. G. Ackland, F.I.A., of two of the larger Bond Investment concerns. In one case a  $3\frac{1}{2}$  per-cent net premium valuation disclosed a deficiency equal to three times the available assets, whilst in the other the deficiency amounted to four times the available assets, even after leaving future expenses as a charge against surplus interest—if any. The premiums charged were apparently adopted without any expert advice, or critical examination, from those charged by similar Offices in America, and, as pointed out by the Report, were hopelessly inadequate to provide the benefits promised, even if the expenses of working the business—which in point of fact were very heavy—were entirely left out of account.

Apparently, the promoters relied upon the profits expected to arise from lapses, surrenders, and death claims, to make up the difference between the benefits promised and those which

the subscriptions were actually able to provide ; but, in practice, the profits arising from the last two sources were almost negligible ; the amount allowed in these cases being so small as to render it more profitable to sell a bond privately, if there was a reasonable chance of its being duly paid on maturity.

The main—indeed, the only—source of profit to which these companies could look was, accordingly, lapses. The data relied upon as to the probable lapse rate likely to be experienced were, however, of the most shadowy description, the rate being invariably grossly over-estimated. One witness, indeed, went so far as confidently to state that it was expected that only 4 per-cent of the bonds issued would ever reach maturity, while another "expert" assumed that out of 100 ten-year bonds eight would lapse each year, thus making the assumption that the lapse rate would increase with the duration of the bond.

In their Report, the Committee state that the evidence before them showed that the lapse rate experienced by various companies during the first year after issue, varied from 12 to 54 per-cent, and during the second year from 9 to 25 per-cent. ; during the third year, the percentage diminished considerably, and practically vanished during the fourth, as after this period surrenders took the place of lapses. Even, however, considering lapses and surrenders together, the percentage apparently became extremely small after a bond had been about five years in force, although, perhaps, this feature was in some measure due to the comparatively short time during which the Companies in question had been working.

The only reliable information about the lapse rate really experienced is given by Mr. Dougherty in the Paper mentioned above, in which the actual experience of two large Companies is shown. The author there remarks "that the actual monthly "lapse rate over the six years' experience was remarkably "small, and could not, therefore, in any circumstances be "relied upon as an element of profit, or even contributing to "solvency, the actual cash profit resulting from the total lapses "over the period named being quite insignificant."

The actual combined lapse experience of the two Companies was as follows :

No. of Monthly Instalments paid	Lapse Rate per-cent	No. of Monthly Instalments paid	Lapse Rate per-cent
* 0	14·2	6	1·1
1	5·5	7	1·0
2	3·2	8	1·4
3	2·4	9	0·8
4	2·2	10	0·8
5	2·1	11	0·6
		12	0·6

\* Application fee only paid.

The rate of lapse after the first year became negligible.

Mr. Dougherty, in remarking on this table, states :

"Compared with the lapse experience of ordinary Life Offices, "that of the bond companies was much less throughout, "and, therefore, much less than the lapse rate obtaining in "the case of industrial companies. The most remarkable "feature shown by the table is the enormous rate of lapse after "the application fee had been paid, and prior to the first "month's instalment, attributable, no doubt, to the hope that "by a stroke of luck a £5 coupon would be redeemed before "the first premium became due. Throughout the evidence "that came before the Bond Investment Committee, the "lapse rate appears to have been discussed as an annual "percentage, rather than a monthly one, so that from the "percentage stated by various persons giving evidence, the "features brought out by the above table were not so apparent. "Indeed, the statement that the lapse rate in a particular "company was 25 per-cent for the first year, was misleading, "for, although 25 per-cent of the bonds might run off in one "year, it did not follow that this percentage actually paid a "year's premium, or anything like it, as would appear to have "been implied."

All the evidence at our disposal goes to show that the lapse rate actually experienced depended greatly upon the following factors :—

1. Duration of the bond, the lapse rate decreasing with each monthly premium paid;
2. Terms for which the bonds were issued, *i.e.*, under a 30-year bond the lapse rate is far greater than under a 10-year bond;
3. Conditions as to surrenders, redemptions, &c.;
4. Class of business, *i.e.*, the lapse rate is lower under quarterly business than under monthly or weekly;

5. Commission paid to the agents and the efforts made to force or to retain business. In one case, for instance, the manager was partially remunerated by commission on the lapses;
6. The locality in which the Company mainly worked, and the general character of its subscribers;

Even if any actual data had been available, therefore, it would have been most dangerous to assume that this would apply to any other Company than the one from which it was actually derived. Indeed, I doubt whether such data could even be assumed to be a reliable guide to the future experience of the Company from which it was derived, unless the data covered a large number of cases, spread over a period of at least twelve years, and were most carefully analysed. Moreover, it is doubtful whether the lapses during the first year of a bond's existence do not involve a loss to the Company, rather than a profit, on account of the initial expenses incurred.

In addition to a grossly over-estimated lapse rate, we also find, in the evidence submitted to the Committee, the familiar and ever-recurring fallacies that a large increase in funds necessarily indicates a state of solvency, and that a Company founded on an unsound basis can be successfully worked by the accession of an ever-increasing number of new members.

As already stated, however, these pitfalls to the small investor have now ceased to exist, and my only object in mentioning them at all is to differentiate between them and the companies belonging to Class B—House Purchase Companies—with which alone the rest of my Paper deals. Too much emphasis cannot be placed on this distinction, as these two classes of business were totally different in nearly every respect, although the fact that both come within\* the somewhat wide definition of a Bond Investment Company given by the 1909 Act, may have the effect of throwing some undeserved opprobrium upon the sole class remaining to-day, the majority of the members of which now carry on their business on lines, the bases at least of which are actuarially sound, and equitable to the public.

#### B. HOUSE PURCHASE COMPANIES.

The system worked by House Purchase Companies—or Property Investment Companies as they are often termed—

\* The doubtful question of whether the contracts issued by House Purchase Companies are life business, within the meaning of the Act, is dealt with later.

has been established in this country for at least twenty years, the two largest and oldest concerns dating from 1877 and 1890 respectively. At the present time, it is difficult to say exactly how many companies are actually carrying on this business, as many apparently did not complete their statements in time for inclusion in the last published volume of the Board of Trade Returns ; but I have the names of about twenty companies at present in existence, and possibly there are a few others of which I have not yet heard, the sums assured exceeding £25,000,000, of which, however, at least 65 per-cent is in the hands of one Office. It will be seen, therefore, that, although the business is small compared with Life Assurance, it cannot be considered as altogether unimportant.

At present, the Companies are of all classes and sizes ; some have been established for a number of years, have for many years been conducted under actuarial advice, and have made periodical valuations, while a much larger number have only been recently established, and have as yet to prove themselves. Some of these latter will, no doubt, disappear in the near future as a result of the requirements of the 1909 Act, but a number of the others appear to be in a satisfactory condition for their age, and may in time, under careful management, develop into large and prosperous concerns.

All the Companies which have been established for any length of time have now, I believe, made the necessary deposit of £20,000, and have acquired powers to transact life business ; so that they have an additional claim upon our notice, as constituting the field from which a number of recent accessions to the ranks of ordinary Life Offices have sprung.

From the definition given on p. 188 and from pp. 214-218, of this Paper, it will be seen that the rate of interest yielded to a subscriber by the bonds of these companies is low ; lower, indeed, in many cases than would be realised if the amount were placed in the Post Office Savings Bank. Against this, however, must be set off the fact that it is extremely unlikely that such small amounts as the monthly—or even quarterly—premiums would be regularly saved, unless they were periodically collected ; so that, apart from the disadvantage, if it be such, that the amount saved cannot be withdrawn when required, without loss, the contracts, from the interest point of view alone, are probably quite as satisfactory as any open to the small investor.

The main inducement offered to the public, however, is the

facility offered for purchasing house or shop property, as an advance, equal to the nominal value of the bond, is made after the latter has been a certain length of time in force, on the joint security of the bond, and the property which it is desired to purchase. In many respects, the contracts are very similar to those issued by Building Societies, to which, indeed, these companies bear a strong resemblance, although the former term includes so many widely differing systems that any general differentiation is impossible. In many cases, however, building societies are more akin to savings banks, the subscriptions, on which interest is paid, being treated as deposits withdrawable on demand or on short notice, and the funds of which are invested in mortgages of house property. The main differences between House Purchase Companies and Building Societies may be summed up the following manner :

#### HOUSE PURCHASE COMPANIES

Are, as a rule, limited companies, carried on for profit-earning purposes, although in some cases the bond-holders take a share of the profits.

Employ agents and have an extensive agency organization.

Subscriptions are treated as the premiums for a pure endowment or sinking fund policy, yielding 2 per-cent to  $2\frac{1}{2}$  per-cent per annum compound interest.

Generally cover the whole country.

#### BUILDING SOCIETIES.

Mutual concerns, so that the members may be liable for any deficit in the event of winding-up, although they share the profits.

Do not employ agents.

Subscriptions are treated as deposits withdrawable on short notice, bearing interest at 3 per-cent to  $3\frac{1}{2}$  per-cent.

Generally limited to one district.

While the rate of interest allowed by building societies is much higher than that allowed by House Purchase Companies, it must be pointed out that there is a corresponding amount of risk, since, although the system of receiving deposits for investment in long-term securities may work well under normal conditions, serious disaster may result, if, for any reason, public confidence in the concern is shaken—a state of affairs well exemplified by a recent financial disaster.

The class of the community by which these contracts are, for the most part, taken up is that often described as the lower-middle class, very few bonds being taken up by the purely industrial classes. It will be seen that the class in question lies between those from which the clients of the industrial and the ordinary life assurance companies are mainly drawn, and, indeed, in many respects, the operations of the Offices now under examination fall midway between these two great main sections of our work.

The average individual amount of the bonds issued is comparatively large, varying, with the Company, from £130 to £300.

*General description of the Bonds issued.*

It is very difficult to give in a few words a general description of the contracts issued by House Purchase Companies, as the conditions vary greatly with different Offices, each of which is practically a law unto itself, but a table showing the main features of the contracts of several of the chief Companies is given on p. 209 in Table I.

In addition to securing a given sum at the expiry of a certain number of years, the certificates, in most cases, provide for the return of all premiums paid, either with, or without, compound interest in the event of death occurring before the maturity of the contract; so that, technically speaking, they are pure endowments with return of premiums, rather than sinking fund policies pure and simple. There are, however, often certain restrictions placed on the payment of this death benefit, so that, in practice, the bonds are usually treated, at any rate, by the Companies, as sinking fund assurances; but this point will be fully dealt with later. A further inducement to take up the bonds also existed until recently in the form of a ballot system, whereby advances without interest, or a payment in cash, were made to the holder of a drawn certificate; but this system has now been forbidden for the future by Section 34e of the 1909 Act.

No enquiry is usually made at the time when the bond is granted, either as to the age of the applicant or his state of health.

The periods for which the bonds are granted vary greatly, each Company, as a rule, having its own favourite terms. Usually quinquennial periods, such as 10, 15, 20, 25 or 30 years, are adopted, but some concerns favour bonds for 7, 14 or 21 years, while others publish rates for all terms up to 10 years, and for

quinquennial intervals thereafter. The vast majority of the bonds issued are probably for thirty years, and I believe that longer periods are seldom, if ever, adopted ; the original form of the contracts issued being a bond for 30 years at a monthly premium of 4*s.* 4*d.* per-cent.\* It is not unusual to find that the bonds for periods of less than 15 years yield a slightly higher rate of interest than those for longer periods. They do not, however, usually carry the right to an advance for the purpose of purchasing house property, and are then often termed "investment" certificates, in contradistinction to "endowment" or "house purchase" certificates, as those for the longer terms are called.

The bonds are usually issued for nominal amounts of £25, £50 and £100, and multiples thereof ; but, in addition to the nominal sums assured, often carry a so-called "bonus" payable at the maturity of the certificate and guaranteed from the outset ; so that, practically speaking, the true sum assured is rarely an integral multiple of £100. In addition to this guaranteed bonus, the bonds are, in some cases, entitled to a share in the profits of the Company.

The premiums are usually payable monthly or quarterly, although a relatively small amount of business is, in some cases, issued at annual, half-yearly, or weekly premiums.

*Return allowed in the event of death before the maturity of the Bond.*

Generally speaking, the return made under a certificate is either :

1. All premiums paid without interest. In at least one case, however, under weekly business only 80 per-cent of the premiums paid are returned.
2. All premiums paid with compound interest. The rate at which the latter is calculated varies from 2 per-cent to 3 per-cent, the rate most usually adopted being  $2\frac{1}{2}$  per-cent. One Company, however, under certain classes of certificates, returns the whole of the premiums paid with compound interest, varying, with the original term of the bond, from  $2\frac{3}{4}$  per-cent on bonds for 7 years to 4 per-cent on bonds for 20 years ; while another Office returns the whole of the premiums paid, with  $2\frac{1}{2}$  per-cent compound interest on the premiums paid after the first five years only.

\* Yielding about  $1\frac{5}{8}$  per-cent per annum and representing 1*s.* per week.

In addition to the premiums paid—with or without interest—under certificates entitled to share in the profits of the Company, the cash value of any vested bonus additions—which may or may not include any bonuses guaranteed—is generally allowed in the event of death; and, as an alternative to drawing the death benefit in cash, the holder's representatives have usually the right to continue to pay the subscriptions until the maturity of the bond, when the full amount assured becomes payable to them. As a rule, too, certain additional restrictions are placed on the payment of the death benefit, as follow:

The death benefit is not payable if—

1. The bond has not been a certain length of time—varying from 1 to 5 years—in force; or if the subscriptions are more than three months in arrear.
2. The bond has been transferred by the original holder to a third party. In some cases, however, if the transferee is approved by the directors, the death benefit becomes payable on his death, instead of on that of the original holder.
3. An advance has been granted on security of the certificate for the purpose of purchasing house property. This provision will be more fully discussed later.

In some cases, also, the representatives of the bondholder do not appear to have any absolute right to a return of premiums in the event of death, this being merely allowed as a matter of grace, at the option of the directors.

At first sight, the last provision may seem unduly harsh, but it must be remembered that no enquiry is made at the time when the bond is granted either as to the age of the applicant or his state of health. In practice, no doubt, it is seldom, if ever, taken advantage of, and at the same time it protects the Company against inequitable claims.

These restrictions on the payment of the death benefit have an important bearing upon the calculation of the premiums for, and the valuation of, these Companies, as well as upon the difficult question of whether or no these contracts are policies of life assurance.

#### *Transfer of Bonds.*

All Companies allow the transfer of their bonds—provided, of course, that the Office has no lien thereon—to third parties, on certain conditions, the most usual of which are as follow:

The transfer is allowed provided :

1. The bond has been a certain length of time—usually one year—in force.
2. All premiums are paid up to date.
3. The proposed transferee is approved by the Directors.
4. Payment is made of a transfer fee varying from 2s. 6d. to 5s., although, I believe, in one or two cases no fee is charged.

In some cases forms of transfer are provided by the Company, or are endorsed by it on the bond.

If the transferee is approved by the Directors, he is usually entitled to the same rights as the original holder of the certificate in regard to an advance for the purpose of purchasing house property ; either immediately, or after the lapse of twelve months from the date of registration of the transfer. But as I have already stated, in some cases the death benefit ceases immediately a bond is transferred, while in others, it is still payable, but on the death of the transferee instead of that of the original holder.

Judged from a purely life standpoint, the above restrictions, particularly No. 3, on the transfer of the bonds may seem unnecessarily stringent. It must be remembered, however, that the main inducement to take up these certificates is the hope of obtaining an advance, and that from the Company's point of view the regular payment of the subscriptions during the probationary period of five years or so is a valuable test of the reliability of the borrower. The latter safeguard would, of course, practically disappear if the bonds were allowed to be transferred indiscriminately, and I think that this fact, and the conditions governing the payment of the death benefit, sufficiently explain their adoption.

#### *Terms upon which Advances are made.*

After a certificate has been a certain length of time in force—usually 3 to 5 years, although in some cases a reduced amount is lent after the first year—the holder has the right of borrowing, out of the available funds of the Company, an amount equal to the nominal\* value of the certificates held by him, for the purpose of purchasing freehold, or leasehold, property.

\* Any bonuses guaranteed, or added to the certificate, are usually disregarded in fixing the amount to be advanced.

*Class of Property accepted.*

The conditions of the certificate or the prospectus of the Company usually clearly state that the only classes of property accepted as security for an advance are freehold and leasehold houses, or shop property, and that advances are not made upon :

1. Leasehold property with less than 60 years to run.
2. Properties in course of erection or in an unfinished state.
3. Uncovered land, licensed premises, warehouses, &c.
4. Property let at a lower weekly rental than 3s. to 4s.

A further safeguard is the invariable rule that all properties submitted must be approved by the Directors, who have, of course, the power of relaxing the above regulations in exceptional cases. Only advances on first mortgage are usually considered, although the accounts of some Offices show some second mortgages outstanding. The amount of the latter is, however, always very small, and generally, I believe, represents cases in which a further advance has been made on security of property, the first mortgage of which is held by the Company. A report by a properly qualified local surveyor as to the value of the property is also always obtained and forms the basis of the Directors' decision.

*Expenses.*

The borrower is required to pay all legal expenses connected with the transaction, as well as the survey fee. In some cases a scale of charges, of which the following may be considered fairly representative, is laid down :

	LOAN NOT EXCEEDING				
	£250	£350	£500	£750	£1,000
Valuer's Fee ...	£ 1 1 0	£ 1 11 6	£ 2 2 0	£ 2 12 6	£ 3 3 0
Legal Charges ...	3 3 0	3 13 6	4 4 0	5 5 0	6 6 0

In these cases, however, it will usually be found that in addition to the charges stated, any disbursement made by the solicitor, such as stamp duties, search fees, and other extra expenses, are payable by the client, so that, on the whole, I think that little is gained by publishing a fixed scale of charges, which

may only prove misleading in practice. The expenses of the transaction are, I believe, always kept within the most reasonable bounds possible, and very few cases of dissatisfaction on this ground are experienced.

The applicant is required to deposit a sum varying from  $\frac{1}{2}$  per-cent to 1 per-cent on the loan required, to cover the necessary preliminary expenses; the balance of this amount being returned if the property is found to be unsuitable. Under such conditions, however, it is very usual to value a second or third property without payment of any further deposit.

#### *Average Individual Amount advanced.*

The Board of Trade Returns do not enable us to ascertain the average individual amount advanced, but loans are not usually made for a larger individual amount than £2,000, or for a smaller amount than £100, a fair average being £300 to £400. No doubt this varies with different Offices, as one Company, for example, has an average individual amount *still outstanding* of at least £850. This is quite exceptional, and probably due to some special circumstances, not revealed by the Returns.

#### *Repayment of Advances.*

After an advance has been granted, several different systems are in use for its repayment, which may be classified as follows :

1. The certificate is kept in force until maturity, the borrower continuing to pay the periodical subscriptions, with interest, at a fixed rate, upon the amount advanced until that date, when the amount of the certificate is applied to wipe off the loan. The rate of interest charged is almost invariably 5 per cent, although in some cases a reduction is made after the interest and subscriptions have been punctually paid for a certain period.
2. The certificate is cancelled and in its place is adopted :
  - a. A scale of equal quarterly instalments of principal and interest calculated to redeem the loan in 10, 15, 20 or 25 years ; or
  - b. A similar scale to "a", the periodical payments being, however, reduced every five years ; or
  - c. A scale of fixed equal periodical repayments of capital to be made throughout a specified term—

usually 10, 15, 20 or 25 years—the interest being proportionately reduced as the loan is repaid. The combined effect is to give a continually decreasing scale of quarterly payments ; but in some cases the repayments are fixed on an annual basis payable by equal quarterly instalments, so that the combined payment only alters annually.

In some cases, two or more of these systems are used by the Company, the borrower having the option of adopting whichever method best meets his requirements.

In addition to the periodical payments, it is usual to accept small amounts (from £1 upwards) at any time, in reduction of the advance, the interest being periodically reduced.

It will be seen that under each of the systems under Heading 2 the bond is cancelled and the loan becomes merely an ordinary mortgage, under which a certain specified scale of repayments of principal and interest has been stipulated for. It might be objected that if this method is adopted the *raison d'être* of the original certificate disappears, but the latter, is, nevertheless, valuable for the following reasons :

1. The regular payment of the subscriptions during the probationary period of three, or five years, tests the reliability of the borrower, and thus improves the Company's security by testing the applicant's fitness for an advance.
2. When the advance is granted, the subscriptions already paid give the Company a certain amount in hand, which provides an additional margin of security for the loan, and at the same time reduces the repayments required from the borrower during the balance of the period.

The equity of the systems under Heading 2, from the borrower's point of view, depends to a very large extent upon the amount with which he is credited in respect of the subscriptions paid during the probationary period, and, unfortunately, this is extremely difficult to discover from the figures published. In some cases it is the total premiums paid, either without interest or accumulated until the end of the period covered by the repayments at about  $2\frac{1}{2}$  per-cent, but in others only the surrender value of the bond—usually  $33\frac{1}{3}$  per-cent of the premiums paid—appears to be allowed; but, generally speaking, the scales in use do not appear to be unfair on the basis of interest at 5 per-cent.

On the whole, although the system is, perhaps, rather more complicated, Method 2 is more favourable to the borrower than Method 1, as in the former case the repayments of capital are practically accumulated at 5 per-cent, as against 2 per-cent to  $2\frac{1}{2}$  per-cent in the latter.

*Termination of the Loan before the Date originally fixed.*

With a few exceptions, very little information is given, either in the various publications of the Companies, or in the Board of Trade Returns, as to the terms on which the borrower may redeem his loan before the date originally fixed for the completion of his payments ; but the usual practice appears to be as follows :

*A. During the Lifetime of the Borrower.*

Under Method of Repayment 1—(certificate kept in force)—the advance and the certificate are treated as two entirely separate transactions, the whole amount of the loan remaining outstanding until the maturity of the certificate. Should the borrower wish to repay the loan before the latter date, credit is only given for the cash surrender value then attaching to the bond, an amount which, as already stated, varies with the period in force from  $33\frac{1}{3}$  per-cent upwards of the total premiums paid.

One or two Offices do not pay cash surrender values, and in these cases—if this method of repayment is used—the borrower has to repay the whole amount advanced, and has the option of either continuing his payments under the bond until maturity, or of taking a fully paid certificate.

Under Method of Repayment 2 (Sections *a* and *b*), a common condition of redemption is to require payment of the instalments still outstanding, under discount at 4 to  $4\frac{1}{4}$  per-cent—an arrangement fairly equitable to all parties.

Under Method of Repayment 2 (Section *c*), the loan is periodically reduced by the fixed equal instalments of principal, the balance outstanding from time to time being readily ascertainable. There is always, of course, the unknown factor of the amount with which the bond-holder is credited in respect of the subscriptions paid during the probationary period, but this presents no difficulty to the Company.

**b. In the event of the Death of the Borrower before the Loan is fully repaid.**

In this event the representatives of the deceased bondholder have the option of :

1. Continuing payment to the Company until the original date fixed for the termination of the advance ; or,
2. Repaying the amount due to the company. (As in the preceding section, however, the amount due is not usually defined so clearly as one could wish.)

In the latter event, if the first method of repayment has been adopted, the death benefit allowed under the certificate will, of course, go to reduce the original amount advanced. Sometimes, however, the death benefit in question is expressly limited to bonds on which an advance has not been made, and in these cases, as well in all those in which one of the methods of repayment described under Heading 2 has been adopted, the position is exactly the same as if it were desired to terminate the transaction during the lifetime of the borrower.

*House Purchase combined with Life Assurance.*

The above remarks, and the fact that many of the more substantial house purchase companies have now made the necessary deposit and acquired power to transact life business, naturally lead us to the advantage of combining a life policy with the ordinary house purchase contract, in order that the advance may be repaid and the property freed from debt in the event of the borrower's death before his repayments are complete, and the advantages of this system are so great that, in spite of the heavier annual payments required, it is probably on these lines that the greatest progress will be made in the future.

The premium for a combined scheme of this nature falls into four parts :

1. Interest on the capital advanced.
2. A sinking fund to repay the advance.
3. The premium for a decreasing term assurance to cover the real amount of capital still outstanding from time to time.
4. An allowance for expenses.

Several formulæ and schemes have been put forward from time to time combining these elements into one premium, but I think that the most convenient way of solving the problem

is to combine Nos. 2, 3, and 4 into the ordinary Office annual premium for a non-profit endowment assurance, and to charge interest annually on the full amount advanced. Competition will not admit of interest being charged at a higher rate than  $4\frac{1}{2}$  per-cent, which is undoubtedly low for the class of mortgage in question, but against this must be set off the fact that the sinking fund portion of the premium is accumulated at the rate used in the calculation of the Office premium—usually  $3\frac{1}{2}$  per-cent—as against 5 per-cent, if 1 and 2 above are combined into the form  $100 \div 1 + a_{n-1}$  at 5 per-cent, and only the decreasing term assurance set up.

Treating these contracts in two separate portions, as described, is undoubtedly simpler and more straightforward for office purposes, and prevents many of the complications which arise if a combined contract is set up under which the premiums cannot be readily separated into their component parts.

#### *Loans on security of the Bonds alone.*

A small amount is also advanced at 5 per-cent on security of the bonds alone, up to the limit of their surrender values.

#### *The Advance System generally.*

From the Company's point of view, it will be seen that under the vast majority of its contracts, it is morally—if not legally—bound to advance the full nominal value of its certificates at the end of five years, if required, and a suitable property is offered as security. In the case of a bond for thirty years, granted at a monthly premium of 4*s. 4d.*, assuming interest is earned at the rate of 5 per-cent, and that the whole of the first year's premium and 5 per-cent of the premiums thereafter are required for expenses—assumptions more favourable to the Company than are likely to occur in practice—the amount in hand at the end of five years will be only about 11 per-cent of the sum assured, or, at the end of 10 years, 28 per-cent, and it is evident, therefore, that a young concern would find it impossible to satisfy the demand for advances, if a large number of its subscribers applied at the end of the probationary period, unless other funds than those derived from the premiums paid were available.

In practice, however, this difficulty is considerably reduced by the following considerations; and it is only fair to say that, so far as my knowledge goes, most of the Companies now in existence have met all requests for advances on security of suitable properties made to them.

1. Many of the bonds issued are for small amounts of less than £100, which are individually useless as security for an advance, but which are taken up in the hope that the holder will be able to add to the total amount held, from time to time, until the aggregate amount is sufficient to cover the loan required. For this, and other reasons, only a small number of those entitled to do so eventually apply for an advance; the exact percentage, judging from the evidence given before the Departmental Committee, being about 20 to 25 per-cent. Even this number is spread over the whole term of the bonds, many holders not applying for an advance for 10 years or more after their certificates were originally effected.

It is difficult to understand why the number of advances applied for is so low, but possibly changed circumstances, altered plans, and difficulty in finding suitable properties, together with the fact that some certificates are taken up purely as an investment, are a sufficient explanation.

2. A certain number of the properties put forward as security will inevitably be found to be unsuitable, and, in addition, there is usually a certain amount accumulated from the subscriptions for short-term bonds, &c., available for investment.
3. The Company may be in a position to borrow some portion of the amount required from outside sources on security of the title deeds to the property, coupled with its own guarantee. From some of the accounts deposited, this course is apparently sometimes adopted, but its safety depends entirely upon the extent to which it is followed, and the terms upon which accommodation can be arranged. There is no information available on either of these points, and I feel extremely doubtful as to the desirability of this system if it is allowed to become an essential feature of the business.
4. Legally, the Company is nearly always safeguarded by a provision that advances will only be made out of the *funds available* for this purpose. The moral responsibility none-the-less remains, and it is to be hoped that this clause is only inserted as "abundant

caution", and that its abuse is sufficiently prevented by publicity and competition.

After a Company has been working for some years the accumulated funds, with incoming premiums, interest, &c., should be sufficient to meet any demands for loans which are reasonably likely to arise; but, at first, it is evident that ample resources in the shape of paid-up capital, in addition to the deposit required by the 1909 Act, are necessary, if a purely House Purchase Company is to be a success. The business in question, therefore, is most satisfactory if carried on as an adjunct to other classes of business, for whose funds it provides a most valuable outlet.

Mortgages of the nature of those in question are, of course, undesirable and troublesome where only a small amount of business is transacted, and no special organisation exists for its management, but where it is worth while setting up the special machinery necessary, it provides an excellent 5 per-cent investment. If the local valuers, on whose reports the decision of the management ultimately rests, are well chosen, and all proposals for advances carefully scrutinized, the security offered is extremely good, as the individual amounts advanced are so small, and so widely distributed, that the risk of a heavy loss is practically *nil*. The effect, too, of a general slump in property is hardly to be feared, even the recent depression affecting the value of the small residential property, generally offered as security, to a much smaller extent than the larger properties usually accepted as security for mortgages by ordinary Life Offices. The market price of the small properties in question is mainly regulated by the demand from a residential point of view, and any disinclination on the part of the public to purchase will probably be more than counterbalanced by an increased tendency on the part of builders and owners to sell, rather than to let. Indeed, a general slump may even tend to increase the value of such property, by stopping the erection of new houses, and so increasing the demand for those already built.

The test imposed on the reliability of the borrower by the regular payment of his subscriptions during the probationary period—if this is adopted; the fact that the advance is in the majority of cases required by the tenant in order to enable him to become his own landlord; and the regular reduction in the amount outstanding from quarter to quarter; all combine to render foreclosure very seldom necessary, and the chance of an ultimate loss on any one transaction very small.

In a "composite" Office, too, there are valuable additional adjuncts to the business in the shape of the fire and burglary policies obtainable, and the connection formed among local valuers.

From the point of view of the public, the system provides a means of purchasing house property on terms which are, probably, as cheap and satisfactory as any others open to the class in question. The system of periodical repayments of capital, so convenient to the borrower, renders an advance of this nature unsuitable for a private lender, in dealing with whom also there is always the risk that the loan may be called in at very short notice. No doubt the terms offered by some building societies are more favourable than those offered by most House Purchase Companies, but the former are often confined to some particular locality, which renders the transaction, in many cases, hardly so private as some borrowers might desire. Sometimes, too, a certain liability attaches to their shares in the event of the society being wound up, and the fact that many of the House Purchase Companies are in a position to issue life policies providing for the repayment of the amount outstanding, in the event of the borrower's death before his payments are completed, is another point in their favour.

In making these comments I must disclaim any intention to minimise the excellent work and careful management which distinguish so many building societies, for both systems have their good points, and it is for the borrower to choose that which suits him best.

#### *Some General Considerations.*

In closing this section of my Paper, may I point out that the business transacted by House Purchase Companies is at present in a transitory stage, and it is difficult to foresee either the lines on which it will develop or the future of the Companies at present transacting it. If it is economically and honestly managed, it is profitable to the Company, and at the same time meets the demand amongst a large section of the public, at present hardly touched by either industrial or ordinary life offices, for some outlet for the small amounts which can be periodically saved, and for some means of becoming the owners of their own houses—a demand proved by the large sums invested with building societies and similar concerns.

From the point of view of the Companies, the sole weak spot in the system described is, I think, the difficulty of finding

sufficient funds adequately to satisfy the demand for advances which is usually fully equal to the spare cash awaiting investment, 80 per-cent to 95 per-cent of the assets of a representative Company of this class being invested in mortgages, as against 22·03 per-cent, the proportion of the total assets of ordinary Offices invested in mortgages within the United Kingdom (Vol. xlii, p. 317, Mr. P. L. Newman, F.I.A.). It might, possibly, be objected that House Purchase Companies place too much of their funds "in one basket," but it must be remembered that their circumstances are quite different to those of ordinary Offices; and that the average individual amount is small, the amount at risk under each advance is steadily reduced year by year, and the majority of the properties mortgaged are occupied by the borrowers.

From the bondholder's point of view, the weak spots are three in number :

1. The low rate of interest yielded on his subscriptions if no advantage is taken of the advance offered—a rate which, however, is usually at least equal to that which could be obtained by him on such sums in any other way.
2. The conditions on which he is able to withdraw from the contract, should he wish to do so, before an advance is granted. No terms can be offered which will satisfy the member who regards the transaction purely from a banking point of view and wishes to withdraw the full amount paid in with interest, at any time convenient to himself; but, at the same time, I think that in some cases the regulations at present in force in this respect are far too onerous, although, in others, very fair surrender values are given.
3. The arrangements in force, should the assured, after obtaining an advance, die, or wish to close the transaction before the date originally fixed for the termination of the loan. If the full death benefit, or a surrender value nearly equivalent thereto, is allowed under Method of Repayment 1 (p. 200), and the future payments are commuted at 4 per-cent to 4½ per-cent under Method 2 (Sections *a* and *b*), there is, perhaps, little cause of complaint, as the Company is entitled to be compensated for any possible loss caused by the earlier repayment of the advance; but in some

Company Number (1)	Class of Certificate	Periods for which Certificates are usually issued	Share of Profits allotted to those Profit Certificates	Are House Purchase Benefits granted?	Method of calculating Paid-up Policy used of those described on p. 225	Rate of Interest used in calculating return of Premiums in the event of death (2)	Are Cash Surrender Values paid?	Which method of Repayment of an Advance of those described on p. 200 is adopted	Rate of Interest used in last published valuation	Proportion of Premium Income reserved for future Expenses	£ s. d.
A	Investment Certificates	3-10 years inclusive	80 per cent of which a portion is guaranteed	Yes	Method II 3 per-cent interest	3 per-cent	Yes	Method I	3 per-cent	19·3 per-cent	4 13 5
B	Endowment Certificates	12, 15, 20, 25 & 30 years	80 per cent of which a portion is guaranteed	Yes	Method I	Nil <sup>(2)</sup> (Cash Value of any bonus allotted is also returned)	Yes	Method I	4·4 per-cent	15 per-cent	4 17 11
C	Investment Certificates	7-20 years inclusive	All certifi- cates are "Without profits"	No	Method IV	Rate varies with original term of certifi- cate from 2 <sup>3</sup> - 4 per-cent	No	Method I	... No	Nil at date of valuation. Only recently formed	
D	Endowment Certificates	30 years	All certifi- cates are "Without profits"	Yes	Method I	Nil	No	Method II a & c	4 per-cent	18 per-cent	
E	Endowment Certificates	15, 20, 25 & 30 years	80 per cent of divisible profits	Yes	Method IV	2 <sup>1</sup> per-cent (Cash Value of any bonuses allotted is also given)	Yes	Method II a & c	3 <sup>1</sup> per-cent	15 per-cent	5 10 0
F	Endowment Certificates	5, 10, 15, 20 & 30 years	All certifi- cates are "Without profits"	Yes	Method III 2 per-cent interest	2 <sup>1</sup> per-cent on premiums paid after first five years	Yes	Method I Method II a & c	3 <sup>1</sup> per-cent	15 per-cent	5 11 2
	Endowment Certificates	29 & 30 years	All certifi- cates are "Without profits"	Yes	Method IIIa 2 <sup>1</sup> per-cent interest	2 <sup>1</sup> per-cent	No	Method II a & b	3 <sup>3</sup> per-cent	17 <sup>1</sup> per-cent	
	Investment Certificates Endowment Certificates	12-20 years 20, 25 & 30 years	Nil Such shares as Directors may decide	Yes	Nil ?	Nil*	Yes	Methods I & IIa.	4 <sup>1</sup> per-cent	15 per-cent	5 19 8

Notes—(1) These letters do not designate the same Company throughout the Paper.  
(2) Death Benefit. When "Nil" is shown the premiums paid are returned without interest.

other cases where these methods are not adopted the conditions in force appear somewhat harsh. In any event, whatever terms are adopted, they should be clearly shown, either on the mortgage or in the prospectus, in order that the borrower may clearly understand his position at any time, as the conditions generally, in some cases, are very intricate, in view of the class to which they are offered, and might with advantage be simplified.

## II.—SOME ACTUARIAL POINTS ARISING OUT OF PROPERTY INVESTMENT BUSINESS.

In the first part of this Paper, I have endeavoured to give a general idea of the business carried on by House Purchase Companies, and of the main features of the certificates issued by them, and will now proceed to deal with some of the actuarial points involved. This, however, can only be done in a very general fashion, as the subsidiary details of the contracts vary so greatly that it is impossible, and, indeed, undesirable, to deal with every point which may arise in practice. I have only attempted, therefore, to deal with the main principles involved, and to test the effect of some of the more important conditions.

It must be remembered that this business is to a very large extent in a somewhat similar position to that of ordinary life assurance before the passing of the Act of 1870, except that up to the present it has not been to any great extent under direct actuarial control; and that this difference of practice arises to some extent, also, from the comparatively short time that many of the Companies have been established, and, perhaps, too, from the absence of competition. Many points of detail, also, have to some extent been regarded as Office secrets, with a view to preventing officials and agents leaving the service of one company in order to found another, no obstacle to which existed until recently.

Practically, the only difficulty in connection with the treatment of the business from an actuarial point of view is caused by the system whereby a death benefit is tacked on to what is primarily a sinking fund policy, without any reference to the age at entry. As will be seen, the difficulties caused by this are great; indeed, I think it is hardly too much to say that an exact solution of the problems involved is impossible, unless data as to the ages at entry are forthcoming; but if it is found that the certi-

ficates can be satisfactorily treated on a purely sinking fund basis the further work involved is not of a complicated nature. In the present section I have endeavoured to show that, as a rule, this course can be adopted, and the death benefit looked on as a special surrender value, payable under certain conditions, rather than as an integral part of the contract; although, in some cases, an addition to the reserves may be necessary.

The consideration of the actuarial side of the subject falls into three parts, as follows :—

1. Calculation of premiums.
2. Paid-up policies and surrender values.
3. Valuation of these contracts.

### I. THE CALCULATION OF RATES.

From a technical point of view, the bonds issued by House Purchase Companies are pure endowments, under which the premiums are to be returned, in the event of death before maturity, either :

- (a) Without interest,
- (b) With compound interest at rate  $j$ ,

according to the system adopted by the particular company in question.

Theoretically, the formulæ for the calculation of the premiums for these benefits are, as given in the text-books :

In the case of (a)

*Premiums returned without Interest,*

$$\text{I. } \pi\{1 + a_{x, \overline{n-1}}\} = \frac{D_{x+n}}{D_x} + \{\pi(1+k) + c\} \left\{ \frac{R_x - R_{x+n} - nM_{x+n}}{D_x} \right\}$$

where, as usual,  $x$  is the age at entry and  $n$  the term of the endowment ; and in the case of (b)

*Premiums returned with Compound Interest at rate  $j$ ,*

$$\text{II. } \pi\{1 + a_{x, \overline{n-1}}\} = \frac{D_{x+n}}{D_x} + \{\pi(1+k) + c\} \left\{ \frac{(1+j)(A'_{\overline{xn}} - A_{\overline{xn}})}{j} \right\}$$

where  $A'_{\overline{xn}}$  is calculated at a rate of interest  $J$ , such that

$$J = \frac{i-j}{1+j} \text{ or } \frac{1}{1+J} = \frac{1+j}{1+i}.$$

Using the approximation given by Mr. H. T. Adlard (Vol. xxxvi, p. 389)\* this formula becomes

$$\text{III. } \pi\{1+a_{x,\overline{n-1}}\} = \frac{D_{x+n}}{D_x}$$

$$+ \frac{\{\pi(1+k) + c\} \{N_x - N_{x+n} - D_{x+n}(1+i)s_{\overline{n}}\} A_{\overline{xn}}}{D_x \cdot A'_{\overline{xn}}}$$

where  $A'_{\overline{xn}}$  is calculated at rate  $j$  and the rest of the formula at rate  $i$ .

Formula III will usually be found more convenient than Formula II, since the values of  $A'_{\overline{xn}}$  at rate  $J$  required by the latter are seldom, if ever, available. As shown by Mr. Adlard, the approximation slightly underestimates the true annual premiums required; but the results are sufficiently accurate for practical purposes, although, as will be seen later, the effect on the reserves, if the formula is adopted for use in a valuation, is more serious. In practice, however, the restrictions placed on the payment of the death benefit, mentioned on p. 197 and the general custom of issuing bonds at level rates, irrespective of the applicant's age, render the theoretical formulæ inapplicable, and the usual custom is to charge net sinking fund premiums at a rate of interest equal to, or slightly higher than, that at which the Death Return is calculated. This course is theoretically correct, since in Formulae II and III above, if net premiums are charged, and  $j=i$ , both forms become  $\pi = \frac{1}{s_{a+1}-1}$ , a result apparent from general reasoning. Indeed, the use of net sinking fund premiums at a low rate of interest operates in favour of the Company, owing to the restrictions on the payment of the death benefit.

If the Office premiums are loaded sinking fund rates calculated at a relatively high rate of interest, such as  $3\frac{3}{4}$  per-cent, theoretically the rate of mortality should be introduced, and one of the above formulæ used; but in practice it will usually be best to find the rate of interest at which the Office premium per unit  $= \frac{1}{s_{n+1}-1}$ † and to use this to fix the maximum death benefit which can be allowed.

\* See also note on p. 233.

† Or for monthly premiums  $[12s_{\overline{1}}^{12} \cdot s_{\overline{n}}]^{-1}$ .

Where only the premiums paid are returned, without interest, in the event of death, the use of net sinking fund rates operates still more in favour of the Company, and would even enable lower subscriptions than usual to be charged, if the age of the insured were taken into account ; although, in practice, I cannot trace that this is ever actually done.

*Rates charged in Practice.*

In Tables II and III will be found a few specimens of the monthly subscriptions charged by various Companies for the terms most frequently adopted, selected from the last volume of the Board of Trade Returns.

In practice, as I have already mentioned, the monthly premiums are usually quoted for a bond of the nominal amount of £100. The latter, in many cases, however, carries with it a guaranteed bonus, varying with the term, so that for our present purpose, the bond may be treated as one for (£100 + guaranteed bonus). In Table II the rates are set out as quoted by the various Offices, the true sum assured being given above, and the monthly premium charged therefor, below. In Table III, these subscriptions have been reduced to the common basis of the monthly rate for a true sum assured of £100, in order that the rates may be easily compared, although the subsidiary details of the contracts issued by different Companies vary so greatly that care must be exercised in making a comparison.

As indicated by the footnote to the Tables, some of the rates shown entitle the bond to a share in the profits of the Company, over and above any bonus guaranteed. Table III apparently shows that the with-profit rates are slightly higher than the without-profit ; but this is solely due to accident, as an examination of all the rates available showed that no general rule could be laid down, and that in many cases the without-profit rates were higher than those charged by another company for a bond participating in profits. At the present time, however, the question of the relative rates for bonds with, and without, profits is of little or no importance, as, with one exception, the companies immediately concerned have not been working long enough to bring the subject of profits within the range of "practical politics."

It will also be noticed that some of the contracts, especially those for the shorter terms, do not carry house purchase benefits, the bonds in question being investment certificates pure and simple. These generally yield a slightly higher rate of interest

than the house purchase contracts of the same company, although this is not invariably the case.

A comparison of the subscriptions charged by any Company for a true sum assured of £100, with the net premiums at various rates of interest in Tables IV and V, will readily show the actual return realized by the holder. For example, the 30-year bonds in Table III, apart from the effect of any bonuses added over and above those guaranteed, yield rates of interest varying from about 2 per-cent to  $2\frac{1}{2}$  per-cent, while the bonds for ten years yield from about 2 per-cent to over 3 per-cent.

TABLE II.

*Specimen Rates of Subscription for various terms : showing (I) amount guaranteed at maturity ; (II) monthly premiums therefor.*

Office	TERM OF YEARS					
	10	12	15	20	25	30
A	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. d.
A	... 0 9 6	... 0 9 6	100 0 0 0 9 6	100 0 0 0 6 8	100 0 0 0 5 0	100 0 0 0 4 0
B	*137 10 8 N 1 0 0	*170 6 0 N 1 0 0	100 0 0 0 9 6	...	...	100 0 0 0 4 0
C	*135 7 6 N 1 0 0	...	121 0 0 0 11 2	...	...	115 6 0 0 4 4
D	139 14 10 1 0 0	104 13 8 0 12 8	105 17 0 0 9 10	107 16 0 0 6 10	109 15 0 0 5 4	111 14 0 0 4 4
E	*131 8 0 1 0 0	...	103 15 2 0 10 0	109 6 9 0 7 6	115 6 2 0 6 0	121 14 1 0 5 0
F	*137 11 4 N 1 0 0	*170 6 1 N 1 0 0	*227 6 5 N 1 0 0	*357 6 9 N 1 0 0	...	109 15 0 N 0 4 4

\* Indicates that House Purchase Benefits are not granted under these Tables.

TABLE III.

*Specimen Monthly Subscriptions to secure £100 at maturity, deduced from Table II, above.*

Office	TERM OF YEARS					
	10	12	15	20	25	30
A	£ s. d. ... ...	£ s. d. ... ...	£ s. d. 0 9 6	£ s. d. 0 6 8	£ s. d. 0 5 0	£ s. d. 0 4 0
B	*0 14 6½ N	*0 11 9 N	0 9 6	...	...	0 4 0
C	*0 14 9½ N	...	0 9 2¾	...	...	0 3 9
D	0 14 3¾	0 12 1¾	0 9 3½	0 6 4¼	0 4 10½	0 3 10¾
E	*0 15 2¾	...	0 9 7¾	0 6 10½	0 5 2½	0 4 1½
F	*0 14 6½ N	*0 11 9 N	*0 8 9¾ N	*0 5 7¼ N	...	0 3 11½ N

\* Indicates that House Purchase Benefits are not granted under these Tables.

All the bonds in the above Tables, except those marked N, appear to be entitled to a share of the profits of the Company, over and above any bonus guaranteed, although as this point is not always clearly stated in the Board of Trade Returns, it is sometimes difficult to determine whether or no any particular class of bond is entitled to participate.

#### *Basis of the rates used in Practice.*

Assuming that the premiums are to be calculated on a net sinking fund basis, the formula for the monthly premium required to provide £100 at the end of  $n$  years is, of course, on the basis of an effective annual rate of interest of  $i$  per annum.

$$\frac{100}{12(1+i)^{\frac{1}{12}} s_{\frac{1}{12}}^{(12)} s_{\frac{n}{12}}}$$

or, generally, for a premium payable " $m$ " times a year

$$\frac{100}{m(1+i)^{\frac{1}{m}} s_{\frac{1}{m}}^{(m)} s_{\frac{n}{m}}}$$

In practice, this form will usually be found more convenient than the neater formula, based on a nominal annual rate of interest, of  $100(s_{m+1} - 1)^{-1}$ , since, if  $m$  is at all large, the required values of  $s_{m+1}$  at  $\frac{j(m)}{m}$  are seldom available.

In the case of monthly premiums, the neglect of the small factor  $(1+i)^{\frac{1}{12}}$  causes hardly any appreciable difference, while the resulting formula  $\frac{100}{12s_1^{(12)} s_n}$  is theoretically correct if it is remembered that the monthly premiums are hardly ever actually received by the office until a month after they fall due. For quarterly premiums, however, the neglect of the corresponding factor  $(1+i)^{\frac{1}{4}}$  does cause an appreciable difference in the rates.

Many of the monthly rates charged are simply  $\frac{100}{12s_n^-}$  at various rates of interest, the neglect of  $s_1^{(12)}$  providing a small extra loading for the trouble involved by monthly payments.

In Table IV will be found the values of

$$\frac{100}{12s_n^-} \text{ and } \frac{100}{12s_1^{12} s_n^-}$$

for the terms and rates of interest in general use, from which it will be seen that the increase in the monthly subscription, caused by the neglect of the factor  $s_1^{(12)}$ , varies, on the basis of an effective annual rate of  $2\frac{1}{2}$  per-cent, from 2d. per month in the case of a bond for 10 years, to  $\frac{1}{2}d.$  per month in the case of one for 30 years, decreasing with the rate of interest and as the term of the bond is increased.

In Table V the quarterly rates by the formula

$$\frac{100}{4(1+i)^{\frac{1}{4}} s_1^{(4)} s_n^-}$$

are given, the values of  $\frac{1}{4s_n^-}$ , if required, being, of course, exactly three times the values of  $\frac{1}{12s_n^-}$  in Table IV.

#### *Provision for Expenses.*

If the subscriptions charged are unloaded sinking fund premiums at a low rate of interest, the only provision for expenses is the margin between the rate of interest actually earned, and

that assumed in the calculations, so that the actual allowance made is practically an unknown quantity. In order, therefore, that a true idea may be formed as to the sufficiency of the scale used in any particular case, it is necessary to construct a table of loaded premiums at a fairly high rate of interest, and this I have endeavoured to do in Tables VI and VII, which give the monthly and quarterly premiums for £100 for different terms, by various methods of calculation, as described below.

TABLE IV.

*Net Monthly Premiums to secure £100 at end of various Terms.*

$$\text{Formulae I. } \frac{100}{12s_{\bar{n}}}$$

$$\text{II. } \frac{100}{12s^{\frac{(12)}{1}} s_{\bar{n}}}$$

Formula No.	TERM OF YEARS									
	5	10	12	14	15	20	21	25	30	
I	£ s. d. 1 12 0 <sub>2</sub>	£ s. d. 0 15 2 <sub>3</sub>	£ s. d. 0 12 5 <sub>4</sub>	£ s. d. 0 10 5 <sub>3</sub>	£ s. d. 0 9 7 <sub>3</sub>	£ s. d. 0 6 10 <sub>2</sub>	£ s. d. 0 6 5 <sub>3</sub>	£ s. d. 0 5 2 <sub>1</sub> <sub>2</sub>	£ s. d. 0 4 1 <sub>2</sub>	
	II 1 11 9	0 15 1 <sub>4</sub>	0 12 4	0 10 4 <sub>1</sub> <sub>4</sub>	0 9 6 <sub>3</sub>	0 6 9 <sub>3</sub>	0 6 5	0 5 2	0 4 1	
I	1 11 10 <sub>2</sub>	0 15 0 <sub>2</sub>	0 12 3 <sub>1</sub>	0 10 3 <sub>1</sub>	0 9 5 <sub>3</sub>	0 6 8 <sub>2</sub>	0 6 3 <sub>4</sub>	0 5 0 <sub>2</sub>	0 3 11 <sub>2</sub>	
	II 1 11 6 <sub>2</sub>	0 14 10 <sub>3</sub>	0 12 1 <sub>3</sub>	0 10 2	0 9 4 <sub>2</sub>	0 6 7 <sub>2</sub>	0 6 3	0 5 0	0 3 11	
I	1 11 8 <sub>2</sub>	0 14 10 <sub>3</sub>	0 12 1	0 10 1 <sub>4</sub>	0 9 3 <sub>4</sub>	0 6 6 <sub>2</sub>	0 6 1 <sub>4</sub>	0 4 10 <sub>3</sub>	0 3 9 <sub>3</sub>	
	II 1 11 4 <sub>1</sub> <sub>2</sub>	0 14 8 <sub>2</sub>	0 11 11 <sub>2</sub>	0 9 11 <sub>2</sub>	0 9 2 <sub>2</sub>	0 6 5 <sub>2</sub>	0 6 0 <sub>2</sub>	0 4 10	0 3 9 <sub>2</sub>	
I	1 11 6 <sub>3</sub>	0 14 8 <sub>2</sub>	0 11 11	0 9 11 <sub>2</sub>	0 9 1 <sub>3</sub>	0 6 4 <sub>2</sub>	0 5 11 <sub>3</sub>	0 4 8 <sub>2</sub>	0 3 8	
	II 1 11 2	0 14 6 <sub>2</sub>	0 11 9 <sub>4</sub>	0 9 9 <sub>4</sub>	0 9 0 <sub>4</sub>	0 6 3 <sub>2</sub>	0 5 10 <sub>3</sub>	0 4 8	0 3 7 <sub>1</sub> <sub>2</sub>	
I	1 11 4 <sub>3</sub>	0 14 6 <sub>2</sub>	0 11 9	0 9 9 <sub>4</sub>	0 8 11 <sub>3</sub>	0 6 2 <sub>2</sub>	0 5 9 <sub>3</sub>	0 4 7	0 3 6 <sub>2</sub>	
	II 1 10 11 <sub>3</sub>	0 14 4	0 11 7 <sub>2</sub>	0 9 7 <sub>2</sub>	0 8 10 <sub>2</sub>	0 6 1 <sub>2</sub>	0 5 8 <sub>2</sub>	0 4 6 <sub>2</sub>	0 3 5 <sub>2</sub>	

TABLE V.

*Net Quarterly Premiums to secure £100 at end of various Terms.*

$$\text{Formula } \frac{100}{4s_{\frac{i}{4}}^4 (1+i)^{\frac{1}{4}} s_u}$$

Rate of Interest	TERM OF YEARS.								
	5	10	12	14	15	20	21	25	30
2 per-cent	£ s. d. 4 14 11	£ s. d. 2 5 1 $\frac{1}{4}$	£ s. d. 1 16 10	£ s. d. 1 10 11 $\frac{1}{4}$	£ s. d. 1 8 6 $\frac{3}{4}$	£ s. d. 1 0 4	£ s. d. 0 19 2	£ s. d. 0 15 5	£ s. d. 0 12 2
2 $\frac{1}{4}$ per-cent	4 14 3	2 4 6	1 16 3	1 10 4 $\frac{1}{4}$	1 8 0	0 19 9 $\frac{1}{2}$	0 18 7 $\frac{1}{2}$	0 14 11	0 11 8
2 $\frac{1}{2}$ per-cent	4 13 8 $\frac{1}{4}$	2 3 11 $\frac{3}{4}$	1 15 8 $\frac{1}{4}$	1 9 9 $\frac{3}{4}$	1 7 5 $\frac{3}{4}$	0 19 3 $\frac{1}{2}$	0 18 2	0 14 5	0 11 2
2 $\frac{3}{4}$ per-cent	4 13 0 $\frac{1}{2}$	2 3 4 $\frac{3}{4}$	1 15 1 $\frac{3}{4}$	1 9 3 $\frac{1}{4}$	1 6 11 $\frac{1}{4}$	0 18 9 $\frac{1}{4}$	0 17 7 $\frac{1}{4}$	0 13 11 $\frac{1}{4}$	0 10 9
3 per-cent	4 12 6	2 2 10 $\frac{1}{4}$	1 14 7 $\frac{1}{4}$	1 8 9	1 6 5	0 18 3 $\frac{1}{2}$	0 17 1 $\frac{1}{2}$	0 13 5 $\frac{3}{4}$	0 10 4

## TABLES VI AND VII.

a. *Terms.*

The terms for which rates are given in Tables VI and VII are 15, 20, 25, and 30 years, under which most of the business is transacted. For terms shorter than 15 years, the methods of loading adopted produce premiums far higher than could be used in practice; but it will usually be found that the bonds in question only form a very small portion of the total business, and that the commission allowed, and expenses generally, are so reduced as to give the same result per bond as under one for a much longer period, the expenses expressed as a percentage of the premium being very small.

b. *Rate of Interest.*

The rate of interest earned on their funds by House Purchase Companies is substantially higher than that earned by ordinary assurance companies, and varies from 4 $\frac{3}{4}$  per-cent to 6 per-cent, but I hardly think that it would be safe to rely upon the continuance for any length of time of some of the high rates apparently earned. In some cases, at least, these may be due

to accidental causes, and there is, moreover, some trace of a tendency for the rate earned to fall as the age of the Company, and the funds available for investment, increase. An average yield of from  $4\frac{1}{2}$  per-cent to 5 per-cent may, however, be safely counted upon so long as the advance system remains, as at present, the keystone of the business, and  $3\frac{3}{4}$  and 4 per-cent may therefore be looked upon as perfectly safe rates at which to calculate a trial set of premiums. I have also added, for purposes of comparison, a set at  $4\frac{1}{4}$  per-cent, although this is a somewhat high rate to assume in practice.

#### *c. Expenses.*

The expenses actually involved in carrying on the business are far more difficult to gauge than any other factor, as there are practically no data available. Many of the Companies have been working for too short a time for their experience to be any guide to the future; while the Board of Trade Returns do not call for any figures which would enable us to allocate the expenses between new business and renewals.

The commission paid to agents varies, of course, greatly in different Companies; but in the case of a bond for 30 years, a representative scale is to allow the agent 50 per-cent of the subscriptions for the first year, and 10 per-cent of the premiums thereafter in the case of monthly business, and 5 per-cent thereafter in the case of quarterly subscriptions. If anything, perhaps this is a trifle on the high side; the problem, however, being further complicated by the salaries and over-riding commission paid in particular cases.

On the whole, I think that the usual assumptions, that the initial expenses vary with the amount of the bond, independently of the term for which it is granted, while the renewal expenses form a percentage of the premium, give the best results, and I have accordingly prepared two sets of tables for each rate of interest.

1. On an assumption of initial expenses amounting to £2 per-cent of the amount of the bond and renewal expenses of 10 per-cent, 15 per-cent, and 20 per-cent of the premium throughout the whole term of the bond, including the first year.
2. Initial expenses of £1 per-cent on the amount of the bond and renewal expenses of 10 per-cent, 15 per-cent and 20 per-cent of the premium.

Six tables in all for each rate of interest.

In the case of a 30-year bond, the whole of the subscription for the first year is generally regarded as going in expenses, but £2 per-cent and 20 per-cent are probably higher than would usually be experienced, whilst £1 per-cent and 10 per-cent are too low.

*d. Formulae used.*

For monthly premiums the formula used was

$$\frac{100 + \text{Initial Expenses } (1+i)^n}{\begin{matrix} \cdot 90 \\ \text{or } \cdot 85 \\ \cdot 80 \end{matrix} \left\{ 12s_{\frac{1}{12}}^{(12)} s_{\bar{n}} \right\}} \quad . . . \text{ I}$$

and for quarterly premiums

$$\frac{100 + \text{Initial Expenses } (1+i)^n}{\begin{matrix} \cdot 90 \\ \text{or } \cdot 85 \\ \cdot 80 \end{matrix} \left\{ 4s_{\frac{1}{4}}^{(4)} (1+i)^{\frac{1}{4}} s_{\bar{n}} \right\}} \quad . . . \text{ II}$$

If desired these formulæ can easily be thrown into the form  $\pi(1+k) + c$ . For instance, at 4 per-cent

$$\frac{100 + 2(1+i)^n}{\cdot 90 \times \frac{(12)}{12} s_n} \text{ becomes } 111.308 \times s_{\bar{n}}^{-1} + \cdot 0873,$$

but there is little, if any, advantage in making the change.

Throughout, the rates have been calculated to the  $\frac{1}{d}$ . above, as it was found that if the nearest penny, or the penny above, was used, the differences did not run so smoothly, and minute variations, of some importance for experimental purposes, were masked.

The neglect of the factor  $(1+i)^{\frac{1}{12}}$  in Formula I was found not to cause an increase anywhere in the tables of more than  $\frac{1}{2}d.$ , most of the rates remaining unchanged.

It will be seen that the subscriptions calculated by Formulae I and II, above, completely ignore the death benefit; but, as indicated on p. 212, the exact allowance which, apart from any restrictions, can be made in this respect for any particular rate in Tables VI and VII, will be given by a comparison with the net rates in Tables IV and V.

*Use of the Tables.*

Tables V and VII, IV and VI, used in conjunction, will, I think, enable :

1. The approximate return obtained by the bondholder,
2. The approximate allowance for expenses,

for any scale of subscriptions, to be readily judged with sufficient accuracy for practical purposes. But it must be remembered that if the bonds for all terms are to yield the same rate of interest to the assured—as is the case if net rates at a low rate of interest are charged—the percentage loading for renewal expenses must be decreased with the term, an assumption, however, which is in accordance with practice. For instance, in the case of one representative Office, the renewal commission allowed to agents on monthly premiums is 10 per-cent under 30-year bonds, 5 per-cent under bonds for 15 or 20 years, and  $2\frac{1}{2}$  per-cent on bonds for shorter periods, and similar arrangements will usually be found to be in force.

As an example of the use of the tables, the following scale of monthly premiums per £100 :

30 years.	25 years.	20 years.	15 years.
3s. 11d.	5s.	6s. 8d.	9s. 5d.

would yield the assured by Table IV about  $2\frac{1}{4}$  per-cent; while by Table VI, on the assumption that the Office can rely upon earning 4 per-cent on its funds, the allowance for expenses contained in such a scale would be initial expenses at the rate of £2 per-cent on the sum assured and the following percentages for renewal expenses :

30 years.	25 years.	20 years.	15 years.
20 per-cent.	$17\frac{1}{2}$ per-cent.	14 per-cent.	10 per-cent.
*3s. $10\frac{3}{4}$ d.	5s $0\frac{1}{4}$ d.	6s. $8\frac{1}{4}$ d.	9s. 5d.

\* Exact rates on the above assumptions.

*Monthly Premiums for £100 Bond.—TABLE VI.*

$$Formula: \quad \frac{100 + \text{Initial Expenses}}{90} (1+i)^n$$

• 90  
 • 85  
 • 80

12<sub>[1]</sub><sup>{12}</sup><sub>[n]</sub>

Term <i>n</i>	INTEREST 3½ PER CENT						INTEREST 4 PER CENT						INTEREST 4½ PER CENT					
	Initial Expenses £2 per-cent.			Initial Expenses £2 per-cent.			Initial Expenses £2 per-cent.			Initial Expenses £2 per-cent.			Initial Expenses £2 per-cent.			Initial Expenses £2 per-cent.		
	10	15	20	10	15	20	10	15	20	10	15	20	10	15	20	10	15	20
s. d. 30	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	d. s. 3	
25	4 5 6 6½	9 10½ 11½ 12	5 6 7 7½	4 11 14 14	7 11 14 14	4 11 14 14	7½ 10½ 13½ 14	4 10½ 13½ 14	6 11 14 14	4 9½ 13½ 14	6 9 12 14	4 9 12 14	6 9 12 14	4 9½ 13½ 14	6 9 12 14	4 9½ 13½ 14	6 9 12 14	4 9½ 13½ 14
20	6 6½ 7 7½	10 10½ 11½ 12	10 11½ 13½ 14	6 9½ 11½ 12	10 14 17½ 20	10 14 17½ 20	7 11½ 17½ 20	10 14 17½ 20	7 11½ 17½ 20	9 13½ 19½ 23	10 14 17½ 20	9 13½ 19½ 23	10 14 17½ 20	9 13½ 19½ 23	10 14 17½ 20	9 13½ 19½ 23	10 14 17½ 20	
15	9 7½ 10 10½	12 10½ 10 9½	12 11½ 10 9½	9 11½ 10 9½	12 14 17½ 20	12 14 17½ 20	7½ 10 14 17½	10 14 17½ 20	7½ 10 14 17½	9 11½ 10 9½	10 14 17½ 20	9 11½ 10 9½	10 14 17½ 20	9 11½ 10 9½	10 14 17½ 20	9 11½ 10 9½	10 14 17½ 20	

TABLE VII.

*Quarterly Premiums for £100 Bond.*Formula :  $100 + \text{Initial Expenses} (1+i)^n$ 

$$\begin{cases} .90 \\ .85 \\ .80 \end{cases} \left\{ 4s_{1|}^{(4)} (1+i) \cdot 4s_{\bar{n}}^{(4)} \right\}$$

Term of Bond <i>n</i>	INTEREST 3½ PER-CENT						INTEREST 4 PER-CENT						INTEREST 4½ PER-CENT					
	Initial Expenses £2			Initial Expenses £1			Initial Expenses £2			Initial Expenses £1			Initial Expenses £2			Initial Expenses £1		
	10 per-cent	15 per-cent	20 per-cent	10 per-cent	15 per-cent	20 per-cent	10 per-cent	15 per-cent	20 per-cent	10 per-cent	15 per-cent	20 per-cent	10 per-cent	15 per-cent	20 per-cent	10 per-cent	15 per-cent	20 per-cent
30	10 8½	11 4	12 0½	10 4¾	11 0¾	11 8½	10 3½	10 10¾	11 7	9 11¾	10 6¾	11 2¾	9 11	10 6	11 1¾	9 7	10 1¾	10 9½
25	14 2	15 0	15 11¾	13 10	14 7¾	15 6½	13 8¾	14 6½	15 5½	13 4½	14 2	15 0½	13 3½	14 1	14 11½	12 11¼	13 8¾	14 6½
20	19 6	1 0 7¾	1 11¾	19 1	1 0 2¾	1 1 6	19 0	1 0 1½	1 1 4¾	18 7½	19 8½	1 0 11¾	18 6½	19 7½	1 0 10¾	18 1½	19 2½	1 0 4¾
15	1 8 7	1 10 3¾	1 12 1¾	1 8	1 9 9	1 11 7	1 8 0½	1 9 8½	1 11 6½	1 7 6¾	1 9 2	1 11	0 1 7	1 9 1¾	1 10 11¾	1 7 0½	1 8 7¾	1 10 5½

## II. SURRENDER VALUES AND PAID-UP POLICIES.

*Surrender Values.*—A cash surrender value, or a paid-up certificate payable at the original date of maturity, is usually granted after a bond has been in force for five years in the case of certificates for 15, 20, 25, or 30 years, or after three years in the case of certificates for shorter periods. Two or three Companies, however, do not pay cash surrender values, on the ground that all the funds are required for the purpose of granting advances to members, and hence are not immediately available, as would be the case were they invested in more readily convertible securities. Under these conditions a bondholder wishing to cease payment of his subscriptions has to accept a paid-up certificate—if these are granted—or to sell his certificate in the open market. The latter alternative is, of course, always open to anyone dissatisfied with the surrender values offered by a company, and the advertisement columns of certain newspapers show that some sort of market does exist; but there is no information available as to the prices realized, and the amount of the transfer fees shown in the published revenue accounts indicates that only a comparatively small number of bonds are so sold.

The guaranteed surrender values attaching to the bonds are, as a rule, only about one-third of the premiums paid, but the values actually paid in practice are higher than this, and the usual course is to grant a percentage of the premiums paid increasing with the duration of the bond, some representative scales being given below.

*Percentage of total premiums paid allowed as Cash Surrender Value under a Bond for 30 years.*

COMPANY	YEARS IN FORCE				
	5	10	15	20	25
A.	33 $\frac{1}{3}$	40	50	66 $\frac{1}{2}$	91
B.	33 $\frac{1}{3}$	40	54	72	96
C.	33 $\frac{1}{3}$	42 $\frac{1}{2}$	55	80	105
D.	43 $\frac{1}{3}$	59	74	92	112 $\frac{1}{2}$

### *Paid-up Policies.*

The amount granted as a paid-up certificate payable on the original date of maturity, after a bond has been a certain length

of time in force, also varies greatly with different companies, the following methods being those chiefly used :

- i. The paid-up certificate granted is equal to the total premiums paid. In symbols, if " $t$ " is the number of annual premium paid, " $n$ " the original term of the bond, and  $P$  the annual premium, or twelve times the monthly payment—

$$\text{Paid-up policy} = P \times t.$$

From Table VIII, and general considerations, it will be seen that the values yielded by this method are extremely low.

- ii. The paid-up certificate is equal to the total premiums paid with compound interest—at  $2\frac{1}{2}$  or 3 per-cent—up to the date of conversion—

$$\text{Paid-up policy} = P \times s_{\bar{i}}^{-(1*)}$$

- iii. The paid-up certificate is equal to the total premiums paid, together with compound interest thereon at  $2$  per-cent, or  $2\frac{1}{2}$  per-cent, for the period from the date of conversion to the date of maturity—

$$P \times t \times [1+i]^{n-t}$$

- iii<sub>a</sub>. In one case  $2\frac{1}{2}$  per-cent is apparently allowed on the premiums paid from the outset, or

$$P \cdot s_{\bar{i}} [1+i]^{n-t} \text{ at } 2\frac{1}{2} \text{ per-cent.}$$

- iv. A paid-up certificate is granted for such proportionate part of the sum assured, as the number of premiums actually paid bears to the number originally contracted for—

$$\text{Paid-up Policy} = \frac{t}{n} \times \text{Sum Assured.}$$

In some cases, the paid-up certificate is payable on the death of the bondholder, should this happen before the original date of maturity, but this concession does not usually extend to a transferee from the original holder. Such a paid-up certificate is, of course, an endowment assurance, and unless the Company has power to transact life business, its issue appears to involve a transgression of the Assurance Companies Act.

(1\*) See p. 228 as to the introduction of the factor  $s_{\bar{i}}^m (1+i)^{\frac{1}{m}}$  necessary for complete accuracy.

*Principles governing the granting of Surrender Values and Paid-up Policies.*

Although the surrender values given by the specimen scales on p. 224 undoubtedly appear low, especially when compared with those granted by ordinary Offices under sinking fund policies, it must be remembered that the contracts issued by House Purchase Companies are for comparatively small individual amounts, under which the expenses are high. Moreover, the fundamental principle of the latter business is that all the funds available should be advanced to the bondholders for the purpose of purchasing house property, so that the assets of the Company are to a very considerable extent locked up in long-term securities, which it would be almost impossible readily to realize in an emergency.

House Purchase Companies, therefore, must guard against the risk of a sudden call on their resources, to which a high scale of guaranteed surrender values might expose them in the event of a panic; otherwise the business would practically become banking, and it would be necessary to keep a considerable portion of the funds invested in readily convertible securities.

For this reason, I think that the most satisfactory course is only to *guarantee* a very low scale of cash surrender values, such as 30 to 40 per-cent of the premiums paid, and to couple this with a relatively high scale of guaranteed paid-up policies, at the same time adopting a scale of surrender values for Office use, under normal conditions, sufficiently high to afford the bondholder little cause for complaint.

There should be a fairly close relationship between the values of the paid-up policies and the surrender values, as otherwise anomalies will arise, particularly as a bond nears its maturity, and due regard must also be paid to the return given in the event of death, and to the basis on which the premiums are calculated. For example, in the rather unlikely event of the paid-up certificate, payable at death or at the original date of maturity, exceeding the amount payable in the event of death at the date of conversion, evidence of health should be obtained, as otherwise a very strong selection would be exercised against the Office. Again, under certain scales, it might be more profitable, as a certificate approaches its maturity, to convert it into a paid-up assurance than to pay the few remaining premiums. In practice, I think that the most satisfactory course to adopt is to fix the scale of paid-up values, and then

to deduce therefrom the corresponding cash surrender values, although, of course, the reverse process could be followed with equally good results.

*Effect of some of the Methods adopted in Practice.*

In Table VIII are set out the results given by various methods of calculating surrender values and paid-up policies in the case of a bond for £100, by monthly payments, payable in 30, 25 and 20 years, for quinquennial periods in force.

The scale of monthly premiums adopted was as follows :

30 years.	25 years.	20 years.
4s. per month.	5s. per month.	6s. 8d. per month.

These are fair average rates, yielding the assured about  $2\frac{1}{4}$  per-cent, from which any subscriptions charged in practice hardly differ sufficiently to affect the value of the general results obtained.

*Paid-up Policies.*—The methods of calculating the paid-up policies, payable at the original date of maturity, values of which are given, are as follow :—

(As usual,  $t$  is the duration and  $n$  the original term of the bond, while  $P$  is the annual payment, or twelve times the monthly subscription.)

Col. 3.—Paid-up policy equal to the total premiums paid without interest.

Col. 4.—Paid-up policy proportionate to the number of premiums paid.

$$\frac{100 \cdot t}{n}$$

Col. 5.—Total premiums paid with  $2\frac{1}{2}$  per-cent compound interest from the inception of the contract to the date of conversion.

$$P \times s_t^- \text{ at } 2\frac{1}{2} \text{ per-cent.}$$

Col. 6.—As per Col. 5, except that 3 per-cent compound interest is allowed.

Col. 7.—Total premiums paid accumulated at  $2\frac{1}{2}$  per-cent from the date of conversion to the date of maturity.

$$t \cdot P(1+i)^{n-t} \text{ at } 2\frac{1}{2} \text{ per-cent.}$$

Col. 8.—Total premiums paid with  $2\frac{1}{2}$  per-cent compound interest from the inception of the contract until the date of maturity.

$$P \times s_t^- \times (1+i)^{n-t}$$

To be strictly accurate, the factor  $s_{\overline{1}}^{(12)} \times (1+i)^{\frac{t}{12}}$  should be introduced into Nos. 5, 6 and 8 above, as the premiums are payable monthly, in advance, but this has been ignored, as being more in accordance with what I believe to be the course usually followed in practice.

For purposes of comparison, the reserve values on a  $3\frac{1}{2}$  per-cent basis are added in Col. 2, as a rough indication of the amount the Office would have in hand under the bonds at various dates. These values are calculated by the formula  $100(v^{n-t} - \pi a_{n-t})$ : the following percentages being thrown off the value of the annual payment in each case as a provision for future expenses; 30-year bonds, 17·79 per-cent; 25 and 20-year bonds, 15 per-cent. The somewhat peculiar percentage in the case of the 30-year bond is due solely to the use of some reserve values which had already been calculated for another purpose.

From this table it will be seen that the highest scale of values is given by Method IIIa,— $P.s_{\overline{1}}.(1+i)^{n-t}$  at  $2\frac{1}{2}$  per-cent (Col. 8); the results being, indeed, so high that it appears doubtful whether this method makes sufficient provision for the expenses involved in placing the bond on the books, unless at the date of conversion it has been a considerable period in force. Method III,— $t.P.(1+i)^{n-t}$  at  $2\frac{1}{2}$  per-cent (Col. 7), gives very high values if the bond is converted during the early years of its existence, but has the disadvantage that the values are extremely low if the certificate has been many years in force.

Method II,— $P \times s_{\overline{1}}$  at  $2\frac{1}{2}$  per-cent, and 3 per-cent (Cols. 5 and 6), on the contrary, gives low values during the early years of a bond's existence, and high ones if it had been some time in force, at the date of conversion.

To be strictly equitable, the paid-up policy given by any formula after the bond has been the full period in force (*i.e.*,  $t=30$ ) must be equal to the sum assured. Tested by this criterion, Methods IIIa— $P.s_{\overline{1}}.(1+i)^{n-t}$ —and II— $P \times s_{\overline{1}}$ —give good results, if the rate used in their calculation is the same as that on which the premiums are based, but not otherwise. But, as stated above, the results given, if the bond has only been a short period in force are unsatisfactory; in case of Method II, since the values given are too low, and, in the case of Method IIIa, for the opposite reason. I think, therefore, that Method IV (proportionate paid-up policy) is the most

satisfactory of those described. It is true that it entirely loses sight of the actual premiums paid; but, on the other hand, it is extremely simple and easily explained to the public, and is, moreover, a method which gives results, equitable to both parties, throughout the whole duration of bonds for all terms. It will be remembered that, as shown in *Text-Book*, Part I, p. 139, the proportionate paid-up policy is less than the theoretical paid-up equivalent.

### *Death Benefit under Paid-up Certificates.*

Very little information is available as to the position of the holder of a paid-up certificate in the event of his death before its maturity, and from the scanty data at hand, it is practically impossible to form any opinion as to the effect of granting in lieu of a bond, a paid-up certificate payable at death, or at the original date of maturity.

Before any exact calculations could be made it would be necessary to know—

1. The average age at entry of the bondholders, as the death benefits given are the same for all ages at entry.
2. The rate of mortality experienced, which is probably nearer to that shown by a population table, such as the English No. VI, than to a table like the O<sup>M</sup>, although, as stated below, in Col. 9, I used the latter for the sake of convenience.
3. The effect of any restrictions placed on the payment of the death benefit, of the nature of those given on p. 197.

If any information on these points were available, a scale of surrender-values—not necessarily the cash values payable to the assured—could be fixed, the application of which, as the single premiums for endowment assurances maturing at the original date of the bond, would give a scale of death benefits for comparison with the actual paid-up policies allowed.

At present no data exist, but in Col. 9 of Table VIII are given the results obtained if the cash surrender values given in Col. 12 are applied as the single premiums for endowment assurances maturing at the original date of the bond, on the basis of the O<sup>M</sup> Table, and an average age at entry of 30, with interest at 4 per-cent, the actual formula used being—

$$\{ \text{Surrender Value} \div A_{30+t} : \overline{n-t} \}$$

So many assumptions are, however, involved in these figures that they cannot be regarded as of any but the very roughest value ; but a comparison with the results obtained by applying the reserves in Col. 2 as single premiums, instead of the surrender values on Col. 12, tended to show that the grant, for instance, of a proportionate paid-up policy, to take an extreme case, payable at death or maturity, is probably within the power of the Office, although the return made to the assured is certainly very liberal. Under Method II (Cols. 5 and 6), the concession could be allowed without difficulty ; while under Methods III and III $a$ , the amount receivable at death would depend entirely upon the period elapsed since the date of conversion, so that the effect of these methods is simply that the premiums are deposited with the Office, and a low rate of interest allowed thereon until they become repayable.

#### *Cash Surrender Values.*

As will be seen from the specimen scales given on p. 224, the cash surrender values given by different Offices vary very greatly, and for this reason it is difficult to discuss them at any length without criticising the actual values allowed in practice by various Offices. Col. 2 of Table VIII shows, however, roughly, the amount which will be in hand under average conditions at the end of various periods, and the exact proportion of this returned to the assured depends entirely upon the views of the management. The problem, too, is further complicated by the almost infinite number of different methods of calculating surrender values which might be adopted in practice ; most of the scales at present in use appearing to be more or less arbitrary. For comparison, however, I have given, in Table VIII (Cols. 10, 15), the surrender values brought out by various methods, as follow :—

Col. 11.—Cash surrender values produced by applying Scale A, on p. 224, to the total premiums paid in Col. 3.

Col. 15.—Similar to Col. 11, except that Scale D on p. 224 is used, the latter being the highest scale with which I am at present acquainted.

Cols. 12 and 13.—The cash surrender values produced by multiplying the proportionate paid-up policy in Col. 4 by the values of  $v^{n-t}$  at 5 per-cent and  $4\frac{1}{2}$  per-cent respectively.

TABLE VIII. Paid-up Policies and Surrender Values by various methods.

## 30-Year Bond. Monthly Premium, 4s. £2 8s. per annum.

Duration $t$	Reserves Total Premiums Paid per-cent	Paid-up Policy						Surrender Value						Scale D Per-cent of Premiums Paid	
		Method IV	Method II	Method III	Method IIIa	Proprietary Paid-up Policy per-cent	$P_s t \{1 + i\}^{n-t}$	Method IIIa $\frac{2}{3}$ per-cent	Hypothetical Death Benefit per-cent	Scale A Per-cent of Premiums Paid	Surrender Value by Scale A	Proprietary Paid-up Policy $\times \frac{1}{5}$	Scale D Per-cent of Premiums Paid		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
5	£ 8.66	£ 12	16.67	12.62	12.74	£ 22.25	23.40	11.29	4.90	4.92	5.5	4.34	5.25	5	
10	21.23	24	33.33	26.88	27.50	39.33	44.06	24.74	40	9.6	12.56	13.8	59	14.20	10
15	36.17	36	50.00	43.03	44.64	62.14	62.32	40.50	50	18.0	21.05	25.8	74	26.60	15
20	53.91	48	66.67	61.32	64.49	78.49	78.72	58.59	66.1	31.9	40.93	42.9	92	44.20	20
25	74.98	60	83.33	81.98	87.50	92.73	92.73	78.72	91	54.6	65.29	66.9	112 <sup>a</sup>	67.50	25
25-Year Bond. Monthly Premium, 5s. £3 per annum.															
5	12.75	15	20.00	15.77	15.93	24.58	25.85	15.17	33 <sup>b</sup>	5.0	7.54	8.3	55	8.25	5
10	29.30	30	40.00	33.61	34.39	43.45	48.67	32.88	47	14.1	19.24	20.7	71	21.30	10
15	48.94	45	60.00	53.80	55.80	57.61	68.87	53.15	64	28.8	36.83	38.6	88	39.60	15
20	72.28	60	80.00	76.64	80.61	87.88	86.70	75.75	88	52.8	62.68	64.2	108	64.80	20
20-Year Bond. Monthly Premium, 6s. 8d. £4 per annum.															
5	19.16	20	25.00	21.02	21.24	28.97	30.44	20.75	33 <sup>b</sup>	6.7	12.02	12.49	65	13.0	5
10	41.62	40	50.00	44.81	45.86	51.20	57.36	44.55	56 <sup>b</sup>	22.6	30.70	32.2	82	32.8	10
15	68.31	60	75.00	71.73	74.40	87.88	81.13	71.14	81	48.6	58.76	60.2	101	60.6	15

The latter system works well in practice, since the actual cash values given may easily be adjusted to varying circumstances and views by altering the rate of interest at which the paid-up policies are discounted. The results, too, are consistent under all conditions, and can easily be expressed in the form of a percentage of the total premiums paid, in order to facilitate their comprehension by the public.

### III. VALUATIONS.

From a theoretical point of view, the return of premiums with compound interest, in the event of death, renders the valuation of the contracts issued by House Purchase Companies extremely complicated, but, as will be seen later, in practice this difficulty is, fortunately, more apparent than real, as in most cases the burden of the death return may be thrown upon the interest margin, and the contracts valued on a purely sinking-fund basis.

As already stated, the subscriptions charged are, usually, net sinking-fund rates, calculated at a rate of interest equal to, or slightly greater than, the rate with which the premiums are returned. If this rate is also used in calculating the reserves, no difficulty, of course, arises, as mortality may be ignored, and the reserve under a bond for " $n$ " years, " $t$ " years in force, is simply

$$\frac{s_t}{s_n}$$

In practice, however,  $j$  varies from 2 per-cent to 3 per-cent, the rate most commonly employed being  $2\frac{1}{2}$  per-cent, and hence the reserves required by this method are so heavy as to be quite beyond the reach of most Offices of the class in question.

If  $P_n^-$  = Office annual premium,

$\pi_n^-$  = valuation net premium,

$t$  = duration, and

$x$  = age at entry,

the reserve at the end of  $t$  years in respect of a bond maturing at age  $x+n$  under which all premiums are to be returned with compound interest at rate  $j$  in the event of death before maturity, is

$$\frac{D_{x+n}}{D_{x+t}} - \pi_n^- a_{x+t} : \bar{n-t} + P_n^- \cdot t F_{xn^-}$$

Here  $tF_{x\bar{n}}$ , the value of the death benefit, is :

$$A. \quad tF_{x\bar{n}} = D_{x+t}^{-1} \left\{ (1+j)s'_{t+1} M_{x+t} - (1+j)s'_{\bar{n}} M_{x+n} + \frac{1}{(1+j)^{x-1}} (R'_{x+t+1} - R'_{x+n}) \right\}$$

or approximately, using the approximation already mentioned on p. 212,

$$B. \quad tF_{x\bar{n}} = A_{x+t : \bar{n-t}}^1 \left\{ (1+j)s'_{\bar{t}} + \frac{(1+j)^t [\mathbb{N}_{x+t} - \mathbb{N}_{x+n} - (1+i)s_{n-t}^{(i)} D_{x+n}]}{D_{x+t} \cdot A_{x+t : \bar{n-t}}^1} \right\}$$

where the accentuated factors, except  $R'_x$ , are calculated at rate  $j$ , and the other factors in the formulæ at the valuation rate  $i$ .

$$\text{Note.} \quad R'_{x+t+1} = \Sigma (1+j)^{(x+t+1)} \times M_{x+t+1} \dots$$

so that

$$R'_{x+t+1} - R'_{x+n} = (1+j)^{x+t+1} M_{x+t+1} + (1+j)^{x+t+2} M_{x+t+2} + \dots + (1+j)^{x+n-1} M_{x+n-1}.$$

These formulæ are better suited, I believe, to the numerical calculations required in the present instance than any previously given, from which they differ slightly.

Formula A may be readily obtained from the expression for the value in question given by Messrs. Ross and Robertson, “Actuarial Theory”, p. 371,  $R'_x$  being substituted for  $\Sigma (1+j)^x M_x$ . The approximation used in Formula B was, I believe, first mentioned by the late Mr. R. P. Hardy in 1876, *Journal*, Vol. xxiii, p. 224, where it is referred to as “well-known.” It was further used by Mr. H. T. Adlard, Vol. xxxvi, p. 389, where it is shown to give very accurate results in the case of annual premiums, although, as stated below, it understates the reserves, if it is used for valuation purposes.

The problem of the reserve to be made to similar contracts is, of course, a very old one, which has been many times dealt with by various writers. Messrs. G. King, Vol. xxxix, p. 151, and E. E. Thomas, Vol. xxxviii, p. 277, have thoroughly investigated a similar, but more complicated (owing to the introduction of a salary scale) form of the same problem, in connection with

the valuation of pension funds ; and Formula A is little more than a modification of their formulæ, on a purely theoretical basis. For reasons which will be seen later, I did not consider it worth while entering into the question of the modifications necessary before Formulae A and B could be used in a valuation, as they are fairly obvious, and as will be seen from the papers mentioned above, present little difficulty in practice.

*Difference in the results produced by Formulae A and B.*

Although the approximation used in Formula B gives very good results when applied to the calculation of annual premiums, I found on investigation that it systematically understated the reserves required, and no further use was accordingly made of it ; Formula A only being used. The alternative form might, perhaps, be useful, however, for the approximate calculation of isolated values, and it is for this reason that it is mentioned here. In the following tables a few specimens of the differences in the values of  $tF_{x\bar{n}}$ , and the reserves produced by the two formulæ are shown ; only reserves for age at entry 40, the highest likely to occur often in practice, being given, as the divergence between the two forms increases with the age at entry. As will be seen, the difference after the few first years of duration is small, but is nevertheless sufficiently marked to render Formula B unsuitable for the purpose in view.

TABLE IX.

*Differences in the Values of  $tF_{x\bar{30}}$  by Formulae A and B.*

*Bond for 30 years.*

$O^{M(5)}$  Mortality,  $i = 3\frac{1}{2}\%$ ;  $j = 2\frac{1}{4}\%$

Duration $t$	AGE AT ENTRY 20			AGE AT ENTRY 30			AGE AT ENTRY 40		
	Formula A	Formula B	Difference	Formula A	Formula B	Difference	Formula A	Formula B	Difference
1	2.638	2.553	.085	3.906	3.753	.153	6.275	5.998	.277
2	2.735	2.652	.083	4.058	3.908	.150	6.540	6.273	.267
3	2.829	2.747	.082	4.209	4.064	.145	6.807	6.551	.256
5	3.005	2.930	.075	4.502	4.369	.133	7.347	7.113	.234
10	3.356	3.300	.056	5.159	5.062	.097	8.672	8.501	.171
15	3.493	3.460	.033	5.569	5.511	.058	9.744	9.641	.103
20	3.245	3.232	.013	5.416	5.391	.025	9.977	9.930	.047
25	2.287	2.285	.002	4.037	4.032	.005	7.946	7.939	.007

The differences in the Reserves produced by the two formulæ are as follows, under the same conditions as above :

$$P_{\overline{30}} = \frac{1}{s_{31} - 1} @ 2\frac{1}{4}\% = £2. 6s. 5d.$$

$$\pi_{\overline{30}} = .85 P_{\overline{30}} = 1.973.$$

*Reserves for £100 bond. Term, 30 years. Age at Entry, 40.*

Duration	Formula A	Formula B	Difference	Formula B as percentage of Formula A
1	1.273	.631	.642	49.57
2	3.343	2.724	.619	81.49
3	5.481	4.887	.594	89.15
5	9.975	9.431	.544	94.56
10	22.572	22.174	.398	98.24
15	37.363	37.124	.239	99.36
20	54.764	54.655	.109	99.82
25	75.312	75.296	.016	99.98

NOTE.—For age 20 at entry, duration 1, the reserve by Formula B is 68.12 % of that produced by Formula A.

#### *Difference between Sinking Fund Reserves and those by Formula A.*

Although, from a theoretical point of view, it is incorrect to value the contracts in question as sinking fund policies, it will usually be found in practice that, quite apart from the cumbrous nature of Formula A, it is practically impossible to treat them in any other way, owing to the difficulties mentioned on p. 212 in connection with the calculation of premiums for these benefits.

The difficulty as to the absence of reliable information as to the age of the bondholder might be overcome by an investigation into the average age at entry in those cases where it is known, upon the results of which an average age at entry might be assumed, since Tables X to XIII show that a variation of a few years in the age at entry has but little effect upon the true reserves, and care could be taken that the error involved was on the safe side. A more serious obstacle, however, to the accurate valuation of the business is the restrictions usually placed upon the payment of the death benefit.

No doubt, in some cases, after a Company has been working for many years, a modified rate of mortality might be deduced from its own experience, which would give effect to all the

factors involved, and enable the actuary to make an approximately correct valuation; but as a rule, the information available is so scanty, and any attempt at a valuation on theoretical lines would involve so many assumptions, that the results would be almost valueless.

### Tables X to XIII.

In order to test the error involved by valuing bonds, under which the premiums are returnable with compound interest in the event of death before maturity, on a purely sinking fund basis, Tables X to XIII have been constructed, in which the reserves required by Formula A and the corresponding reserves on a sinking fund basis, are compared.

In these tables are given—

1. The reserves required by Formula A for a 30-year bond for £100, for three ages at entry 20, 30 and 40, and quinquennial durations, the values at the end of the first, second and third years of “insurance” being also given.
2. The corresponding reserves on a sinking fund basis  $tV_{\bar{n}}$ .

The investigation was confined to the case of bonds for 30 years, for two reasons. First, I have reason to believe that bonds for this period form the vast majority of the business at present in force; and secondly, it is in the case of bonds for this or longer periods that the effect of the death benefit on the reserves is most marked; an investigation showing that the differences between the reserves by Formula A and  $tV_{\bar{n}}$  are much less marked, and diminish very rapidly, as the term of the bond is decreased.

The rate of mortality used in the calculation of the true reserves, *i.e.*, those by Formula A, was that of the O<sup>M(5)</sup> Table, which I judged to be the most appropriate, in the total absence of any information as to the rate of mortality actually experienced by these Companies. From the nature of the contracts, moreover, I do not anticipate that the rate of mortality actually experienced will have any very marked effect on the reserves required.

The Office premiums assumed were net annual sinking fund rates  $\frac{100}{s_{311} - 1}$  at the rate of interest used in the calculation of the death benefit, adjusted to the penny above, and

the loading thrown off for future expenses was 15 per-cent of the Office premium, so that throughout

$$\pi_{\bar{n}} = .85 \cdot P_{\bar{n}}.$$

The conditions assumed in the Tables may be summed up as follow :

*Reserves by Formula A for a Bond for 30 years, under which all Premiums are to be returned with compound interest at rate  $j$  in the event of death before maturity, on the the following assumptions :*

**O<sup>M(5)</sup> Mortality.**

Table	Valuation rate of interest $i$	Rate used in calculating Death return $j$	Office Premium	Net Valuation Premium
X	$3\frac{1}{2}$ per-cent	$2\frac{1}{4}$ per-cent	2.321 per-cent	1.973 per-cent
XI	$3\frac{1}{2}$ “	2 “	2.417 “	2.054 “
XII	4 “	$2\frac{1}{4}$ “	2.321 “	1.973 “
XIII	4 “	2 “	2.417 “	2.054 “

The values of  $tF_{x:\bar{30}}$  used in the calculations are also given separately in Table XIV, as they may possibly be of assistance in any future investigation on the above lines.

#### *Effect of any alteration in the Premiums assumed above.*

An alteration in the percentage of the subscriptions reserved as a provision for future expenses will not affect the relative values of the “true” reserves and  $tV_{\bar{n}}$  to any great extent, and may easily be given effect to, if required, by “adding” the values of  $(\pi - \pi')a_{x+t:\bar{n-t}}$  to the reserves by Formula A, and  $(\pi - \pi')a_{\bar{n-t}}$  to the sinking fund reserves, where  $\pi'$  is the new net premium. If the subscriptions charged are different from those assumed above, the allowance for future expenses being still 15 per-cent, the reserves by Formula A will be “increased” by

$$(P' - P)(tF_{x:\bar{30}} - .85a_{x+t:\bar{30-t}});$$

the general effect of an increase in the Office rates assumed being thus to *diminish* the true reserves up to a certain point, i.e., so long as  $.85a_{x+t:\bar{30-t}} > tF_{x:\bar{30}}$ , and thereafter to *increase* them. If both the subscriptions charged, and the allowance for expenses are altered, the true reserves will be “increased” by

$$(P' - P) \cdot tF_{x:\bar{30}} - (\pi' - \pi)a_{x+t:\bar{30-t}}.$$

The values of  $tV_{\bar{s}0}$  are, of course, only affected by any alteration in the net premium valued, and in the last case would be "diminished" by  $[\pi' - \pi] \cdot a_{n-t}$ .

### *Description of Tables X to XIII.*

In each Table, Cols. 2, 3, and 4, show the reserves required by Formula A for ages at entry 20, 30 and 40.

Col. 5 contains the difference between Cols. 2 and 4, and shows the effect of the age at entry upon the reserves, which, as would be expected, increase with the age at entry; the actual difference increasing with the duration of the bond up to about duration 10, and thereafter diminishing. Expressed as a percentage of the reserve at each duration, the difference is heavy at first, but rapidly decreases.

Col. 6 gives the average of the reserves required by Formula A for the three ages at entry, for comparison with the sinking-fund reserves, and thus roughly corresponds to an age at entry of 36 or 37. This is probably a trifle higher than would usually be met with, and thus the general effect is to slightly exaggerate the difference between the true reserves and  $tV_n$ .

Col. 7 gives the reserves required on the assumptions and rate of interest used in Cols. 2 to 4, except that mortality is ignored, and the reserves are calculated by the formula  $v^{n-t} - \pi_{\bar{n}} \cdot a_{n-t}$ . In Col. 8 is given the difference between these reserves and the average "true" reserves in Col. 6; and thus shows the effect of ignoring the death benefit in valuing these contracts, and treating them as purely sinking fund policies.

As would have been expected from general considerations, the effect of such an assumption is to under-estimate the true reserves throughout, the difference being very great during the first few years of a bond's existence, but rapidly decreasing, until at maturity, of course, the two reserves are identical.

In any particular case the exact error involved by valuing the bonds as sinking fund policies will depend upon :

1. Any restrictions placed upon the payment of the death benefit, particularly during the first few years of "assurance."
2. The average duration of the bonds in force.
3. The average age at entry.
4. The premiums charged, the reserve made for expenses, and the rate of mortality experienced.
5. The difference between  $i$  and  $j$ .

The remaining columns of Tables X to XIII show the sinking fund reserves,  $v^{n-t} - \pi_n \mathbf{a}_{n-t}$ , at a lower rate of interest than that employed in the calculation of the true reserves, using the same net premiums. These, I think, reveal two facts :

1. That although it is impossible exactly to represent the "true" reserves by decreasing the rate of interest, and using a sinking fund formula (the shape of the two curves being totally different), yet a fairly close approximation may be obtained.
2. The difference between the sinking fund reserves and the average "true" reserves varies mainly with the difference between the valuation rate of interest  $i$ , and the rate  $j$  used in the calculation of the premiums and the death return. The general effect of the four tables in this respect may be briefly summarised as follows : (a still closer approximation to the true reserves, and a more regular progression in the values of  $i-I$ , being obtained, if the values of  $I$  used in the investigation proceed by differences of  $\frac{1}{32}$  per-cent) :

Valuation Rate of interest $i$	Rate used in calculation of Death Return $j$	$i-j$	Rate of interest I at which the Sinking Fund Reserves on the average approximate to "true" reserves	$i-I$
$3\frac{1}{2}$ per-cent	$2\frac{1}{4}$ per-cent	$1\frac{1}{4}$ per-cent	$3\frac{6}{7}$ per-cent	$\frac{2}{9}$ per-cent
$3\frac{1}{2}$ "	2 "	$1\frac{1}{2}$ "	$3\frac{5}{7}$ "	* $\frac{3}{16}$ "
4 "	$2\frac{1}{4}$ "	$1\frac{1}{4}$ "	$3\frac{13}{14}$ "	* $\frac{1}{16}$ "
4 ",	2 ",	2 ",	$3\frac{1}{2}$ "	$\frac{4}{9}$ "

\* Obtained by interpolation.

As already stated, the difference between the true reserves by Formula A, and those on a sinking fund basis at the same rate of interest, will, no doubt, be greatly diminished in many cases by the effect of any restrictions placed on the payment of the death benefit, particularly during the early years of "assurance"; and provided, therefore, that a proper allowance has been made for future expenses, and the valuation rate of interest is, at least,  $\frac{1}{2}$  per-cent to  $\frac{3}{4}$  per-cent below that which may safely be assumed to be earned in the future, I think that until some better method of dealing with the problem is evolved, or the data necessary for an exact valuation are available, the only

practical method which can be adopted in practice is to value the contracts by the formula  $v^{n-t} - \pi \cdot a_{n-t}$ , and to leave the death benefit as a charge upon the margin between the rate of interest actually earned and that assumed in the valuation. If, however, for any reason, the average age at entry is unusually high, or the average duration of the contracts unusually low, it may be necessary to make some additional reserve.

The details of these conclusions, though not the general principles, would, of course, be altered by any great variation in the assumptions on which the Tables are based, and hence the latter can only be regarded as giving a general indication of the results to be expected in any particular case. It is, however, comparatively easy to investigate the question on the above lines with special regard to the exact circumstances of the Company under immediate consideration, and as the results already obtained accord well with those expected from general considerations, I did not deem it necessary to go further into the matter on the present occasion. I feel, too, that in order thoroughly to investigate the problem it would be necessary construct a “model” Office on the usual lines, and to test the effect of valuing this by various methods; but the necessary data are not at hand, and so many assumptions would have to be made that the results would be almost valueless from a practical point of view.

*Premiums returned in the event of death WITHOUT interest.*

The preceding investigation has been confined to the case where the premiums paid are returned with compound interest in the event of death before maturity. If, however, only the premiums paid are returned in the event of death *without* interest, the value of  $tF_{x_n^-}$  becomes

$$\frac{tM_{x+t} + R_{x+t} - R_{x+n} - uM_{x+n}}{D_{x+t}}$$

and the effect of valuing the contracts as sinking fund policies is to over-estimate the true reserves required, unless the age at entry is very high.

TABLE X.

*Differences between the Reserves for a Bond for £100, Term 30 Years,  
by Formula A, p. 233, and those on a Staking Fund Basis.*

Duration $\ell$	Reserves by FORMULA A						$v^{n-t} - \pi \bar{a}_{n-t}^{3\frac{1}{2}}$ $3\frac{1}{2}$ per-cent ( $\pi = 1.973$ )	S.E. Reserve $v^{n-t} - \pi \bar{a}_{n-t}^{3\frac{1}{2}}$ $3\frac{1}{2}$ per-cent	Average Increase in Reserve due to Death Benefit, Col. (6) - Col. (7)	Difference between Reserves at 3 $\frac{1}{2}$ per-cent and Average Reserves Col. (9) - Col. (6)				
	Ages at Entry			Difference between Reserves for Ages 20 and 40 at Entry Col. (4) - Col. (2)										
	20	30	10	(4)	(5)	(6)								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	S.E. Reserve $v^{n-t} - \pi \bar{a}_{n-t}^{3\frac{1}{2}}$ $3\frac{1}{2}$ per-cent	Average Increase in Reserve due to Death Benefit, Col. (6) - Col. (7)	Difference between Reserves at 3 $\frac{1}{2}$ per-cent and Average Reserves Col. (9) - Col. (6)					
1	.618	.827	1.273	.655	.906	.015								
2	2.670	2.887	3.343	.673	2.967	2.088								
3	4.792	5.013	5.481	.689	5.095	4.204								
5	9.215	9.477	9.975	.730	9.566	8.659								
10	21.777	22.027	22.572	.795	22.125	21.234								
15	36.605	36.845	37.363	.758	36.938	36.169								
20	51.185	51.363	51.764	.759	54.437	53.968								
25	75.073	75.146	75.312	.739	75.177	74.973								

TABLE XI.

Differences between the Reserves for a Bond for £100, Term 30 Years, by Formula A, p. 233, and those on a Sinking Fund Basis.

Duration $\ell$	RESERVES BY FORMULA A						S.F. Reserve $r^n - \pi \bar{a}_{n-\ell}$ 3½ per cent $= \frac{3\frac{1}{2}\% + 3\frac{1}{4}\%}{2}$	Difference between Reserves at 3½ per cent and Average Reserves Col.(9)–Col.(6)		
	Age at Entry			Difference between Reserves for Ages 20 & 40 at Entry Col.(4)–Col.(2)	(6)	(7)				
	20	30	40							
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)		
1	·689*	– ·415*	+ 1·174	·863 + 2·277	– 310* + 1·780	– 1·467* + 1·607	1·157 1·173	– 293* + 1·819		
2	+ 1·389	+ 1·673	+ 3·828	4·451 8·356	·912 9·010	2·755 8·472	1·184 1·194	+ 0·39 ·063		
3	3·539	8·051	20·748	21·767 36·061	1·019 36·718	7·278 20·043	8·585 1·151	·113 ·244		
5				21·767 53·776	·958 54·270	21·194 53·868	21·438 53·212	36·567 ·656		
10				36·061 53·558	·712 74·995	36·180 74·599	·977 74·599	·387 ·506		
15								·439		
20										
25										

\* Negative values.

(+) Note.—The effect of calculating the S.F. Reserves at 3½ per cent would be to under-estimate the true reserves up to duration 20 years.

TABLE XII.

*Differences between the Reserves for a Bond for £100, Term 30 Years,  
by Formula A, p. 233, and those on a Sinking Fund Basis.*

$\ell$	RESERVES BY FORMULA A			S.I.F. Reserve $v^{n-t} - \pi \bar{a}_{n-t}$			Average Increase in Reserve due to Death Benefit Col.(6) - Col.(7)			S.I.F. Reserve $v^{n-t} - \pi \bar{a}_{n-t}$ ( $= \frac{3\frac{1}{4}\% + 3\frac{3}{4}\%}{2}$ )			Difference between Reserve at 3½ per-cent and Average Reserves Col.(9) - Col.(6)	
	Duration			S.I.F. Reserve $v^{n-t} - \pi \bar{a}_{n-t}$			4 per-cent			S.I.F. Reserve $v^{n-t} - \pi \bar{a}_{n-t}$				
	$\text{O.M.}^{(3)}$	$j = 2\frac{1}{4}$ per-cent	$P_{30} = 2.321$	$\pi = .85P = 1.973$	Avg. at Entry	Difference between Reserves for Ages 20 and 40 at Entity Col.(4) - Col.(2)	Avg. of Reserves for 3 Ages at Entry	(6)	(7)	(8)	(9)	(10)		
20	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(6)	(7)	(8)	(9)	(10)		
25	-1.878*	-1.560*	-.886*	.992	-1.441*	-2.781*	1.343	-1.781*	-3.39					
30	+ .069	+ .397	+ 1.093	1.024	+ .520	- .844*	1.366	+ .201	-.319					
35	2.089	2.427	3.145	1.056	2.554	+ 1.175	1.379	2.256	.298					
40	6.364	6.709	7.480	1.116	6.851	5.457	1.394	6.605	.216					
45	18.955	18.965	19.792	1.207	19.114	17.753	1.361	19.015	.099					
50	33.381	33.738	34.529	1.148	33.883	32.712	1.171	33.978	.095					
55	51.335	51.604	52.207	.872	51.715	50.914	.801	52.019	.304					
60	73.201	73.307	73.559	.358	73.356	73.058	.298	73.772	.416					

\* Negative values.

TABLE XIII.

Differences between the Reserves for a Bond for £100, Term 30 years  
by Formula A, p. 233, and those on a Sinking Fund Basis.

Duration <i>t</i>	Reserves by Formula A						S. F. Reserves $\rho^{vt} - \pi\bar{a}_{vt}$ 4 per-cent			S. F. Reserves $\rho^{vt} - \pi\bar{a}_{vt}$ 3½ per-cent			Difference between Reserves at 3½ per-cent and Average Reserves Col. (9) — Col. (6)		
	Age at Entry			Averages of Reserves for 3 Ages at Entry			S. F. Reserves $\rho^{vt} - \pi\bar{a}_{vt}$ 4 per-cent	Average Increase in Reserve due to Death Benefit Col. (6) — Col. (7)	(8)	(9)	(10)				
	20	30	40	(4)	(5)	(6)									
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)		
1	- 3·121*	- 2·714*	- 1·935*	1·186	- 2·600*	- 4·215*	1·615	- 2·905*	- 3·05	- 2·72	- 8·83*	-	-		
2	- 1·148*	- 759*	+ 0·75	1·223	- 611*	- 2·248*	1·637	-	-	-	- 2·215	- 235	-		
3	+ .897	+ 1·298	2·155	1·258	+ 1·450	- .201*	1·651	+ 1·651	+ 1·215	+ 1·649	- 1·59	-	-		
5	5·226	5·647	6·552	1·326	5·808	+ 4·41	1·667	1·667	18·276	18·276	+ .060	-	-		
10	17·594	18·010	19·014	1·420	18·216	16·608	1·608	1·608	33·454	33·454	+ 308	-	-		
15	32·559	32·976	33·933	1·344	33·146	31·776	1·370	1·370	51·700	51·700	+ 544	-	-		
20	50·718	51·028	51·723	1·005	51·156	50·231	.925	.925	73·635	73·635	+ 619	-	-		
25	72·843	72·962	73·243	.400	73·016	72·683	.333	.333	-	-	-	-	-		

\* Negative Values.

TABLE XIV.

*Specimen Values of  $\bar{W}_{x0}$  - Formula A, p. 233 for various Rates of Interest and Ages at Entry.*

Mortality.  $O^{(5)}$  Table.

Term $t$	Age at Entry ( $x$ )			Age at Entry ( $x$ )			Age at Entry ( $x$ )			Age at Entry ( $x$ )		
	20	30	40	20	30	40	20	30	40	20	30	40
$i = 3\frac{1}{2}$ per-cent $j = 2$ per-cent	2.565	3.795	6.094	2.638	3.906	6.275	2.339	3.448	5.533	2.404	3.549	5.695
$i = 3\frac{1}{2}$ per-cent $j = 2\frac{1}{4}$ per-cent	2.559	3.942	6.351	2.735	4.058	6.540	2.435	3.595	5.792	2.504	3.703	5.962
<i>i = 4 per-cent</i>												
$i = 4$ per-cent $j = 2$ per-cent	2.750	4.087	6.610	2.829	4.209	6.807	2.528	3.747	6.055	2.600	3.857	6.234
$i = 4$ per-cent $j = 2\frac{1}{4}$ per-cent	2.920	4.371	7.132	3.005	4.502	7.347	2.706	4.010	6.558	2.784	4.160	6.785
<i>j = 2<math>\frac{1}{4}</math> per-cent</i>												
$i = 4$ per-cent $j = 2$ per-cent	3.255	5.001	8.406	3.356	5.159	8.672	3.070	4.708	7.914	3.164	4.856	8.163
$i = 4$ per-cent $j = 2\frac{1}{4}$ per-cent	3.381	5.387	9.426	3.493	5.569	9.744	3.239	5.156	9.025	3.347	5.330	9.328
$i = 4$ per-cent $j = 2$ per-cent	3.132	5.226	9.625	3.245	5.416	9.977	3.044	5.078	9.356	3.154	5.263	9.697
$i = 4$ per-cent $j = 2\frac{1}{4}$ per-cent	2.200	3.881	7.643	2.287	4.037	7.946	2.167	3.824	7.529	2.254	3.976	7.829

*Methods of Valuation at present adopted in Practice.*

As might be anticipated from the short time many of the Offices have been working, and the fact that it is only since 1911 that House Purchase Companies have been compelled to make regular valuations, an examination of the reports already filed shows a great variation of practice in regard to the principles and methods adopted in their valuations.

With one exception, all the Companies whose Returns are available have ignored the rate of mortality and age at entry, and valued their contracts as endowments-certain or sinking-fund policies, but most of those returning the premiums in the event of death with compound interest have made an additional reserve in respect of this benefit. The principles upon which this is based are seldom given, however, and appear in most cases to be little more than arbitrary.

In fixing the rate of interest which should be used in a valuation of a House Purchase Company it is necessary to consider :

1. The rate of interest which may be safely assumed to be earned in the future for the term of the bond at present in force, at the most about 30 years. [As stated in connection with the calculation of Premiums I think that a yield of  $4\frac{1}{2}$  to 5 per cent may safely be counted upon for many years to come.]
2. The interest margin over and above this, which it is desirable to retain.

These points differ slightly from those to be considered in the case of ordinary life business, inasmuch as the contracts will not run for so long a period, and there is usually no necessity for valuing at a low rate of interest to provide a reserve for the maintenance of future bonuses.

The interest margin to be retained depends to a very large extent upon whether any charges, such as the death benefit, are thrown upon it. If this is the case, I think that at least a margin of 1 per-cent should be maintained, and that  $\frac{1}{2}$  per-cent should be regarded as the minimum in all cases. Assuming a future yield of  $4\frac{1}{2}$  to 5 per-cent, this indicates that  $4\frac{1}{2}$  per-cent is the highest valuation rate which can be assumed under any circumstances, and  $3\frac{1}{2}$  per-cent to 4 per-cent as rates of interest which it is safest to adopt, if the strength of the Office permits.

The rate of interest actually used by different Offices whose

Returns are available, varies from  $2\frac{1}{2}$  to  $4\frac{1}{2}$  per-cent;  $2\frac{1}{2}$ , 3,  $3\frac{1}{2}$ , and  $3\frac{3}{4}$  per-cent being used by one Office each; three use 4 per-cent; one uses  $4\frac{1}{4}$  per-cent; and three use  $4\frac{1}{2}$  per-cent. One tiny concern valued at 5 per-cent, but in this case the actuary's fee appears to have swallowed up the whole of the Bond Investment funds.

#### *Reserve for future Expenses.*

With the exception of one Company, which makes a net premium valuation, the provision made for future expenses takes the form of a percentage of the Office premiums valued, the actual percentage reserved varying from 10 to 20 per-cent. The allowance to be made in any particular case must, of course, be fixed with due regard to the expenses likely to be incurred, and the premiums charged; but in the case of 30-year bonds by monthly premiums, 15 per-cent should be regarded as a minimum under ordinary conditions, as the renewal commission paid under such contracts is usually 10 per-cent. In the case of bonds for shorter periods, however, the latter item is much reduced, often running down to  $2\frac{1}{2}$  per-cent in the case of bonds for 5 and 10 years, and hence a reduction in the percentage reserved, varying with the term of the bond, can be defended. As a rule, however, the amount of business involved under short-term contracts is comparatively small, and it is hardly worth while differentiating between bonds for different periods in this respect.

The question of the relative merits of the net and gross premium methods of valuation, as applied to house purchase contracts, is as difficult to decide as in the case of ordinary life business, and is entirely a matter of degree, since, if a sufficiently high percentage of the gross premiums is reserved, the latter method can be made to produce the same, or higher, reserves than a net premium valuation at the same rate of interest.

The net premium method has the great advantage that the allowance for expenses is automatically increased with the valuation rate of interest, and thus the combination of a high rate of interest, and an insufficient allowance for future expenses, is rendered impossible. A further advantage is that, if the subscriptions charged are net rates, calculated at a low rate of interest, the valuation of the net annual premiums, at the valuation rate of interest, automatically increases the reserve made for monthly and quarterly business, as compared with annual, and diminishes the percentage of the Office premiums reserved

for future expenses in the case of short period bonds. This will be seen from the following example.

Term of Bond	30 years	20 years	15 years	10 years	5 years
Office Prems. $\frac{1}{s_{n+1}-1} 2\frac{1}{2}\% P.$	2.318	3.926	5.554	8.830	18.700
Valn. Prems. $\frac{1}{s_{n+1}-1} 3\frac{1}{2}\% \pi$	1.872	3.417	5.007	8.236	18.017
Allowance for future expenses $P - \pi \dots \dots \dots$	.446	.509	.547	.594	.683
Allowance for future expenses per-cent of the Office Prems.	19.24	12.96	9.85	6.73	3.65

On the other hand, the allowance for future expenses may become almost absurdly high; for example, if the net premiums in the above table had been calculated at 4 per-cent, instead of  $3\frac{1}{2}$  per-cent, the allowance for expenses would have been 26.06 per-cent of the Office premium, under bonds for 30 years.

It may, of course, be objected, that a net premium valuation is inelastic and loses sight entirely of the actual premium payable, and that the system of reserving a percentage of the Office premiums is easier to apply in practice and can be more easily adapted to varying conditions. The very elasticity of the last method is, however, its chief source of weakness, as there are practically no means whereby an independent check may be applied as to the sufficiency of the percentage reserved for expenses in any particular case.

A net premium valuation, however, while an excellent standard to aim at, is often one which a young Company could not afford to set up, and as a compromise, therefore, I suggest that in such cases it may be used for the bonds for terms of 15 years or less, and a percentage of the Office premiums, varying from 15 to 20 per-cent, according to circumstances, reserved for the rest of the business. The net premium method may then be gradually extended, as the strength of the Office increases, until it is used for all the contracts.

#### *A few Points of Detail.*

If the contracts are valued as sinking-fund policies, the actual work involved in the valuation is extremely simple, and presents no features of very special interest. It is only necessary

to group the business according to the period unexpired at the date of valuation, and in doing this almost any desired degree of accuracy may be obtained.

In practice there are three methods of procedure which may be followed :

1. The bonds may be grouped according to the period still to run at quarterly or half-yearly intervals.
2. If the valuation is made as at December 31, all those contracts maturing between July 1 in one year and June 30 in the next may be grouped together.
3. The bonds may be grouped according to the calendar year of maturity, and the average date, in the year on which they mature, determined by an examination of a few of the chief groups.

Of these the first is the most accurate : but as it considerably increases all the work involved, I think the last is, perhaps, that most suitable for use under normal conditions, especially as any error involved will be but slight.

It is, of course, easy to value the bonds for different original terms separately, but unless the method of valuation, or the reserve made for future expenses, is to be varied in the case of short term contracts, there is usually no advantage in adopting this course. A valuation of this nature generally involves a very large number of contracts for individually small amounts, and as much of the preliminary work of collecting the data has to be performed by a comparatively unskilled staff, it is desirable that the sub-groupings required should be as few and simple as possible.

If the bonds are grouped according to the calendar year of maturity, and the average duration in the year of maturity is " $t$ " months, the valuation factor for the sums assured and bonuses will be, of course,  $v^{n+\frac{t}{12}}$  for a valuation at December 31 ; the most convenient course being to use  $v^n$  and to apply the adjustment of  $v^{\frac{t}{12}}$  to the final totals. As regards the premiums, monthly, quarterly, and annual business may be valued separately with appropriate factors, and there is much to be said for this course, as the last two classes do not appear to come within the statutory definition of "bond investment" business at all ; but unless the amount is large, compared with that at monthly premiums, no very great error is involved by grouping all classes together and valuing the premiums by the continuous annuity  $\bar{a}_{n+\frac{t}{12}}$ .

This is equal to  $\frac{v^{\frac{t}{12}}}{\delta} \{(1+i)^{\frac{t}{12}} - v^n\}$  and in this form is easily calculated from the ordinary tables, since  $(1+i)^{\frac{t}{12}} - v^n$  may be written down by inspection, and then only requires to be multiplied by the constant quantity  $\frac{v^{\frac{t}{12}}}{\delta}$ . The latter is independent of  $n$ , and hence the comparatively small quantity  $(1+i)^{\frac{t}{12}} - v^n$  could be used as the valuation factor for the premiums, and the total of these products multiplied by  $\frac{v^{\frac{t}{12}}}{\delta}$ .

A simpler and better method of procedure, which also lends itself to the use of the equated time of payment, is, however, to use the same valuation factor  $v^n$  for the sums assured and the premiums, and then to obtain the value of the latter from the formula :

$$\text{Value of premiums} = \frac{1}{\delta} [\Sigma P - v^{\frac{t}{12}} \Sigma v^n P].$$

A short examination will be necessary to disclose the terms and durations under which "negative values" will arise, as it is hardly necessary to say that these should always be eliminated. As a rule they do not appear except in the case of bonds for 25 and 30 years, but in the latter case, as will be seen by a reference to Tables X-XIII, the reserve values may, under certain circumstances, be negative for three years after the date of issue.

*Drawings.*—Although the issue of bonds benefiting by a system of ballots for advances without interest, or for the immediate payment of a part, or whole, of the sum assured, is now forbidden, it is not unusual to find that this system is still in force in the case of some of the older contracts, and that periodical drawings in respect of these are still being made. The fixing of the annual amount to be expended in this way is, however, as a rule entirely in the hands of the management, and hence the system cannot be taken into account in valuing the liabilities of the Company, except in so far as it increases the general expenses of working the business, although it should be borne in mind in fixing the provision made for future expenses.

*Paid-up Policies.*—Amongst the liabilities of a House Purchase Company will usually be found a certain number of fully paid bonds for small individual amounts; but the exact nature of these varies so considerably that I cannot do more than barely

touch on their treatment here. The most difficult to value are probably those which are payable at a fixed date, or at previous death, with compound interest from the date of conversion to the date of payment, as a double grouping—

(a) according to the term unexpired,

(b) according to the period elapsed since conversion,

is necessary, and a further complication is often introduced by the absence of any reliable information as to the ages. In such a case, sufficient data may sometimes be obtained to arrive at an average age-attained for the whole business, or better still, for each group, when the value will be :

(Sum Assured + accumulation of interest)  $\times A'_{\bar{x}n}$  where

$$A'_{\bar{x}n} \text{ is calculated at rate } I = \frac{i-j}{1+j}.$$

If, however, no information at all as to the ages is to be had, the only course is to ignore the rate of mortality, and to use the formula

(Sum Assured + accumulation of interest)  $\times [1+I]^{-n}$ ,

lowering the value of I as found above, by  $\frac{1}{4}$  per-cent— $\frac{1}{2}$  per-cent as a very rough allowance for the effect of mortality.

#### *Use of "equated time of payment."*

The results of a valuation can easily be roughly checked by the use of the equated time of payment, if the equated time  $\bar{n}$  obtained from the usual formula

$$\frac{n_1S_1 + n_2S_2 + \dots + n_rS_r}{S_1 + S_2 + \dots + S_r}$$

is arbitrarily reduced by about three months as a correction for the error involved, and the business is divided into groups of about five to ten years each ; the difference between the approximate result on this basis, and the true reserve in a large batch of bonds recently valued, being well under 1 per-cent on a true net liability of over £300,000. In this particular case, the equated times of payment obtained from the sums assured were used for valuing the premiums ; but more accurate results would be obtained if the equated times were calculated separately for each.

If desired, great accuracy could be obtained by the use of the second approximation to  $\bar{n}$  given by Mr. G. J. Lidstone, F.I.A., vol. xlvi, p. 483, and it would then be hardly necessary to split the business up into groups, although the work involved would be almost as great as in the original valuation, owing to the necessity of calculating both  $\Sigma n \cdot S_n$  and  $\Sigma n^2 \cdot S_n$ .

### III.—THE APPLICATION OF THE ASSURANCE COMPANIES ACT, 1909, TO "BOND INVESTMENT" BUSINESS.

The requirements of the 1909 Act in respect of Bond Investment Companies are very similar to those for life business, which have already been thoroughly dealt with before the Institute by Mr. A. R. Barrand, vol. xlvi, p. 257. There is accordingly little or nothing fresh to be said regarding the greater part of the Act, which is applicable to all classes of insurance alike, but there are a few points of interest in those sections relating solely to the business at present under consideration, which it may be useful briefly to touch upon, the first of which is, naturally, the definition of bond investment business.

#### *Sec. I.—Definition of "Bond Investment" business.*

Sec. I defines bond investment business as "the issuing of bonds or endowment certificates by which the company in return for subscriptions payable at periodical intervals of two months or less contract to pay the bondholder a sum at a future date and not being life assurance business as hereinbefore defined", i.e., "the issue of policies of assurance upon human life or the granting of annuities upon human life."

The title "Bond investment business" appears at the present day to be rather unfortunate, since, as stated above, the class of company to which this term properly applies has entirely ceased to exist. Beyond this there are two points of interest in the definition.

1. Unless the subscriptions are payable at intervals of two months or less, the business does not fall directly within the scope of the Act, unless it can be held to be life insurance.

It will be readily understood that the limitation of the Act to business by monthly or weekly premiums—which is what the section means in practice—must lead to many anomalies, since if two bonds are issued to one holder, identical in every

respect except the intervals at which the premiums are payable, the Act applies to the one contract, but not to the other.

Apparently, a complete separation should be made between monthly and quarterly business, separate funds established, and separate returns made throughout. This course is adopted by at least one Company, but the inconvenience entailed is obvious, and the Board of Trade have, I believe, shown a tendency to disencourage any difference in the treatment of bonds at monthly and quarterly subscriptions. The inclusion of all classes of certificates under one head is at any rate in accordance with commonsense, and hence the course followed by the majority of these Companies is perhaps permissible.

If this course is not adopted, "Bond Investment" business, at subscriptions payable otherwise than monthly or weekly, only comes within the provision of the Act in so far as the latter relate to sinking fund policies. These, it will be remembered, are nowhere mentioned in the body of the Act itself; but if a Company, making returns in respect of any other class of assurance business, also issues capital redemption policies :—

1. The heading to Schedule I requires the income and expenditure relating thereto to be stated in a separate account.
2. The heading to Schedule IV requires a separate statement signed—not "made and signed"—by the actuary showing :
  - (a) Number of capital redemption policies valued.
  - (b) Total sums assured.
  - (c) Total Office yearly premiums.
  - (d) Total net liability in respect of such business.
  - (e) Basis on which the said liability is calculated.
3. The heading to Schedule V requires a separate statement, signed by the actuary, showing the total sums assured maturing in each calendar year, and the corresponding Office premiums.

These conditions are much less onerous than those governing bond investment business. Moreover, no deposit is required before sinking fund policies may be issued, so that apparently, if a Company were formed to transact bond investment business on the basis of quarterly subscriptions, and were to carry on no other class of insurance business, it would be quite outside the scope

of the Act, and need make neither deposit nor returns. This point is, however, more of theoretical than practical interest, and the further question of the exact legal position of the holder of a bond at quarterly subscriptions is too complicated to be discussed by laymen.

If this definition is ever amended it will, probably, be thought necessary to differentiate between capital redemption policies, as issued by ordinary life offices, and “House Purchase” business, but this should not be difficult.

2. The second point of interest in the definition is the words “not being life assurance business.”

It will be seen that the application of the sections of the Act now under consideration are expressly limited to contracts which are not life assurance business. The certificates issued by House Purchase Companies, however, almost without exception provide for the return of all premiums paid, either with or without, compound interest, in the event of the holder’s death before the maturity of the bond, and the question accordingly arises whether such contracts are, or are not, life assurance business within the meaning of the Act.

In a recent case, certain policies of this nature, under which all premiums paid were returned *without* interest in the event of death before maturity, were issued by a Company, which had no power to transact life assurance business. In a friendly action to settle the point, a shareholder asked for a declaration by the Court that the contracts in question were policies of assurance on human life, and as such, illegal and void, since their issue was outside the powers of the Company; and in the Lancashire Chancery Court, the Deputy Chancellor decided that the contract under the policies was properly described as a means of enabling thrifty persons to accumulate savings, that the policies were not illegal or void, and that the Company did not carry on life assurance business within the meaning of the Act. This decision was, however, overturned on appeal, the Master of the Rolls holding that “the policies in question were contingent upon the duration of human life, and that the Company in issuing them acted contrary to its memorandum of association.” On the question of a deposit of £20,000 under the Assurance Companies Act, he did not consider it material to enter into the construction of the Act, seeing that the question was decided on the broad point that the Company had no power to transact this business,

and that before doing so, it would require to obtain the powers, and thus, as a natural consequence, comply with the Act.

So far as I am aware, the case is not to be carried further, and the Company in question has taken steps to obtain the power to transact life business, and has made the necessary deposit.

Unless this decision is eventually over-ruled by a higher Court the consequences will be most sweeping, as, although a full report of the case is not obtainable, there was apparently no essential difference between the particular contracts in question, and those issued by many other companies.

No doubt its most serious effects are mitigated by the fact that the majority of the companies now transacting house purchase business have obtained, or are obtaining, powers to transact life business, and have made the necessary deposit ; but a more regrettable feature is that, if it holds good, all those sections of the 1909 Act relating to "Bond Investment" Companies would appear to be obsolete.\* Practically all the contracts at present returned thereunder would in future be included therefore in the returns relating to Life Assurance, which are unsuitable to the business in question, and do not call for information on many of the points upon which it is very desirable that light should be thrown.

As laymen it is, of course, quite outside our province to discuss the propriety of the decision in question, but unless some essential feature of the case has been omitted in the short reports available, I must confess that personally I feel some difficulty in accepting it. It is true that, technically speaking, these bonds are pure endowments with return of premium, which are usually included by ordinary Offices amongst their life business in making returns under the Act, but on the other hand, the Inland Revenue authorities have always refused to recognize them as such for the purpose of calculating the abatement of income in respect of life assurance premiums for Income Tax purposes.

In the case of the General Accident Assurance Company *v.* I.R., reported by Mr. A. R. Barrand, vol. xli, p. 114, it will be remembered that the question arose whether an accident policy, under which a portion of the premiums were returnable on the assured attaining a specified age or dying earlier, provided no payment had been made under certain clauses relating to

\* Indeed it appears very doubtful whether these sections were ever required at all, as the return of premiums in the event of death was a feature of most of the contracts issued by the now defunct companies, at which this part of the Act was mainly aimed.

payment for accident involving death or total disablement, was or was not a policy of life assurance. The test applied in this case was whether the second so-called "life" contract could stand alone, and if this is applied to the certificates issued by House Purchase Companies, and the restrictions often placed on the payment of the death benefit considered, the correctness of describing them as life policies appears doubtful, as the death benefit in question is certainly quite subsidiary to the main contract and could not stand alone.

*Sec. II (Deposits).*—Under this section, as modified by Sec. 34, sub-sec. "b", a deposit of £20,000—instead of £10,000 recommended by the Departmental Committee as a minimum—is required from all new Bond Investment Companies, whether or no a separate deposit has already been made in respect of any other class of business. No deposit is required if the Company commenced to carry on bond investment business before the passing of the Act; and it is provided that the deposit shall be returned:

1. So soon as the Bond Investment Fund set apart and secured for the satisfaction of policyholders of that class amounts to £40,000, provided that,
2. The Company has made a deposit in respect of any other class of assurance business.

The general rules regulating the management, &c., of the deposits are identical with those governing life business.

*Annual Accounts.*—As for life business,

1. A Revenue Account,
  2. A Profit and Loss Account,
  3. A Balance-sheet,
- } in the prescribed form

are required at the end of each financial year.

The last two forms are, of course, identical for all classes of business, but the Revenue Account required from Bond Investment Companies differs in several interesting respects from that required from Life Offices.

1. In view of the nature of bond investment companies, no distinction is made or required between ordinary and industrial business, or between business in and out of the United Kingdom.
2. The income side is practically identical with that required from Life Offices, except that space is left for any additional reserve made in addition to the Bond Investment Fund.

3. The details required in respect of the outgo are not nearly so minutely subdivided.

(a) The items "Bonuses in cash and reduction of premium" and "Annuities paid away", are not usually required for the class of business under consideration, and are accordingly omitted.

(b) The item "Claims under bonds paid and outstanding" is not subdivided into—

1. Claims by maturity.
2. Claims by death.

This is, I think, regrettable, as it would be, at least, interesting to trace the amounts paid away by those companies in respect of the death benefit included in their certificates, although the wording used for life business might need some slight modification.

(c) Another regrettable omission is the absence of any enquiry as to the amount paid away in respect of surrenders.

The information mentioned in "b" and "c" above may, of course, be given individually under "Other payments"; but this is not often done, and, I think, it would be better if these items were specifically asked for.

One large Company sets an excellent example, which might with advantage be followed in every case, by splitting the item "Commission" into "Agents' Collecting Commission" and "Special new business charges", and the item "Expenses of Management" into several items such as Advertising, Posting, Salaries, &c.

A curious omission from these returns, in view of the information required from Life Offices, is the absence of any requirement of a statement as to the amount of new business effected during the year the inclusion of which would :

1. Indicate the progress made by a Company from year to year, and the extent to which the business as a whole is expanding, and its real condition.
2. Enable a rough estimate to be made of the proper apportionment of the total expenses between initial and renewal expenditure.

*Sec. III.—Separation of Accounts.*

Certain Companies accept deposits, repayable on demand, or on 3 or 6 months' notice, upon which interest is paid varying from 3 to  $4\frac{1}{2}$  per-cent, according to the notice of withdrawal required. It appears rather doubtful whether a separate Revenue Account, and a separate Depositor's Fund, should not be set up in regard to these, as the business is banking pure and simple. The amounts in question are, however, usually very small, and, so long as the totals do not exceed their present moderate dimensions, will not affect the working of the Companies one way or another, but should a Company unwisely allow them to reach a large amount, they might be a cause of serious embarrassment, the funds being usually locked up in mortgages which could not be realised in a sudden emergency.

*Secs. V and VI.—Periodical Valuations.*

Secs. V and VI requiring an actuarial valuation to be made at least once in five years are identical, except as regards the Schedules, for Life and Bond Investment business. It may be of interest, however, to mention an interesting point as to the construction of the words "with a view to the distribution of profits," in Sec. V, Sub-sec. 2, which recently came under my notice. The Board of Trade contended that an annual valuation made for the private guidance of the management of a certain Office, in order to ascertain the amount by which the "Bond Investment Fund" should be increased to meet the normal addition to the liabilities, fell within the meaning of this section, on the ground that the valuation in question was made with a view to distribution of profit to the shareholders. They did not press their point, however, when the objection was raised that compliance with their request would cause some inconvenience, as the valuation had not been made up in exactly the specified form.

If their contention was correct, it would appear that Schedule IV must be furnished whenever a valuation is made, as the ultimate end of every operation of a Company is the distribution of profit.

*Schedules IV and V.*

In examining these forms, in the first place, it will be observed that, as for life business, Schedule IV must be made and signed by the actuary, while Schedule V is only to be signed by him.

As pointed out by Mr. T. G. Ackland, vol. xlvi, p. 331, in the discussion on Mr. A. R. Barrand's Paper, if the valuation is made by a consulting actuary, as will usually be the case with the Companies now under discussion, Schedule V must be signed by him, although he cannot personally vouch for much of the information given. Apparently, however, in view of Sec. 24, the actuary is only required to certify that the statements in question are true to the best of his knowledge and belief, and probably, therefore, it is only necessary for him to satisfy himself that the information in question is in accordance with the data supplied to him by the Company, and those contained in its general literature. In most cases it would be practically impossible for a consulting actuary to verify all the details given, and I should be glad, therefore, of some expression of opinion as to the exact extent to which this is necessary.

#### *Schedule IV.*

The information required by Schedule IV from "Bond Investment" Companies is very similar to that required from Life Offices, with certain obvious omissions and simplifications occasioned by the peculiar nature of the business under consideration, the main differences between the two forms being as follows :

##### *Question 2.—Principles adopted in the valuation.*

In the Bond Investment Schedule no reference is made to the method of calculating the net premiums. Indeed, the latter are not mentioned in the returns at all, the authorities recognising and acquiescing in the system of providing for future expenses by reserving a percentage of the Office premiums : although in Schedule VI e, in the event of the winding up of a Bond Investment Company, it is laid down that the contracts shall be valued on a net premium basis. The Act, moreover, does not contemplate the necessity of introducing the rate of mortality into the valuation, and no question is inserted in Schedule IV e dealing therewith. If, however, contracts with return of premiums at compound interest are correctly treated as falling within the scope of the bond investment sections, I think it would have been better if some specific information as to the exact method of treating these, and of arriving at any extra reserve made had been called for, although, perhaps, the point is covered by the words "principles adopted in the valuation."

*Question 4.*—The provision for future expenses and profits is not required to be given separately for bonds with and without profits, although this must be separately stated in the valuation schedule required by Question 6.

*Question 5.*—As for life business, the form of Consolidated Revenue Account provided is similar to that required by Schedule I and covers the period since the last valuation, but there is a rather curious difference in the wording of the two questions.

In the case of life business the last part of the question reads, "No return under this heading will be required where a statement under this schedule (IV) is *deposited* annually", while in the case of bond investment business the wording is, "no return under this heading will be required where the valuation is *made* annually."

In the latter case there is no reference to the deposit of the result of the annual valuation, and this might give rise to the contention that the Consolidated Revenue Account was not required where an annual valuation had been made, even if the results had not been made public. This is doubtless correct in the case of the first valuation made after the passing of the Act, but I hardly imagine that it is intended to apply to subsequent valuations, and probably this point is covered by the expression "the valuation," *i.e.*, the statutory valuation, returns in respect of which must be deposited, no other valuation being recognised by the Act. The point is not, of course, of any very great practical importance, as the desired information can be readily obtained from the returns for the preceding 5 years, even if a Consolidated Revenue Account is not deposited.

*Question 6.—Summary and Valuation of the Certificates in force.*

The specified form of return requires the information to be given separately for bonds with and without profit, but otherwise the business is not required to be separated into classes. The amount of the net annual premiums and their value are also not required, and the latter is replaced by an enquiry as to the "Provision for future expenses and profits" for each class of bond. The wording "Office yearly premiums" is replaced by the alternative expression "full yearly premiums", which is, perhaps, better suited to this class of business. With these exceptions the form is very simple, and calls for little comment.

*Question 7* is practically identical with the corresponding

questions for life business, except that the wording "number of years premiums to be paid before a bonus (*a*) is allotted, and (*b*) vests" is replaced by "the time during which a bond investment policy or endowment certificate must be in force to entitle it to share in profits."

*Question 8* is practically identical with the life form, except that the specimen bonuses required are more indefinite, no terms and ages being specified.

It will be observed that owing to the omission of the question regarding the rate of mortality employed, Schedule IV, for bond investment business, contains one question less than the life schedule.

#### *Schedule V.*

The Schedule just dealt with, IV *e.*, differs but little from the form required for life business, the modifications introduced dealing solely with comparatively unimportant points of detail; but the second schedule of the valuation returns required from Bond Investment Companies introduces many new features, and is aimed at elucidating the exact principles on which the business is carried on, and the conditions attaching to the certificates issued, as well as providing the data for an independent verification of the valuation.

*Question 1* requires a statement of :

- (*a*) The published table or tables of rates of contributions for bond investment policies, and endowment certificates, which are in use at the date of valuation;
- (*b*) Full particulars as to the terms and conditions on which advances are made under such policies, or certificates, whether on security of house property or land, or otherwise;

both of which require little comment.

Questions 2, and 3 and 4, require a detailed statement of the business in force at the date of valuation in the following form :

Complete years unexpired	WITH PROFITS				WITHOUT PROFITS		
	Total sums insured	Total bonus additions	Amount of premiums receivable annually	Total amount of premiums received from the commencement	Total sums insured	Amount of premiums receivable annually	Total amount of premiums received from the commencement

In the case of a valuation made as at 31 December, it is immaterial whether the business is arranged according to the complete years unexpired, or according to the calendar year of maturity, and the latter method has, in consequence, been adopted by the majority of companies and accepted by the Board of Trade.

The information given by the answers to these questions affords the material for making a completely independent valuation of the company as a check on the net liability shown in the answer to Question 6, Schedule IV., and in view of this fact the item, "Total amount of premiums received from the commencement," seems hardly required. The number of contracts for individually small amounts renders the labour of calculating this information very great, even if a system of continuous classification is used, and out of all proportion to any benefit obtained by its publication.

Question 5, as to the average rate of interest realized by the assets invested, or uninvested, constituting the Bond Investment Fund of the Company, is identical with that in the Life Schedule, except that no enquiry is made as to the treatment of reversionary investments, which are seldom made by these Companies. It sometimes happens that no specified assets are allocated to the Bond Investment Fund, the total interest earnings being included under one head in the Revenue Account, and that hence it is difficult to determine exactly how much of the total interest earnings of the Company was earned by the fund in question, in order to calculate the average rate of interest yielded by it. In such a case it would, probably, be best to give the average rate earned on the total assets of the whole Company, although, perhaps, this course tends to under-estimate the apparent strength of the valuation.

*Questions 6, 7, 8, and 9,* require full particulars of the terms and conditions upon which :

Question 6. Surrenders of policies and certificates are granted.

Question 7. Allowances are made upon the death of the policy or certificate holder.

Question 8. Transfers of the interest in a policy or certificate are granted—

(a) On the death of a policyholder.

(b) During his lifetime.

Question 9. Redemption of advances is allowed.

In each case, except No. 8, specimens of the values allowed in respect of bonds for different durations and different unexpired terms are required to be given. When the Act is revised, it will be an advantage if these values are required in respect of certain specified terms and durations, as, although in many cases the fullest information is given, in others, the specimen values given are so scanty as to be practically useless for the purpose of forming any idea as to the equity of the arrangements in force.

Dealing with these questions *seriatim*, the wording of Question 6 apparently includes both the terms upon which—

- (a) Cash surrender values are paid, and
- (b) Paid-up certificates are granted,

and it has, as a rule, been interpreted in this sense. In some cases, however, no information is given as to the terms upon which paid-up certificates are granted, and perhaps it would be better if these were specifically asked for.

*Question 7.* Here, again, fuller and more explicit information appears desirable if the true effect of the conditions in force are to be brought out, especially as regards any restrictions placed on the payment of the death benefit, since, as already stated, this point is of considerable importance in connection with the valuation of these Companies. For example, information might be required as to :

- (a) The death benefit allowed under a bond after it has been exchanged for a paid-up certificate.
- (b) Whether the death benefit is allowed under a bond upon the partial security of which an advance has been made.
- (c) Whether the death benefit is payable in the case of a bond transferred to a third party during the lifetime of the original holder, and if so, whether upon the latter's death or upon that of the transferee.

*Question 8* needs no comment.

*Question 9.* Terms upon which redemption of advances are granted; that is, the amount outstanding under an advance at the end of various periods, or the terms on which the advance may be terminated by the borrower before the date originally fixed upon for the extinction of the loan. This question, and the second part of Question 1, might well have been incorporated into one, as the two subjects are intimately connected, and the terms of redemption obviously depend upon the terms of repayment originally

arranged. Information as to the terms on which redemption is allowed is most important if a clear idea of the equity of the arrangements in force is to be obtained, and from this point of view it is regrettable to find that in some cases only the most meagre information is given. Where the certificate is kept in force, no difficulty arises, but in cases where one of the systems described under Heading 2, on p. 200, is adopted, the information desired can, as a rule, only be obtained approximately, and after a long investigation, from the published Returns.

*Question 10* is especially noteworthy, as for the first time in the insurance legislation of this country a Return is required of the certificates lapsed or surrendered ; the exact requirements of the question being shown by the following form :

*Certificates Lapsed since last Investigation.*

Year lapsed	Year of issue	No. of cases	Amount assured	Annual premiums	Total premiums received from commencement
1912	1895				
	1896				
	:				
1911	1895				
	1896				
	:				
etc.,	1912				

A similar statement is required in respect of certificates surrendered during the period.

Certificates which have lapsed, and been revived, are not to be treated as lapses.

This question is inserted in accordance with the Report of the Departmental Committee, which showed that the promoters of the now defunct Bond Investment Companies proper relied almost entirely for solvency upon a very considerable proportion of surrenders and lapses. From the Returns now required it is impossible, however, to form an exact idea as to either

1. The actual lapse rate experienced by a Company, since no statement is required of the amount of new business obtained in the year of issue of the bonds lapsed, or surrendered, in any given year or
2. The actual profit derived from this source, on account of the difficulty of gauging how much of the total premiums received has been absorbed in expenses.

The present Returns therefore only enable us to form a very rough idea of the part played by lapses and surrenders in a Company's management. Apparently this varies considerably, the ratio between the total sums assured under lapsed and surrendered certificates during the year preceding the date of valuation, and the total sums assured in force at the end of the year, ranging in various Companies from 5 to 15 per-cent. The latter amount appears, however, to be somewhat exceptional, and a fair average ratio is probably about 6 per-cent. The ratio in question is, of course, only of the very roughest value for testing the importance of these items, and probably the majority of the lapsed certificates had been less than one year in force, and hence involved very little profit, if not an actual loss, to the Company.

The comparison of the figures of different Companies is further vitiated by the following considerations :

In the first place, no definition of what constitutes a lapsed bond is given by the Act, and as most Companies are prepared to revive certificates, however long a period may have elapsed since the date when the last subscription was paid, it is probable that the term has been differently understood by different Companies. As an alternative, I think, therefore, that it would be an improvement if a Return were required of certificates under which the subscriptions were three or more months in arrear, at the date to which the Returns are made up.

There is, also, apparently, a great difference of practice as to what certificates are included as surrendered. In some cases the statement expressly includes :

1. All bonds which had acquired a surrender value at the date of lapsing, whether or no such surrender value had been actually paid over to the assured.
2. All bonds which had been exchanged for fully paid certificates.
3. All bonds in respect of which a return of premiums had been made under the death benefit.

While in other cases no information is given on this point, which, by itself, would prevent any true estimate of the relative profit arising from surrenders.

As regards the proper course to be followed, the withdrawals all into four different classes, as follow :

1. Surrender values paid away in cash, or applied to keep the bond in force for a period beyond the actual date of lapsing,

2. Surrender values attaching to lapsed bonds which are still unclaimed,
3. Bonds exchanged for fully paid certificates,
4. Bonds under which payments had been made under the death benefit,

of which the second might properly be included under the statement of lapsed bonds.

Possibly, the wording of the question might be held to indicate that Nos. 1, 3, and 4 should be included under the one heading of surrenders, as in 3 the surrender value might be held to have been applied as a single premium, and in 4 the death benefit is practically a special surrender value; but I personally think that a far preferable course would be to give separate statements for each class. The Returns for each Company should certainly state exactly what is and what is not, included, and it would be better still if this point were definitely settled by the Act.

While I do not see how the Returns can be amended so as to give all the information desirable without causing an amount of labour out of all proportion to the results obtained, should a Company use a lapse rate either in connection with the calculation of its premiums or in its valuations, I think that the Returns should require a full statement as to :

1. The actual lapse rate experienced.
2. The lapse rate assumed, and how the latter was introduced into the calculations.

In its most valuable form, the first of these would require more data than could reasonably be expected, but as lapses are only important for the first five years after entry, during which this is practically the only cause of exit, a sufficiently accurate idea of the rate experienced would be given by a statement showing the number and amount of the bonds issued in each year, and the number and amount of these lapsing in each of the following years. The point, however, is not of very great importance, as only two concerns, so far, have introduced the lapse rate unto their calculations, and then only to a very small extent. So many wild statements are current as to the "vast" profits made from lapses and surrenders, that probably it would be to the advantage of the Companies to publish the fullest information possible in respect of these.

*Question 21* also introduces an entirely new feature into insurance returns by requiring a statement of—

1. The total *number* of advances made under policies or certificates to the holder thereof, whether on security of house property, or lands, or otherwise.
2. The total *amount* of such mortgages outstanding at the date of valuation,

distinguishing the advances on first mortgages, and these on second and subsequent mortgages.

It will be observed that the question only deals with advances made to bondholders, and includes any advances made on personal security or on security of the surrender values of certificates, the last two being, however, in practice, usually negligible ; and that the first part of the question apparently requires a statement of the total number of advances made since the formation of the Company, whether or no these have since been repaid.

In regard to the second part of the question, it is doubtful whether in calculating the amount outstanding at the date of valuation, account should be taken of the periodical reduction of the loan by the subscriptions paid by the assured. The true amount outstanding at any time under an advance is reduced quarter by quarter until the loan is finally terminated at the end of the period agreed upon, but in practice it is not unusual to find that for Office purposes the original amount lent is treated as outstanding throughout the whole of the period, especially where the loan is to be repaid by a certificate as maturity. If the loan is written down quarter by quarter, or year by year, the true amount outstanding at any date is readily ascertainable, but if this course is not adopted, a tremendous amount of labour would be involved in making such a Return, as each case would have to be investigated individually. Probably, therefore, the question is intended to be answered by a statement of the nominal amount outstanding at the date in question.

From the somewhat scanty information required, the only object of the question is, apparently, roughly to ascertain to what extent the house purchase element forms part of the Company's operations, and how far the promises of advances, held out to bond-holders, have been fulfilled, but in view of the very large portion of the funds of these Companies invested in mortgages, it would not be unreasonable if fuller information than usual were required respecting the latter's position. This might be similar to that required from building societies, and in something of the following form :

	FIRST MORTGAGE		SECOND MORTGAGE		TOTAL	
	No.	Amount	No.	Amount	No.	Amount
Total advances made to Bondholders on security of House Property since formation of Company ...		£		£		£
		£		£		£
<b>ADVANCES AS ABOVE NOW OUTSTANDING.</b>						
I. Of greater individual amount than £1,000.						
(a) Interest and instalments fully paid up...						
(b) Interest and instalments less than 6 months in arrear ...						
(c) Interest and instalments more than 6 months in arrear ...						
II. Of lesser individual amount than £1,000.						
(a) Interest and instalments fully paid up...						
(b) Interest and instalments less than 6 months in arrear ...						
(c) Interest and instalment more than 6 months in arrear ...						
Total advances now outstanding ... ... ...						
		£		£		£
Properties in possession.						
(a) Let* ... ... ...						
(b) Unlet ... ... ...						

\* Excluding those used as Offices.

The publication of such figures would probably be to the Companies' advantage, as showing the position of the investments in which the greater part of the assets are placed, a position which is, I believe, on the whole, extremely satisfactory.

The only remaining section of the Act regarding these Companies of special interest is Sec. 34 (e), which renders the ballot

system illegal in the case of any policy issued after the passing of the Act. The section goes on to say that this provision shall not be construed as anywise prejudicing any question of the application of the law relating to lotteries to such transactions, but this question has, I believe, never been finally tested, and in a few cases, drawings are still made in respect of bonds issued before the Act came into force.

Although a few alterations might be made in the provisions of the Act, as regards "bond investment" business, with a view to obtaining more precise information in regard to certain details, and in order to settle a few obscure points, the Act is, as a whole, very well suited to the business in question, the only serious flaw being, I think, the definition of bond investment business. It must be remembered that when it was framed an entirely fresh set of conditions had to be faced in connection with a business, the exact requirements of which would only be gauged after the Act had been some little time in force, and that these sections were primarily framed to deal with a class of company which is now defunct, so that, on the whole, there is very little to cavil at in its requirements.

#### CONCLUSION.

In writing this Paper, I have endeavoured to the best of my ability to give a general survey of the business carried on by House Purchase Companies; but, in dealing with such a subject, it will be readily understood that many points of practical interest had to be omitted, or only inadequately dealt with, if the Paper was to be kept within reasonable limits. I have, too, endeavoured to deal only with the broad lines of the subject, and have avoided, as much as possible, direct criticism of details peculiar to individual Companies.

In conclusion, I wish to express my thanks to my colleague Mr. C. W. Winstanley, for the very able and valuable assistance he has given me in connection with the calculations involved by the Paper.

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#### ABSTRACT OF THE DISCUSSION.

Mr. L. E. CLINTON said that it was only right that in a Paper on this subject the system of the real bond investment companies should be distinguished from that of house purchase companies, for there could be no doubt that at one time the public at large were apt to condemn the latter because of the former. The out-

standing feature of house purchase business was, of course, as its title indicated, that of granting loans to certificate-holders for the purchase of house or shop property for occupation, and the fact that one Company alone had advanced in this fashion over £7,000,000, on the security of nearly 30,000 houses, was sufficient justification for saying that the system is a popular one with persons of limited means who are desirous of purchasing their own homes by a series of small payments. As against the Building Society system, the principal point in its favour was that a loan for the full amount of the certificate was guaranteed at the end of a certain period—nearly always five years—provided the property offered as security were eligible and of equal value. From the point of view of the Company, the advances had proved excellent investments. The individual amounts being small and the properties situated all over the country, the loss on realisation and depreciation was easily borne, the net yield before deduction of income tax, but after allowing for all losses, being about  $4\frac{1}{2}$  per-cent. The system of guaranteeing that loans would be granted and the further guarantee not to call in the loan so long as the conditions of the mortgage are duly fulfilled, had a most important bearing on the cash transactions of the office, and it was obvious that the company must protect itself against any possible severe demands on the cash reserves. It was necessary to appreciate this point in considering what some people called the onerous conditions attaching to these contracts.

On pp. 195-203 the author passed under review the various forms of contract issued by house purchase companies. He did not propose to discuss the merits of all the various types, as there was really only one that mattered, the others representing only a tiny proportion of the total business. The contract he referred to was one for 30 years, at a monthly premium of 4s. 4d per-cent. This contract represented about 95 per-cent of the bond investment business of the largest of the companies, which held nearly 70 per-cent of the total business of all companies, and he believed the percentage was nearly the same in all offices with a few exceptions, so that it might be regarded as the typical contract. Originally all these contracts were issued with the right to participate in periodical drawings for the purpose of determining priority in the allotment of advances free of interest, by which system the payment of the amount due at maturity was anticipated on satisfactory security, in the form of house property, being provided for the due payment of future premiums.

At a later period contracts were issued without ballot rights, but with the alternative privilege of obtaining an advance at a reducing rate of interest, instead of the usual uniform 5 per-cent, the rates being  $4\frac{1}{2}$  per-cent for the first ten years, 4 per-cent for the second ten years, and  $3\frac{1}{2}$  per-cent thereafter. Finally, when it appeared likely that the issue of further ballot contracts would be restricted by Statute, a with-profit contract was introduced, carrying guaranteed bonuses and the right to participate in profits

to the extent of 80 per-cent, subject to deduction of any vested guaranteed bonuses. The author, in referring to this guaranteed bonus as a "so-called bonus," stated that it was really an addition to the sum assured, but a reference to the Board of Trade Returns would have revealed to him that—in the case of the largest company, at any rate—the guaranteed bonuses as and when declared were valued as real bonuses and were, therefore, a direct charge on surplus. In addition, a further reserve was made for future guaranteed bonuses. The reason for guaranteeing the bonuses was that the department was a new one and no profits could possibly accrue therein in the early years.

After five years' premiums had been paid—this period corresponding with the point at which a loan would be granted—these certificates might be surrendered for either a fully-paid certificate, payable on the original date of maturity, or for the cash surrender value. The amount of the fully-paid certificate was equal to the total premiums paid, without interest. When the extremely heavy initial expenditure was taken into account, he thought that, on the whole, the amount allowed was a fair one. The author appeared to under-estimate the expenses of the business, which were probably more nearly equivalent to the whole of the first premium, one-third of the second, one-fourth of the third, and 15 per-cent thereafter. The scale of cash surrender values allowed was not quite so arbitrary as Mr. Maltby thought. At the end of five years the surrender value was £4. 6s. 8d. per-cent, and this was equal to the discounted value of the fully-paid certificate of £13 at  $4\frac{1}{2}$  per-cent for the outstanding term. From this point onwards the scale gradually rose by a graduated percentage until, at maturity, the sum payable was, of course, equal to the face-amount of the certificate.

With regard to the question of transfers, a transfer of these certificates could take place either during the lifetime of the certificate-holder or at his death. The usual condition applicable to transfers during lifetime was as follows:—"Certificates upon which not less than twelve monthly premiums have been paid and upon which all premiums due have been paid to date, may be sold, and are transferable, subject to the approval of the proposed transferee by the Directors. The transferee becomes entitled to the same rights as the original holder of the certificate in respect of the purchase of a house twelve months from the date of registration of the transfer and to all other rights immediately. A transfer fee of five shillings is charged, and transfers can be made only upon the form provided by the company."

It would be noted that under this condition the transferee became the registered holder of the contract and a transfer of these certificates differed from an absolute assignment of a life policy, inasmuch as the original certificate-holder passed completely out of the contract and his death or survivance had no effect whatever in the determination of any payment thereunder. Moreover, there was no limit to the possible number of transfers, and whoever

happened to be the registered holder at maturity would receive the sum then payable. It would, therefore, be seen that the payment of this amount did not depend in any way upon the survivance of any particular life. The due payment of all premiums was the only contingency affecting the payment of the maturity amount. He wished to emphasize this point, because it appeared to him to dispose altogether of the author's contention that these certificates were, technically, pure endowments with return, and that, consequently, they should be valued by means of mortality functions. The technical definition of a "pure endowment" was a sum payable only in the event of a particular person surviving to a specified age. He had shown that the amount payable at maturity did not depend in any way on the survivance of any particular person, and it, therefore, seemed that to introduce mortality functions in the valuation of these contracts was little short of a farce, first, because it incorrectly interpreted the character of the contract, and secondly, because it was impossible; for, although the age of the original holder was usually known, only a prophet could possibly ascertain the number and respective ages of the possible transferees that might be the registered holders of the contract during its currency.

The second mode of transfer was as follows:—In the event of the death of the registered holder, the legal representatives might, at their option, elect to continue the payment of the premiums and so secure the payment of the amount due at maturity—in which case the certificate was transferred free of cost—or they might elect to take a cash payment equal to the total premiums paid under the certificate. From this it would be seen that death did not necessarily determine the contract, and, in fact, the legal representatives did frequently exercise their option to continue. Mr. Maltby described this payment as a death benefit, but, personally, he failed to see where the benefit came in. The payment was nothing more or less than a guaranteed surrender value, payable in special circumstances. It was true that in the early years the payment was greater than the ordinary surrender value, but then the circumstances were different. In the case of an ordinary surrender the certificate-holder exercised an option against the cash reserves which it was vital to these offices to protect, and he must be penalised. In the case of a bondholder dying, no such option could have been exercised, and, consequently, what was more natural—if the legal representatives decided not to continue the contract—than that it should be cancelled and a return made of exactly what had been paid? The payment of this amount, which, after all, was merely the result of an option, was no more a benefit dependent on mortality than the payment by a banker of the amount standing to the credit of the deposit account of any depositor at his death.

From these remarks it would be gathered that he entirely disagreed with the author's contentions (first) that these contracts were really pure endowments with return, (secondly) that the theoretically correct reserves must be determined on a mortality

basis, and (thirdly) that it was theoretically incorrect to value upon a sinking-fund basis. So far as the contracts he had been describing were concerned—and they were typical—the only correct and proper way to value them was on a sinking-fund basis. The late Mr. R. P. Hardy, who was connected with one of these companies for nearly 25 years, always made the valuation on an interest basis, not because it was convenient—for in the case of the company in question the age of the original holder was always required for other purposes, and heavy labour would have been no bar to Mr. Hardy—but because, in his considered opinion, they were interest contracts pure and simple, and he thought it would be generally admitted that such an opinion, emanating from so eminent an actuarial authority on these contracts, must carry very great weight. He thought that the author must have been led to the opposite view, either because the conditions of the contracts with which he had to deal were such that the death or survivance of the original holder really did determine the contract, or because, during his investigation for the purpose of this Paper, he found that the basis adopted by some of the companies brought out practically no reserves for contracts of short duration to meet the liability to make the return on the cancellation of the contract, and the question naturally arose as to what additional reserve was required. It might appear reasonable at first sight to try and estimate the additional reserve required upon a mortality basis, but since the cancellation of the contract was merely the result of an option, he did not see how a theoretically correct reserve could be made in this manner. The rate to be used was, undoubtedly, the rate of cancellation, and to use the rate of mortality, even as a substitute for the rate of cancellation, was erroneous for the following reason:—The additional liability was equal to the difference between the total premiums paid at each duration and the corresponding interest reserves, and it was obvious that these differences would be positive for the shorter durations and negative for the longer durations; that is to say, the greatest additional liability occurred in the early years. Now the rate of cancellation was also greatest in the early years, and diminished with the duration. Consequently, to use the rate of mortality, which was lowest in the early years and increased with the duration, would very likely have the effect of under-estimating the true additional reserve required, because they would have a high rate instead of a low rate operating on the negative differences at the longer durations.

It might seem that he had gone into this matter with unnecessary detail, but he had purposely done so because he regarded it as particularly unfortunate at the present time, when there appeared to be some doubt as to the legal nature of similar contracts, that a Paper on this subject, which would be subsequently officially published by the Institute, should contain such very definite statements that, technically, all these contracts were pure endowments with return, that the theoretically correct reserves could only be ascertained by the use of mortality functions, and that it was

theoretically incorrect to value them on a sinking-fund basis, thereby implying that these contracts were of the nature of life assurance, whereas, as regards the great mass, eminent actuarial and legal authorities had declared that they were interest-contracts pure and simple.

Before leaving this subject he would like to point out that on p. 263, the author, in discussing the questions in the Valuation Returns to the Board of Trade under the 1909 Act, stated that Question (8) of the Fifth Schedule needed no comment. This question required "Full particulars as to the terms and conditions upon which transfers of the interest in a certificate are granted, whether on the death of the holder of the certificate or during his lifetime." These were the very conditions on which he had laid such great stress, and it appeared to him that, far from needing no comment, they really furnished the whole answer as to the real character of these contracts.

He would now like to say something on the general question of the use of the ballot. Section 34 (e) of the 1909 Act stated :— "The company shall not give the holder of any policy issued after the passing of this Act any advantage dependent on lot or chance, but this provision shall not be construed as in anywise prejudicing any question as to the application to any such transaction, whether in respect of a policy issued before or after the passing of this Act, of the law relating to lotteries." He wanted, first, to point out that the limitation was only placed upon contracts issued after December, 1909, and not upon contracts issued before that date. In the original draft of the Act the restriction was placed on all contracts, but was removed as regards those then existing upon the representations of a leading company.

The ballot was used merely for the purpose of determining priority in the allotment of advances free of interest, and it was a very common practice amongst building societies, until, in 1894, the Building Societies Act, Section 12, enacted : "(1) A society under the Building Societies Acts established after the passing of this Act shall not cause or permit the applicants for advances to ballot for precedence or in any way make the granting of an advance depend on any chance or lot. (2) Where the rules of a society under the Building Societies Acts established before the passing of this Act provide that advances may be balloted for, the society may, notwithstanding anything in its rules, resolve, by a majority of its members present or voting by voting papers, at a meeting called for the purpose, upon a scheme for the discontinuance of advances by ballot and for making other provision in lieu thereof, provided that the scheme is supported by a majority of such of the members present, or voting by voting papers at the meeting as have not at the date of the meeting received their advances by ballot ; and every scheme so resolved on, and every alteration thereof, shall be registered in the manner provided for the registration of rules, and shall be binding on all members and officers of the society."

A fairly reasonable interpretation of this section would be

that in respect to societies established before 1894, where a majority of the members decided to continue the ballot, it should be perfectly legal to do so. Generally, it might be said that for the purpose of determination of priority the ballot was used almost universally, and, amongst others, he might instance the following bodies which make use of it:—The House of Commons, financial houses in connection with the drawing of bonds, and our own Institute. In fact, balloting for priority is such a common practice that he did not think that house-purchase companies had much to fear from the second part of Section 34 (e) of the Assurance Companies Act, 1909. There was one other point bearing on this subject that he would like to mention, and that was that when a certificate had been drawn and an advance free of interest had been taken up, one large company made a reserve in the valuation, in addition to the ordinary reserve, of an annuity of the interest foregone for the outstanding term, and he thought this was a proper course to adopt.

Mr. T. G. ACKLAND said that an important question was raised in the Paper, as to the definition in the Act of Bond Investment Companies. That, undoubtedly, had been a source of some trouble, and when the time arrived for amendment of the 1909 Act it might be deemed advisable that that point should be dealt with, because, as the author stated, in the present position of affairs many of the contracts of companies that were carrying on that class of business were apparently not within the strict definition in the Act. He had not had time to look fully at the actuarial formulas in the Paper, but they were evidently the fruit of much study. He had listened with great interest to Mr. Clinton's remarks as to whether a mortality table should be employed in the valuation of the contracts of bond investment companies, in consideration of the fact that a return was frequently made of some portion of the premiums (usually a large one) on the death of the assured or bondholder. Personally, he would be disposed to say that a mortality table would hardly be appropriate, but the author seemed to give cogent reasons for his view, and no doubt the question was one which was open to discussion. Most of the companies that had come under his notice had been valued on the lines of sinking fund policies, and it was, he believed, the practice of many life assurance companies to value their pure endowments, with a return of 75 or 80 per-cent of the premiums, by interest functions alone, either prospectively or retrospectively. That plan seemed to have worked well, and it was somewhat difficult to see why it would not work well in the case of bond investment companies, where the contract, although not identical, seemed at least to be analogous.

He had been much interested in that portion of the Paper which dealt with the sections and schedules of the 1909 Act, because those were matters with which he was presumed to have some little acquaintance. With regard to the legal question, and the decision of the Court of Appeal, that the inclusion of a return of premiums in connection with that sort of contract was of the nature of life

assurance, he thought the matter might, perhaps, be further considered by the Courts, and he had no doubt that the subject would receive the careful consideration of the authorities who had to deal with the Returns, for if that decision should be maintained, the bond investment schedules of the Act would either be inapplicable, or would require material modifications. He believed a case had been decided in Ireland on a somewhat similar, although not an identical question. This case was in connection with a company that had no power to transact life assurance business, and words were added in the Memorandum of Association somewhat to the effect, "Nor anything of the nature of life insurance business." The company proceeded to transact what actuaries would call pure endowments with return of premiums. The case was brought before the Court, not on a "friendly" action (as in the case cited in the Paper) but on a distinctly unfriendly action; and it was decided by the Irish judges, and, he thought, ultimately by an English Court, that if the case were not life assurance, pure and simple, as to which the judges had some doubt, it certainly came within the clause that forbade the company to do anything that savoured, or was of the nature of, life assurance. On that ground it was decided, not that the company were transacting life assurance business and must make a deposit, but that the whole business was *ultra vires*; that the contributions paid by the assured were advances made to the company, without any valid contracts existing, and that the company must proceed to return those contributions to the assured, or such portion of them as their assets would permit. That had some bearing on, although it was not in very direct relation to, the question dealt with by Mr. Maltby. He would further point out that the authorities had a ways refused to recognize similar contracts as life assurance contracts for the purpose of allowing abatement of income tax, and he (the speaker) thought it was also the case that in connection with stamping the contract there was some important distinction made between life assurance and returnable endowment policies.

There was a point which, he thought, rather called for comment in connection with Schedule IV, and the principles adopted in the valuation, in regard to which the author referred to Schedule VI (e) of the Act of 1909, dealing with the method by which the contracts of a bond investment company should be valued, under order of the Court, and appeared to infer that the contracts must be valued on a net premium basis. That was not the case, if the author meant that such a valuation must be made as a test of solvency, because Schedule VI and the sections relative thereto applied only to the cases where the Court had already ascertained, on actuarial or other evidence, the insolvency of the company and where the company was being wound up, in which case the individual rights of the parties entitled were to be ascertained, on the basis of a net premium valuation. The same remark applied to the rules for valuation in Schedule VI, in respect of life assurance and other branches of insurance. That had, he thought, already been pointed

out in the *Journal*, but not in its special application to bond investment companies. For what reason the net premium valuation was laid down to decide the rights of the parties on a winding-up he did not know, but it certainly had not been laid down, so far as his understanding of the Act and schedules went, that there was to be a net premium valuation for the ascertainment of solvency, which, indeed, would hardly be considered reasonable. He thought the suggestions the author made as to the expansion of the schedules were well-conceived and worthy of consideration; but he did not know what view the managers and actuaries of the different offices, who had to make such enlarged returns, would take as to these particular suggestions.

Mr. D. C. FRASER said that he had no special knowledge of the subject of the Paper, but that he wished to refer to a remark made by the author in connection with Schedules IV and V. Speaking of Schedule V, the author pointed out that if the valuation was made by a consulting actuary it must be signed by the actuary, although he could not personally vouch for much of the information given. He then said: "Apparently, however, in view of Section 24, the actuary is only required to certify that the statements in question are true to the best of his knowledge and belief, and probably, therefore, it is only necessary for him to satisfy himself that the information in question is in accordance with the data supplied to him by the company, and those contained in its general literature." He thought that raised a question of principle, and one which bore upon the standards of professional conduct. He was quite sure that when the Act required the actuary's signature to Schedule V it meant that the actuary took responsibility for the statements made in the schedule. On looking at the schedule, with which Mr. Maltby had dealt in some detail, it was quite clear that the actuary must feel satisfied that he had full information on the matters referred to in the schedule before he could properly make the valuation—before he could be quite sure that his valuation covered all the liabilities of the company. The schedule must necessarily be made up by the officers of the company, but it must be made, he thought, under the actuary's instructions and directions, and must satisfy his requirements, and he thought that was involved in the actuary's signature to the schedule.

On referring to Section 24 of the Act, which the author had mentioned, he found there that if any statement in any schedule to the Act was false to the knowledge of the person signing it, that person was liable to fine and imprisonment. The author's suggestion might save the actuary from fine and imprisonment, but he was quite sure that no actuary would be satisfied in escaping the prison door by a hair's breadth. He referred to the matter not because he thought those present required instruction on such a point, but actuaries in the provinces and all over the world, in the Colonies and Dependencies of the Empire and even in foreign countries, looked to the Institute to lay down the standards for professional conduct, and he feared that if the point were left without

remark, some remote company, in, perhaps, some other country, might take advantage of the opinion expressed in the Paper to produce a valuation where the surplus was entirely a fictitious one. They might give the actuary instructions under which he might think he had to deal with the liabilities only, while the company, on its part, would deal with the assets and with the information in Schedule V, and would combine the information so as to produce a surplus where a surplus did not exist. The whole point was that the actuary had to certify the surplus, and had to be satisfied, not merely that he had made his calculations with arithmetical accuracy, but that, if he did declare a surplus, it really existed.

Mr. GEORGE KING said that although there had not been a long discussion it had been a very useful one, but it did not give him very much to say in closing it. He might, however, be permitted to make a few remarks on his own account. In the first place, he wished to explain that since he accepted the honour done him when he was asked to close the discussion, the conditions, so far as he himself was concerned, had been greatly changed through the loss of his very old friend, Mr. Ralph Price Hardy. Owing to his death he had been appointed consulting actuary to one of the bond investment companies, and, therefore, he could not speak with so much freedom as he would have done under other circumstances. In mentioning Mr. R. P. Hardy's name he thought it should always be remembered that he was for many years an advocate of sound finance for bond investment companies, and the Institute owed him a deep debt of gratitude on that account. The late Mr. Hardy laid down for his own company good rules, and saw that they were fully carried out, and other companies had made strenuous efforts to follow the example. With regard to the Paper itself, he thought thanks were due to Mr. Maltby for his monumental and very useful contribution, and he could say that all the more freely when he added that he by no means agreed with all that the author said. The Paper was a mine of information, but there were so many points in it that it was impossible for one speaker in the time available to discuss them all. He was glad to notice the sympathy that the author had shown towards bond investment companies. At one time that was not the general feeling. They were not fully understood, and about eight years ago there was very good cause for distrust. Within his own experience, dozens of new companies came to him, but he would not touch them because they were all founded on unsound finance. They expected to make their profits out of lapses, and they built their premiums upon that basis, and made promises that it was altogether impossible to perform. A very similar feeling of distrust prevailed many years ago with regard to industrial insurance, which at one time was looked on very askance; in fact, even the then Chancellor of the Exchequer said that a company, which is now, perhaps, one of the grandest institutions in the world, was insolvent and a scandal. That was only because the system was not understood. In the early days of bond investment companies similar things were said, but that

opinion was dying away. The Act of 1909 would bring out the real facts better than they had ever been known before, and would show that bond investment companies could be worked on sound and useful lines. It was true that at present it was, perhaps, an expensive form of thrift, but it was educational thrift ; and just as industrial companies had developed great ordinary branches by teaching people thrift in the industrial branches, so he was perfectly sure that as the public learned thrift, the bond companies would fit themselves to the new conditions, and would give opportunities for thrift on improved lines and at reduced expense.

From one point of view the Paper was premature—in fact the author said so himself—because, necessarily there would be great changes. Only one Blue Book had yet been published, and in the next Blue Book a very great deal more information would probably be available, and when two or three more Annual Returns had been published a great deal of information would be gained. Possibly some of the companies that appeared in the present Blue Book would cease to appear ; or, on the other hand, they might develop very much.

One point of very great importance dealt with in the Paper was whether bond investment contracts were of the nature of life assurance. He was very glad that Mr. Clinton had called attention so emphatically in his remarks to the question of the return of premiums on the death of a bondholder, and to the theory that a condition in the bonds providing for such return brought transactions of that kind into the category of life assurance transactions, and placed them under the provisions relating to life assurance in the Assurance Companies Act, 1909. That theory seemed to be supported by the author, because he said that, technically speaking, those bonds were pure endowments with return of premium, which were usually included by ordinary offices amongst their life business in making Returns under the Act. He wished to enter a protest against allowing it to go out from the Hall of the Institute that actuaries, in general, took that view. It was true that in the case of the Law Integrity Company, referred to by the author, it was decided, on 23 August last, in the Court of Appeal, that certain policies of that company were life assurance policies, but the reports of that case were very meagre. So far as he knew, there was no record of the exact terms of the contract which was before the Court, and, moreover, he had not seen any report of the evidence taken. The usual form of contract, with which alone he was at present dealing, was, in consideration of certain periodical subscriptions, payable during a definite term of years independent of human life, to pay a fixed sum at the end of the term ; and it provided, in the conditions endorsed, that if the bondholder desired to terminate the contract he should, on delivering up the bond, receive a fully paid-up certificate for the amount of the subscriptions paid, or for an amount calculated in some other defined way, that certificate maturing at the end of the original term of years, and that that fully paid-up certificate might be surrendered for a sum in cash. It also

provided that, in the event of the death of the bondholder, his legal representatives should have the right to continue the payment of future subscriptions, or, if they preferred, they might have returned to them, practically as surrender-value, all subscriptions previously paid, either without interest or with some interest. If the Law Integrity contract was in that usual form, then he could not help thinking that the case was very badly laid before the Court, and that the actuarial evidence must have been very defective, and in the event of further litigation over other policies in other companies, whatever might happen in the inferior Courts, personally he had very little doubt that the House of Lords would decide that the contracts did not partake of the nature of life assurance. If the matter were looked at dispassionately they must come to the conclusion that in no sense did those bonds partake of the nature of life assurance, and Mr Clinton had shown that the legislature, in passing the Assurance Companies Act, 1909, did not look upon them in that light. That Act, when still a Bill, was carefully scrutinized by sharp actuarial and legal eyes, and yet the life assurance element was never detected. Those ordinary bonds were not terminated by death. The representatives of the bondholder had a right themselves to continue the bond, or they might sell the bond to another, and have him substituted as the holder, in which case the death return, should the purchaser die after a short period, would be extended to his representatives ; or, if more convenient to the parties, the representatives of the deceased bondholder might elect to surrender the bond for a sum, perhaps a little larger than would be paid in other circumstances as surrender-value. That did not create a death benefit, but merely a permission to the representatives of the deceased to have the contract cancelled with a minimum loss to the estate. Then the permission to transfer the bond during the lifetime of the holder, and to continue to the transferee the condition regarding surrender, whether during life or on death, again showed that there was no death benefit involved, and that the bond did not in any way depend on any life ; but that point had been so fully dealt with by Mr. Clinton that he need not elaborate it further. His view thus being that there was no death benefit, he could not admit the propriety of bringing in probabilities of life, either in the calculation of contributions or in the periodical valuations, and he could not admit that it would be right, as suggested by the author, to separate in the revenue account the claims into (a) claims by maturity, and (b) claims by death ; and it followed that, in his opinion, all that portion of Mr. Maltby's Paper which dealt with the matter from the life-policy point of view was beside the question.

There was one other point to which he desired to call attention. The author said : "The net premium method has the great advantage that the allowance for expenses is automatically increased with the valuation rate of interest, and thus the combination of a high rate of interest and an insufficient allowance for future expenses is rendered impossible." He ventured to think that Mr. Maltby

might reconsider that point a little, and, perhaps, amplify it, because, if the statement as it stood was carried to its legitimate conclusion, it meant that a company which found it had not enough margin for expenses had only to value at a higher rate of interest to get that margin. That was absurd, because it weakened its reserves instead of giving itself a larger margin; in fact, he did not think they could separate the premiums from the reserves and allocate a fixed amount for expenses. A company valuing at a low rate of interest, with a narrow margin of loading, had a greater provision for expenses (if it charged the same premium) than another company that valued at a higher rate of interest and with a larger margin of loading. With regard to the question of changes in the law, he ventured to submit it was a little premature even to discuss them. It was impossible to say what alterations could be beneficial until the working of the Act had been watched for a little longer.

The PRESIDENT, in moving a hearty vote of thanks to Mr. Maltby, said that the Paper brought comparatively new problems before the Institute, and broke fresh ground. The departmental enquiry that was held into the subject of bond companies did exceedingly good work, and the Institute could congratulate itself upon the part which it played in the investigations through the medium of some of its prominent members. The danger of depending upon a lapse rate for solvency was fully exposed, and, as the author pointed out, the legislation which followed the report protected the small investor from placing his money in unsound schemes.

Mr. C. H. MALTBY, in reply, said that the chief point in the discussion was whether the bonds were, or were not, contracts of life assurance. Most of the speakers seemed to think that he considered they were contracts of life assurance, but he strongly dissented from that view. He thought he had made it perfectly clear in his Paper that he held exactly the opposite opinion. He had dealt with the point in several places, and he thought if the Paper was carefully read the point was quite obvious. Personally, he had no doubt whatever that they were not life assurance contracts, and he thought the case referred to by Mr. King was decided on altogether incorrect lines. In dealing with the subject of the valuation, he mentioned the possible effect that the restrictions placed on the transfer of the bonds might have on the valuation. The investigations referred to in Tables X to XIII were purely theoretical, and it was quite impossible to give effect to the assumptions upon which they were based. With regard to the question discussed by Mr. Fraser as to how far actuaries were responsible for the data on which the valuation was based, his point was that a consulting actuary was given certain figures which the company said were the whole of the contracts, and without going through the books it was almost impossible for him to say whether they were the whole or not. If a company kept out half of its business, he did not see how an actuary, without going through the books, was going to find it out. Personally, he took the figures signed by the auditors, so as to make sure of their accuracy, but he thought it was just as well to

mention the matter with a view to ascertaining what precautions were generally taken.

(Mr. Maltby has since communicated to us the following further comments on the discussion.—ED. J.I.A.)

The most important point raised in the discussion is, I think, the question of whether the certificates issued by house purchase companies are policies of life assurance within the meaning of the 1909 Act. After the decision of the Court of Appeal in the Law Integrity case, I felt some hesitation in expressing a definite opinion on the point, although I thought that the language used on pp. 255 and 256, in summing up both sides of the case, would sufficiently indicate my views. Having regard, however, to the general trend of the discussion, I now think it well to state my personal opinion, which is that they are certainly not policies of life assurance, and that the result of applying the test used in the General Accident case is sufficient proof of this.

I think that the apparent conflict of opinion between Mr. Clinton and myself, on several points, is due to the fact that his remarks deal entirely with the particular form of contract issued by one company, whereas I was dealing with the business as a whole. No doubt the contracts he describes do constitute, at the present time, a large proportion of the business; but, while recognizing this, I felt that, from the actuarial point of view, the other forms of certificate issued needed fuller attention; firstly, because the actuarial points involved by them are more complicated, and, secondly, because it is with the certificates issued by other Offices than that mentioned that actuaries, as a class, are most likely to come in contact.

As regards the theoretically-correct method of valuing those contracts under which the premiums paid are returnable, under certain contingencies, *with compound interest*, I still think that to arrive at an absolutely correct result it would be necessary to introduce what I may call "a rate of payment of return of premium", which would give effect to the combined influences of mortality, transfers, and any other regulations in force governing the payment of the return.

Such a rate was in my mind when I referred to "a modified rate of mortality" on p. 235. As stated, such a valuation is impossible in practice, and the only practical method is to deal with the certificate on a purely interest-basis.

The investigation contained in pp. 232-240—the only part of the Paper in which mortality functions are introduced—aimed at elucidating the error involved by valuing bonds, *with return of premium with compound interest*, on an interest-basis. As pointed out in the Paper, the investigation is purely theoretical, and the rate of mortality was used alone as it was impossible to make allowance for any regulations in force governing the payment of return. No two sets of these are alike and, as stated on p. 239, the difference between my "true reserves" and those on a sinking-fund basis, at the same rate of interest, would in many cases be diminished.

I believe that, under the certificates described by Mr. Clinton, the premiums paid are returnable in the event of death, *without interest*, and if this is the case, I would point out that the investigation into the correct method of valuation, and the remarks on pp. 211 and 212, as to the calculation of premiums, expressly only refer to bonds under which the premiums are returned *with compound interest*. On p. 240, I deal briefly with the valuation of contracts with return of premium *without interest*, and agree with their treatment on a purely interest-basis.

If the premiums are returnable with compound interest on the death of the original holder or of any transferee, the effect of a transfer on the theoretical reserves will vary according as it has the effect of increasing or diminishing the chance of being called upon to make the return; the reserves remaining unaltered if the transferee should be identical, in every respect, with the original holder.

The valuation basis, too, of the particular Office referred to by Mr. Clinton is, I believe, such that it practically conforms to the conditions mentioned on p. 232, under which sinking-fund reserves are theoretically correct for bonds with return at compound interest. This is by no means always the case, and where such a stringent basis of valuation is not adopted the total premiums paid with interest, at each duration, are often greater throughout practically *the whole duration* of the bond than the corresponding interest reserves. (See Columns 2, 5 and 6 of Table VIII.) Some additional reserve is, therefore, theoretically necessary, although, as stated by me on p. 239, in practice this may be usually disregarded, provided that a proper allowance has been made for expenses, and the interest-margin is at least  $\frac{1}{2}$  per-cent.

My statement that house purchase certificates are, technically speaking, pure endowments, with return, must be read with the rest of my remarks, and particularly with the rest of the paragraph on p. 195, in which the phrase is first used. As shown, these contracts really fall into a category of their own, lying between ordinary pure endowments with return, and sinking-fund policies pure and simple; and approximate to the one or the other class, according to the exact conditions under which the return of premiums is payable.

Mr. Clinton appears to be under a misapprehension in stating that “I appear to under-estimate the expenses of the business.” On the basis of interest at 4 per-cent, loading for the scale of expenses he mentions gives a monthly premium of 3s.  $8\frac{1}{2}d.$  per-cent for a 30-year bond, which coincides with the result of an allowance of £2 per-cent of the sum assured, and about 16 per-cent of the premium throughout, on the system of loading I adopted. (See Table VI.) On p. 220, I state “In the case of a 30-year bond, the whole of the subscription for the first year is generally regarded as going in expenses, but £2 per-cent and 20 per-cent are probably higher than would usually be experienced, whilst £1 per-cent and 10 per-cent are too low,” so that, on the whole, our estimates of the expenses practically agree.

I am much obliged to Mr. George King for pointing out the obscurity of the expression on p. 247. I had in mind the case of a company valuing at 5 per-cent, to take an extreme case. If a net premium valuation is made, the provision for future expenses is greatly in excess of what is really required, and to this extent, the Office is in a far better position than a similar company also valuing at 5 per-cent, and only reserving, say, 12 per-cent of the gross premium-income for expenses. In both cases, of course, the interest-margin, necessary for safety, would be under-estimated.

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*On the Graduation of Mortality Tables by Interpolation.*  
By S. T. SHOVELTON, M.A., formerly Fellow of Merton College, Oxford.

1. IN the several Papers on graduation of mortality tables which have been published from time to time in the *Journal of the Institute of Actuaries* by Mr. King, Dr. Buchanan, and others, the general method adopted has been that of osculatory interpolation. In osculatory interpolation two curves are drawn passing through points whose ordinates are  $u_a, u_{a+1}, \dots u_b$  and  $u_{a+1} \dots u_{b+1}$  respectively. If these curves are denoted by A and B and the central interval is between  $u_c$  and  $u_{c+1}$ , the osculatory curve has the same slope and curvature as A at the point "c," and the same slope and curvature as B at the point "c+1." In this Paper a formula is given for a curve which has the same slopes as A and B at the points "c" and "c+1" respectively, and which is such that the average value of the ordinate between these points is equal to the average value of the mean of the ordinates of the curves A and B between the same points. It is subsequently shown that, when unadjusted values of the ordinates are known and adjusted quinquennial values are obtained by a fifth-difference interpolation formula and the intermediate values by the new formula, the average value of the ordinates so obtained is equal to the average value of the corresponding ordinates given by the unadjusted data to a high degree of accuracy. As there is one condition less to be satisfied than in osculatory interpolation, the general effect will be to obtain a curve of one degree less than that obtained by the latter method. The formulæ used are those of central differences, either in the ordinary form or in that of Everett's formula (*J.I.A.*, xxxv, 452). In the latter case the very convenient notation introduced by Dr. Sheppard

and explained by Dr. Buchanan (*J.I.A.*, xlvi, 393) is used. No confusion arises from the use of the symbol  $\delta$  since  $\Delta$  throughout refers to intervals of unity, and methods are given which avoid the necessity of calculating the leading differences for subdivided intervals for which the symbol  $\delta$  has hitherto been used.

2. The ordinary central difference formulæ are

$$u_x = u_0 + x\Delta u_0 + \frac{x(x-1)}{[2]} \Delta^2 u_{-1} + \frac{x(x^2-1)}{[3]} \Delta^3 u_{-1} \\ + \frac{x(x^2-1)(x-2)}{[4]} \Delta^4 u_{-2} + \dots$$

and

$$u_x = u_0 + x\Delta u_{-1} + \frac{x(x+1)}{[2]} \Delta^2 u_{-1} + \frac{x(x^2-1)}{[3]} \Delta^3 u_{-2} \\ + \frac{x(x^2-1)(x+2)}{[4]} \Delta^4 u_{-2} + \dots$$

together with Stirling's formula, which is obtained by taking the mean of  $u_x$  as given by these two formulæ. If we stop at any even difference the three formulæ are identically equivalent. For example, if we stop at  $\Delta^4 u_{-2}$  any one of the three will give the equation of the curve of the fourth degree passing through the five points  $u_{-2}, u_{-1}, u_0, u_1$  and  $u_2$ . It will be found convenient to use the first of these formulæ, and we then have, as the equation of the curve through the points  $u_{-2} \dots u_2$ ,

$$y = \phi(x) = u_0 + x\Delta u_0 + \dots + \frac{x(x^2-1)(x-2)}{[4]} \Delta^4 u_{-2} \dots \quad (1)$$

Similarly, the curve of the fourth degree through the points  $u_{-1} \dots u_3$  may be written

$$y = f(x) = u_1 + (x-1)\Delta u_1 + \dots + \frac{(x-1)(x-1^2-1)(x-3)}{[4]} \Delta^4 u_{-1} \quad (2)$$

Now, it is easily seen that

$$f(x) = \phi(x) + \frac{x(x^2-1)(x-2)}{[4]} \Delta^5 u_{-2}$$

for these expressions are equal to one another when  $x$  takes the values  $-1, 0, 1$  and  $2$  (the corresponding values being

$u_{-1}, u_0, u_1$  and  $u_2$ ), and the coefficients of  $x^4$  are equal. Hence, since the expressions are of the fourth degree, they are identically equivalent. Therefore,

$$y=f(x)=\phi(x)+\frac{x(x^2-1)(x-2)\Delta^5u_{-2}}{|\underline{4}|}$$

is the curve of the fourth degree which passes through  $u_{-1}, u_0, u_1, u_2$  and  $u_3$ .

3. We now seek to find a curve of the fourth degree which shall have the same ordinate and slope as the curve  $y=\phi(x)$  at  $x=0$  and the same ordinate and slope as  $y=f(x)$  at  $x=1$ . In addition we have to impose as a further condition that the average value of the ordinate of this curve between 0 and 1 shall be the mean between the average values of the ordinates of the curves (1) and (2) between the same points. We shall give effect to this condition by equating areas between the extreme ordinates. Proceeding on lines similar to those adopted by Mr. Lidstone in his Note to Dr. Buchanan's Paper (*J.I.A.*, xlvi, 397), we assume the required curve to have the equation  $y=\phi(x)+\psi(x)$ , and obtain the following conditions :

$$\phi(0)+\psi(0)=u_0=\phi(0) \quad . \quad . \quad . \quad . \quad (a)$$

$$\phi'(0)+\psi'(0)=u'_0=\phi'(0) \quad . \quad . \quad . \quad . \quad (b)$$

$$\phi(1)+\psi(1)=u_1=f(1)=\phi(1) \quad . \quad . \quad . \quad . \quad (c)$$

$$\phi'(1)+\psi'(1)=u'_1=f'(1) \quad . \quad . \quad . \quad . \quad (d)$$

$$\int_0^1 \{\phi(x)+\psi(x)\} dx = \frac{1}{2} \int_0^1 \{\phi(x)+f(x)\} dx$$

or  $\int_0^1 \psi(x) dx = \frac{1}{2} \cdot \frac{\Delta^5 u_{-2}}{|\underline{4}|} \int_0^1 x(x^2-1)(x-2) dx \quad (e)$

It is readily seen that  $\psi(x)=x^2(x-1)(ax+b)$  and it follows at once from (d) and (e) that  $a=\frac{1}{2} \frac{\Delta^5 u_{-2}}{|\underline{4}|}$  and  $b=-\frac{5}{2} \frac{\Delta^5 u_{-2}}{|\underline{4}|}$ .

Therefore, the required interpolation curve is

$$y=\phi(x)+\frac{x^2(x-1)(x-5)}{2|\underline{4}|} \Delta^5 u_{-2} \quad . \quad . \quad . \quad (3)$$

4. If we employ Everett's formula we find that the curve through  $u_{-2}, \dots, u_2$  is

$$\begin{aligned} y = \phi(x) = & \left[ \xi + \frac{\xi(\xi^2 - 1)}{3} \delta^2 \right] u_0 \\ & + \left[ x + \frac{x(x^2 - 1)}{3} \delta^2 \right] u_1 + \frac{x(x^2 - 1)(x - 2)}{4} \delta^4 u_0, \end{aligned}$$

when  $\xi = 1 - x$  and  $\delta^2 u_0, \delta^4 u_0, \&c., = \Delta^2 u_{-1}, \Delta^4 u_{-2}, \&c.$ .

Now

$$\psi(x) = \frac{x^2(x-1)(x-5)}{2 \underline{4}} \Delta^5 u_{-2} = \frac{x^2(x-1)(x-5)}{2 \underline{4}} \delta^4(u_1 - u_0),$$

and

$$\frac{x(x^2 - 1)(x - 2)}{4} - \frac{x^2(x - 1)(x - 5)}{2 \underline{4}} = \frac{\xi^2(\xi - 1)(\xi - 5)}{2 \underline{4}}.$$

Hence, we may write equation (3) in the symmetrical form,

$$\begin{aligned} y = & \left[ \xi + \frac{\xi(\xi^2 - 1)}{3} \delta^2 + \frac{\xi^2(\xi - 1)(\xi - 5)}{2 \underline{4}} \delta^4 \right] u_0 \\ & + \left[ x + \frac{x(x^2 - 1)}{3} \delta^2 + \frac{x^2(x - 1)(x - 5)}{2 \underline{4}} \delta^4 \right] u_1 . . . \quad (4) \end{aligned}$$

If this formula be used every set of calculations will do duty twice over, as in the case of Dr. Buchanan's formula (*J.I.A.*, xlvi, 369), and the labour is reduced by half.

5. So far, the problem of interpolation only has been considered, but when we pass on to graduation we have, first of all, to find from the given data suitable values of  $u$  at equidistant intervals between which to make the interpolations. If the given data consist of values  $U_0, U_1, \dots, U_r, \dots$  and these are replaced by interpolation and graduation by  $u_0, u_1, \dots, u_r, \dots$ , and if the series  $U$  represents, say, a table of  $E_x$  or  $\theta_x$ ,  $L_x$  or  $d_x$ , it follows that  $\Sigma U$  must be equal to  $\Sigma u$ . Formulae are, therefore, required which will not only provide a smooth graduation but also satisfy the above condition. In order that differences may always represent an interval of unity, it will be convenient to take the given  $U$  series in the form  $U_0, U_{\frac{1}{2}}, U_{\frac{2}{3}}, \dots$

Let

$$Y_0 = U_{-\frac{2}{3}} + U_{-\frac{1}{3}} + U_0 + \dots$$

$$Y_{-1} = U_{-\frac{5}{3}} + U_{-\frac{4}{3}} + \dots, \text{ and so on.}$$

Let  $\omega_0 = Y_0 - Y_1 = U_{-\frac{3}{5}} + U_{-\frac{1}{5}} + U_0 + U_{\frac{1}{5}} + U_{\frac{3}{5}}$ ,

with corresponding values for  $\omega_1, \omega_2, \dots$ . Hence,  $U_0$  is the central term of the 15 terms involved in  $\omega_{-1}, \omega_0$  and  $\omega_1$ , and of the 25 terms involved in  $\omega_{-2} \dots \omega_2$ .

The general value of  $y$  involving fifth differences of  $Y$ , or fourth differences of  $\omega$ , is

$$\begin{aligned} y &= Y_0 + x\Delta Y_0 + \frac{x(x-1)}{2}\Delta^2 Y_{-1} + \frac{x(x^2-1)}{3}\Delta^3 Y_{-1} \\ &\quad + \frac{x(x^2-1)(x-2)}{4}\Delta^4 Y_{-2} + \frac{x(x^2-1)(x^2-4)}{5}\Delta^5 Y_{-2}. \end{aligned} \quad (5)$$

Now  $U_0 = Y_{\frac{3}{5}} - Y_{\frac{1}{5}}$ , and we shall, therefore, obtain a new central term  $u_0$ , correct to fifth differences of  $Y$  by the application of this formula.

We have

$$y_{\frac{3}{5}} = Y_0 + \frac{2}{5}\Delta Y_0 - \frac{3}{5^2}\Delta^2 Y_{-1} - \frac{7}{5^3}\Delta^3 Y_{-1} + \frac{14}{5^4}\Delta^4 Y_{-2} + \frac{168}{5^6}\Delta^5 Y_{-2},$$

and

$$y_{\frac{1}{5}} = Y_0 + \frac{3}{5}\Delta Y_0 - \frac{3}{5^2}\Delta^2 Y_{-1} - \frac{8}{5^3}\Delta^3 Y_{-1} + \frac{14}{5^4}\Delta^4 Y_{-2} + \frac{182}{5^6}\Delta^5 Y_{-2}.$$

Remembering that  $\Delta Y_0 = -\omega_0, \Delta^2 Y_0 = -\Delta \omega_0, \dots$ , we have

$$u_0 = y_{\frac{3}{5}} - y_{\frac{1}{5}} = \frac{\omega_0}{5} - \frac{\Delta^2 \omega_{-1}}{5^3} + \frac{14}{5^6} \Delta^5 \omega_{-2} \dots \quad (6)$$

Similarly, if we stop at third differences of  $Y$ , we have

$$u_0 = \frac{\omega_0}{5} - \frac{\Delta^2 \omega_{-1}}{5^3} \dots \quad (7)$$

These formulæ are respectively (iiia) and (v)a in Mr. King's Paper (*J.I.A.*, xlili, 109). They are actually correct up to sixth and fourth differences of  $Y$ , since the coefficients of even differences in (5) for  $x = \frac{2}{5}$  are equal to the corresponding coefficients for  $x = \frac{3}{5}$ .

We thus obtain graduated values of the function  $U$  for quinquennial intervals, the resulting values being  $\dots u_{-1}, u_0, u_1 \dots$ , &c. The intermediate values  $u_{\frac{1}{5}}, u_{\frac{3}{5}}, \dots$  must now be inserted by some interpolation formula, and that formula

will be best which gives a smooth graduation and, at the same time, makes  $u_0 + u_{\frac{1}{5}} + \dots + u_{\frac{4}{5}}$  as nearly as possible equal to the corresponding sum in U. It will be found that a formula which gives good results when  $u$  is obtained by (6) is often not the best to employ when  $u$  is obtained by (7).

6. Using the same notation as in Sections 2, 3, let the curve  $y = \phi(x) + \psi(x)$  have the same ordinates and slopes as  $y = \phi(x)$  at  $x = 0$  and  $y = f(x)$  at  $x = 1$ . If, in addition, we impose the conditions that the sum of the ordinates at  $x = 0$ ,  $\frac{1}{5}$ ,  $\frac{2}{5}$ ,  $\frac{3}{5}$  and  $\frac{4}{5}$  shall be equal to  $U_0 + U_{\frac{1}{5}} + \dots + U_{\frac{4}{5}}$ , at least up to seventh differences, we shall obviously have a good graduation formula leading to results with an aggregate deviation depending on seventh differences. As before,

$$\psi(x) = x^2(x-1)(ax+b), \text{ and } a+b = -\frac{1}{12} \Delta^5 u_{-2}.$$

The sum of the ordinates at  $x = 0$ ,  $\frac{1}{5}$ ,  $\dots$ ,  $\frac{4}{5}$  is

$$5u_0 + 2\Delta u_0 - \frac{2}{5}\Delta^2 u_{-1} - \frac{1}{5}\Delta^3 u_{-1} + \frac{46}{5^4}\Delta^4 u_{-2} - \left(\frac{146a}{5^4} + \frac{2b}{5}\right).$$

Giving  $u_0, u_{-1}, \dots$  the values given by the formula (6), this sum is

$$\begin{aligned} \omega_0 + \frac{2}{5}\Delta\omega_0 - \frac{3}{5^2}\Delta^2\omega_{-1} - \frac{7}{5^3}\Delta^3\omega_{-1} + \frac{14}{5^4}\Delta^4\omega_{-2} + \frac{53}{5^6}\Delta^5\omega_{-2} \\ - \left(\frac{146a}{5^4} + \frac{2b}{5}\right), \end{aligned}$$

neglecting differences higher than the fifth.

Now,  $U_0 + \dots + U_{\frac{4}{5}} = Y_{\frac{1}{5}} - Y_{\frac{4}{5}}$  which, by (5) et seq., is equal to

$$\omega_0 + \frac{2}{5}\Delta\omega_0 - \frac{3}{5^2}\Delta^2\omega_{-1} - \frac{7}{5^3}\Delta^3\omega_{-1} + \frac{14}{5^4}\Delta^4\omega_{-2} + \frac{168}{5^6}\Delta^5\omega_{-2},$$

as far as fifth differences.

Hence,

$$-\left(\frac{146a}{5^4} + \frac{2b}{5}\right) = \frac{23}{5^5}\Delta^5\omega_{-2} = \frac{23}{5^4}\Delta^5u_{-2}$$

Therefore,

$$a = \frac{1}{48}\Delta^5u_{-2} \text{ and } b = \frac{-5}{48}\Delta^5u_{-2},$$

and the required interpolation formula is identical with (3).

If this formula be employed in conjunction with (6) the resulting deviation in the value of  $u_0 + \dots + u_{\frac{1}{5}}$  is  $\frac{290}{5^7} \Delta^6 \omega_{-3}$ , and therefore depends on seventh differences of  $Y$ . If, however, this formula be employed with (7) there will be errors in the fourth and fifth differences of  $\omega$ . The actual amount of the deviation in this case is

$$-\frac{14}{5^5} \left( \Delta^4 + \frac{2}{5} \Delta^5 \right) \omega_{-2}$$

as far as fifth differences.

7. If we proceed on the same lines to find an interpolation formula involving third differences and satisfying the same conditions as in Section 3 or Section 6, we find in both cases, assuming

$$\phi(x) = u_0 + x \Delta u_0 + \frac{x(x-1)}{[2]} \Delta^2 u_{-1},$$

that the interpolation curve has the equation

$$y = \phi(x) + \frac{x^2(x-1)}{[2]} \Delta^3 u_{-1} . . . . . \quad (8)$$

It will be seen that this equation is a modified form of the one given by Mr. King (*J.I.A.*, xli, 580).

The deviations resulting from the use of this formula with (7) and (6) are, respectively,

$$-\frac{12}{5^4} \Delta^4 \omega_{-2} - \frac{143}{5^6} \Delta^5 \omega_{-2} \text{ and } -\frac{46}{5^5} \Delta^4 \omega_{-2} - \frac{23}{5^5} \Delta^5 \omega_{-2}.$$

Thus, cases might be constructed, and might possibly occur in practice, in which the use of the two short formulæ (7) and (8) give a better graduation than that given by (6) and (8) or (6) and (3). For example, if  $\Delta^4 \omega_{-2} = \frac{12}{25} \Delta^5 \omega_{-2}$ , the deviations are, in these three cases,

$$\frac{1}{5^6} \Delta^5 \omega_{-2}, -\frac{23}{5^7} \Delta^5 \omega_{-2} \text{ and } \frac{28}{5^7} \Delta^5 \omega_{-2}.$$

8. It is interesting also to compare the deviation in the value of  $u_{-\frac{2}{5}} + u_{-\frac{1}{5}} + u_0 + u_{\frac{1}{5}} + u_{\frac{2}{5}}$  from  $\omega_0$  when obtained by various combinations of formulæ. Taking first the case of  $u$  given by (6) and the interpolations performed by (3), we

must first of all find the expression corresponding to (3) for the interval  $-1$  to  $0$ . Referring to the point  $0$  as origin the formula is clearly

$$y = \phi_1(x) + \frac{(x+1)^2 x(x-4)}{2 \underline{\underline{4}}} \Delta^5 u_{-3},$$

where  $y = \phi_1(x)$  is the curve of the fourth degree passing through the points  $-3, -2, -1, 0$  and  $1$ . Hence

$$\phi_1(x) = \phi(x) - \frac{(x+2)(x+1)(x)(x-1)}{\underline{\underline{4}}} \Delta^5 u_{-3}$$

as is seen by giving  $x$  the values  $-2, -1, 0$  and  $1$ , and comparing the coefficients of  $x^4$ .

Therefore the interpolation curve for the section  $-1$  to  $0$  is

$$\begin{aligned} y &= \phi(x) - \frac{x(x^2-1)(x+2)}{\underline{\underline{4}}} \Delta^5 u_{-3} + \frac{x(x+1)^2(x-4)}{2 \underline{\underline{4}}} \Delta^5 u_{-3} \\ &= \phi(x) - \frac{x^2(x+1)(x+5)}{2 \underline{\underline{4}}} \Delta^5 u_{-3} \quad \dots \dots \dots \dots \quad (9) \end{aligned}$$

This will give the values of  $u_{-\frac{2}{5}}$  and  $u_{-\frac{1}{5}}$ , while  $u_{\frac{1}{5}}$  and  $u_{\frac{2}{5}}$  are given by (3).

We thus get

$$u_{-\frac{2}{5}} + \dots + u_{\frac{2}{5}} = 5u_0 + \frac{\Delta^2 u_{-1}}{5} - \frac{9}{5^4} \Delta^4 u_{-2} + \frac{31}{4 \cdot 5^4} \Delta^6 u_{-3},$$

which, from (6), is equal to  $\omega_0 + \frac{867}{4 \cdot 5^7} \Delta^6 \omega_{-3} + \dots$

The deviation resulting from the use of (7) instead of (6) is  $-\frac{14}{5^5} \Delta^4 \omega_{-2}$ .

The formula corresponding to (8) for the section  $-1$  to  $0$  is

$$y = u_0 + x \Delta u_0 + \frac{x(x-1)}{\underline{\underline{2}}} \Delta^2 u_{-1} + \frac{x^2(x+1)}{\underline{\underline{2}}} \Delta^3 u_{-2} \quad \dots \quad (10)$$

Hence,

$$u_{-\frac{2}{5}} + \dots + u_{\frac{2}{5}} = 5u_0 + \frac{\Delta^2 u_{-1}}{5} - \frac{8}{5^3} \Delta^4 u_{-2}.$$

The deviation from  $\omega_0$  is  $-\frac{31}{5^5} \Delta^4 \omega_{-2}$  or  $-\frac{9}{5^4} \Delta^4 \omega_{-2}$  according as  $u$  is given by (6) or (7).

9. Since we now know the deviation in any isolated group of five terms, we can find the deviation over any required portion of a table without first constructing the table. This information will tell us at once whether it is necessary to employ formulæ (6) and (3) or whether the two short formulæ (7) and (8) will give sufficiently good results. For example, suppose we graduate a table of  $U$  from  $U_{-\frac{3}{2}}$  to  $U_{r+\frac{3}{2}}$ . The resulting deviation by (7) and (8) is

$$-\frac{9}{5^4} \Delta^4(\omega_{-2} + \omega_{-1} + \dots + \omega_{r-2}) = -\frac{9}{5^4} \Delta^3(\omega_{r-1} - \omega_{-2}).$$

If, then,  $\Delta^3\omega_{r-1}$  is approximately equal to  $\Delta^3\omega_{-2}$  the result will be practically as good as regards the aggregate, but probably not in smoothness, as if the longer formulæ had been employed.

10. It is interesting to compare formula (3) with Karup's formula, from which it differs only in the last term,

$$\frac{x^2(x-1)(x-5)}{2 \underbrace{|4|}_{\underline{4}}} \Delta^5 u_{-2}$$

taking the place of

$$\frac{x^3(x-1)(5x-7)}{\underline{|4|}} \Delta^5 u_{-2}.$$

Since

$$\int_0^1 x^3(x-1)(5x-7)dx = \frac{1}{2} \int_0^1 x^2(x-1)(x-5)dx,$$

it follows that Karup's formula satisfies the same condition as (3) as regards average value. Further, it can readily be verified that the value of  $y_0 + y_{\frac{1}{2}} + \dots + y_{\frac{3}{2}}$  as given by Karup's curve is identical with the value given by (3) and by the mean curve  $y = \frac{1}{2}\{\phi(x) + f(x)\}$ . The differences between the ordinates of Karup's curve and the mean curve, and between those of (3) and the same mean curve are respectively

$$x(x-1)(2x-1)(5x^2-5x-2) \frac{\Delta^5 u_{-2}}{48} \text{ and } x(x-1)(2-4x) \frac{\Delta^5 u_{-2}}{48}.$$

It will therefore be seen that, if  $\Delta^5 u_{-2}$  is positive, both (3) and the Karup curve lie above or below the mean curve according as  $x$  is  $>$  or  $< \frac{1}{2}$ , and also that the Karup curve lies above

or below (3) according as  $x$  is  $>$  or  $< \frac{1}{2}$ . If  $\Delta^5 u_{-2}$  is negative

the conditions are reversed. Karup's curve, therefore, diverges more sharply than (3) from the mean curve. In the examples which are appended it will be seen that the interpolations by the use of (3) are at least quite as smooth as those resulting from the use of Karup's formula. In section 8 it was found that the deviation of  $u_{-\frac{2}{3}} + \dots + u_{\frac{2}{3}}$  from  $\omega_0$  as obtained from (6) and (3) was  $\frac{867}{457} \Delta^6 \omega_{-3}$ . If Karup's formula be used instead of (3)

the deviation is  $\frac{173}{57} \Delta^6 \omega_{-3}$ , while the deviation is  $u_0 + \dots + u_{\frac{4}{3}}$  is  $\frac{290}{57} \Delta^6 \omega_{-3}$ , which is the same as that given by (3).

11. In the application of such formulæ as Karup's or (3) of this Paper to the construction of tables the leading differences for each interval may be formed, as was done by Mr. King in his Papers, but the formulæ are capable of such modification that the work may be reduced very considerably.

The formula for the interval 0 to 1 is of the form  $y = \phi(x) + \theta(x)$ , where  $\phi(x)$  is the same in both case and  $\theta(x)$  is equal to  $\frac{x^3(x-1)(5x-7)}{4} \Delta^5 u_{-2}$  in Karup's formula and to

$\frac{x^2(x-1)(x-5)}{2 \underline{4}} \Delta^5 u_{-2}$  in (3). It was shown in section 8 (9)

that the formula for the interval -1 to 0 may be expressed, with reference to the same origin, in the form  $y = \phi(x) + \theta_1(x)$  when  $\theta_1(x)$  is equal to  $-\frac{x^2(x+1)(x+5)}{2 \underline{4}} \Delta^5 u_{-3}$  in the case of (3)

and may easily be shown to be equal to  $\frac{x^3(x+1)(5x+7)}{4} \Delta^5 u_{-3}$  in the case of Karup's formula. Separating  $\phi(x)$  into even and odd terms, we have

$$\begin{aligned}\phi(x) &= \left[ u_0 + \frac{x^2}{2} \Delta^2 u_{-1} + \frac{x^2(x^2-1)}{4} \Delta^4 u_{-2} \right] \\ &\quad + \left[ x \Delta u_0 - \frac{1}{2} x \Delta^2 u_{-1} + \frac{x(x^2-1)}{3} \Delta^3 u_{-1} - \frac{x(x^2-1)}{2 \underline{3}} \Delta^4 u_{-2} \right] \\ &= \left[ u_0 + \frac{x^2}{2} \Delta^2 u_{-1} + \frac{x^2(x^2-1)}{4} \Delta^4 u_{-2} \right] \\ &\quad + \left[ \frac{x}{2} (\Delta u_0 + \Delta u_{-1}) + \frac{x(x^2-1)}{12} (\Delta^3 u_{-1} + \Delta^3 u_{-2}) \right]\end{aligned}$$

Hence,

$$\phi\left(\pm \frac{1}{5}\right) = [u_0 + \cdot02\Delta^2 u_{-1} - \cdot0016\Delta^4 u_{-2}] \\ \pm [\cdot1(\Delta u_0 + \Delta u_{-1}) - \cdot016(\Delta^3 u_{-1} + \Delta^3 u_{-2})]$$

and

$$\phi\left(\pm \frac{2}{5}\right) = [u_0 + \cdot08\Delta^2 u_{-1} - \cdot0056\Delta^4 u_{-2}] \\ \pm [\cdot2(\Delta u_0 + \Delta u_{-1}) - \cdot028(\Delta^3 u_{-1} + \Delta^5 u_{-2})].$$

Thus the expressions in the brackets are each used twice over in calculating  $u_{-\frac{1}{5}}$ ,  $u_{-\frac{1}{2}}$ ,  $u_{\frac{1}{5}}$  and  $u_{\frac{2}{5}}$ . Further, the values of  $\theta(x)$  may each be used twice over. For when  $x = -\frac{1}{5}$ ,  $-\frac{x^2(x+1)(x+5)}{2|4} \Delta^5 u_{-3}$  is equal to  $-\cdot0032\Delta^5 u_{-3}$ , and this term occurs in  $u_{-\frac{1}{5}}$ , while  $+\cdot0032\Delta^5 u_{-3}$  occurs in  $u_{-\frac{1}{2}}$ . The corresponding terms in  $u_{-\frac{1}{5}}$  and  $u_{-\frac{1}{2}}$  are  $\mp \cdot0092\Delta^5 u_{-3}$ . Hence the interpolation may be performed according to the following scheme :

$$u_{-\frac{1}{5}} = C_0 - D_0 - \cdot0092\Delta^5 u_{-3}$$

$$u_{-\frac{1}{2}} = A_0 - B_0 - \cdot0032\Delta^5 u_{-3}$$

$$u_0$$

$$u_{\frac{1}{5}} = A_0 + B_0 + \cdot0032\Delta^5 u_{-2}$$

$$u_{\frac{1}{2}} = C_0 + D_0 + \cdot0092\Delta^5 u_{-2}$$

$$u_{\frac{2}{5}} = C_1 - D_1 - \cdot0092\Delta^5 u_{-2}$$

$$u_{\frac{3}{5}} = A_1 - B_1 - \cdot0032\Delta^5 u_{-2}, \text{ and so on.}$$

It will be seen that the amount of calculation involved is the same as in the scheme given by Dr. Buchanan (*J.I.A.*, xlii, 381). The latter scheme would, of course, be followed if formula (4) were used instead of (3).

12. An example of the application of this method to a portion of the Government Female Annuitants (1883) Ultimate Table is appended. The graduated values of  $E_x$  and  $\theta_x$  for ages 32, 37, 42, &c., are first obtained by means of formula (6), and the values for the intervening ages are formed from (3). Here  $u_{-2}$ ,  $u_{-1}$ ,  $u_0$ , &c., correspond to ages 32, 37, 42, &c. The final values of  $q_x$  are then found from  $q_x = \theta_x/E_x$ .

The graduated quinquennial values as given by (6) are as follows :

Age	$E_x$	$\theta_x$
32	219.846	1.916
37	418.887	3.986
42	838.403	11.077
47	1580.093	17.464
52	2659.164	41.560
57	4479.498	86.886
62	6433.780	157.025
67	8070.002	304.841
72	8146.109	456.949

The differences of each are now put down centrally, and in the case of the odd differences the sum of each pair of consecutive terms is put down between them in order to facilitate the calculation of the odd terms in  $\phi(x)$ . In the case of  $\theta_x$  we have the following scheme of values :

Age		$u$	$\Delta(u_0 + u_{-1})$	$\Delta^2 u_{-1}$	$\Delta^3(u_{-1} + u_{-2})$	$\Delta^4 u_{-2}$	$\Delta^5 u_{-2}$
42	$u_0$	11.077	13.478	- .704	12.688	24.138	- 39.030
47	$u_1$	17.464	30.483	17.709	21.934	- 14.892	14.954
52	$u_2$	41.560	69.422	21.230	7.104	.062	49.219
57	$u_3$	86.886	115.465	24.813	56.447	49.281	- 175.530
-	-	-	-	-	-	-	-

The detailed calculation for  $\theta_x$  by (3) is as follows :

Age	(1) Even Terms in $\phi(x)$	(2) Odd Terms in $\phi(x)$	(3) Sum of (1) and (2) = $\phi(x)$	(4) Fifth difference Correction	(5) Sum of (3) and (4) = $\theta_x$	Age
40	10.88550	- 2.34034	8.54516	...	...	40
41	11.02429	- 1.14479	9.87950	...	...	41
42	11.077	...	...	...	11.077	42
43	11.02429	+ 1.14479	12.16908	- .12490	12.044	43
44	10.88550	+ 2.34034	13.22584	- .35908	12.867	44
45	18.96410	- 5.48245	13.48165	+ .35908	13.841	45
46	17.84200	- 2.69736	15.14464	+ .12490	15.270	46
47	17.464	...	...	...	17.464	47
48	17.84200	+ 2.69736	20.53936	+ .04785	20.587	48
49	18.96410	+ 5.48245	24.44655	+ .13758	24.584	49
50	43.25805	- 13.68549	29.57256	- .13758	29.435	50
51	41.98450	- 6.82854	35.15596	- .04785	35.108	51
52	41.560	...	...	...	41.560	52
53	41.98450	+ 6.82854	48.81304	+ .15750	48.971	53
54	43.25805	+ 13.68549	56.94354	+ .45281	57.396	54
55	88.59507	- 21.51248	67.08259	- .45281	66.630	55
56	87.30341	- 10.64335	76.66006	- .15750	76.503	56
57	86.886	...	...	...	86.886	57
58	87.30341	+ 10.64335	97.94676	- .56170	97.385	58
59	88.59507	+ 21.51248	110.10755	- 1.61488	108.493	59

Performing the calculations for  $E_x$  in the same way we have finally :

Age	$E_x$	$\theta_x$	$q_x$	$\Delta^3 q_x \times 10^5$	Expected Deaths	Deviation
42	838.403	11.077	.01321	33	11	- 1
43	960.756	12.044	.01254	18	11.8	2.8
44	1098.001	12.867	.01172	11	12.9	.9
45	1248.118	13.841	.01109	- 3	13.9	- 3.1
46	1409.328	15.270	.01083	- 20	15.4	- 5.6
47	1580.093	17.464	.01105	- 13	17.4	4.4
48	1757.110	20.587	.01172	- 13	20.6	6.6
49	1944.445	24.584	.01264	- 10	24.8	- 1.2
50	2151.430	29.435	.01368	3	29.5	7.5
51	2386.892	35.108	.01471	6	35.1	- 1.9
52	2659.164	41.560	.01563	- 26	41.7	- 1.3
53	2972.535	48.971	.01647	59	48	- 3
54	3320.437	57.396	.01729	- 44	57.5	- 1.5
55	3736.891	66.630	.01783	- 5	65.7	8.7
56	4094.501	76.503	.01868	14	76.7	- 6.3
57	4479.498	86.886	.01940	...	86.8	.8
58	4883.998	97.385	.01994	...	97.4	7.4
59	5308.002	108.493	.02044	...	107.9	- 15.1

Taking the age groups, 45-49, 50-54, 55-59, the deviations in  $\Sigma E_x$  and  $\Sigma \theta_x$  are  $-25.9$ ,  $35.5$ ,  $78$ , and  $.8$ ,  $.5$  and  $-3.1$  respectively. These are under  $.4$  per-cent in the case of  $E_x$  and under  $.9$  per-cent in the case of  $\theta_x$ . When  $q_x$  is found in this way from graduated values of  $E_x$  and  $\theta_x$  it is hardly to be expected that the result will be as smooth as if the values of  $q_x$  had been found for immediate values by the application of an interpolation formula to quinquennial values of  $q_x$ . The results, however, do not compare unfavourably with the latter method, as will be evident from a comparison of this table with the one given by Mr. King, *J.I.A.* xlivi, 140.

The total deviations for the age groups 42-46, 47-51, 52-56 are  $-6$ ,  $15.4$  and  $-3.4$  as against  $-3.6$ ,  $12.6$  and  $-2.2$  by Mr. King's construction.

13. In order to test the smoothing effect of formula (3) as compared with Karup's formula or Dr. Buchanan's modification of it when applied to interpolation between given quinquennial values of a function, I have applied it to the construction of  $q_x$  in the English Life Table, No. 6, Males. The values of  $q_x$  at ages 20, 25, &c., are those given by Mr. King and employed by Dr. Buchanan in forming his Table (*J.I.A.* xlvi, 398). Up to age 49 the values are the same. From that point the values are as follows :

Age	$qx$	$10^5 \Delta^3 qx$	Age	$qx$	$10^5 \Delta^3 qx$
50	.01935	4	65	.04949	-19
51	.02040	3	66	.05347	-16
52	.02151	5	67	.05797	-18
53	.02272	4	68	.06280	-5
54	.02406	-3	69	.06780	20
55	.02558	-2	70	.07279	26
56	.02732	-7	71	.07772	25
57	.02925	-2	72	.08279	30
58	.03135	-6	73	.08826	16
59	.03355	8	74	.09438	1
60	.03583	11	75	.10145	13
61	.03813	13	76	.10963	4
62	.04053	11	77	.11893	...
63	.04314	13	78	.12948	...
64	.04609	-6	79	.14132	...

It will be seen that the table is at least as smooth as that of Dr. Buchanan, and considerably smoother than those given by Mr. King (*J.I.A.*, xlvi, 275, and xlvi, 151).

As a final example, formula (3) is applied to the calculation of intermediate values of  $q_{[x]+3}$  between the values given for ages at entry 30, 35, 40. The results are compared with those given by Dr. Buchanan, working with his formula (III) (*J.I.A.*, xlvi, 376) and with Everett's formula (*J.I.A.*, xxxv, 454).

$x$	VALUES OF $10^6 q_{[x]+3}$ AND THEIR FOURTH DIFFERENCES CALCULATED BY					
	Formula (3) or (4)		Everett's Formula		Buchanan's Formula	
30	9,225	-5	9,225	8	9,225	-17
31	9,336	7	9,338	-5	9,329	20
32	9,512	6	9,519	-45	9,507	27
33	9,729	34	9,721	120	9,733	5
34	9,958	6	9,945	-49	9,964	24
35	10,177	3	10,177	3	10,177	19
36	10,370	-1	10,358	-1	10,376	-15
37	10,555	...	10,549	...	10,560	...
38	10,756	...	10,762	...	10,752	...
39	11,000	...	11,012	...	10,994	...
40	11,313	...	11,313	...	11,313	...

Formula (3) gives a slightly better interpolation than Dr. Buchanan's formula. This appears to be due to the fact that the fifth difference term involves  $x$  to the fourth degree

only instead of the fifth degree as in Dr. Buchanan's and Dr. Karup's formulæ, and that the interpolation curve does not depart so sharply from the mean curve as that given by the other formulæ.

### LEGAL NOTES.

*By ARTHUR RHYS BARRAND, F.I.A., Barrister-at-Law.*

What constitutes a policy of life assurance. TWO cases have recently been decided dealing with the question as to whether certain forms of endowment policies can properly be described as policies of assurance on human life. The first of these was decided by the Court of Appeal in Ireland, under the name of *Flood v. Irish Provident Assurance Company Limited*, [1912] 46 I.L.T. 214, [1912] 2 Ch. 597. This was an action brought by certain policyholders against the company for a declaration that the company was carrying on the business of a life assurance company, and to restrain it from doing so, the objects of the company, as set out in its memorandum of association, expressly excluding "the issue of policies of assurance upon or in any way relating to human life."

The company had issued to the plaintiffs endowment policies for fixed sums, payable at fixed dates at fixed weekly premiums, which provided that if the assured should die before the expiration of the endowment period, the company would return to the person legally entitled to the benefit of the policy, a percentage of the amount of all premiums which should have become due up to the date of the death of the assured, and have been duly paid.

The case came before the Irish Court of Appeal by way of appeal by the plaintiffs against the judgment of the Master of the Rolls in Ireland (Meredith, M.R.). In the course of his judgment, Walker, L.C., said: "It will be observed that all "that a policyholder can ever get under the policies with which "we have to deal is a payment of some proportion, according "to the circumstances, of the premiums actually paid by himself, "so that this company trades without any risk. For the purpose "of coming to a conclusion on the second question (*i.e.*, whether, "if the policies were void as being *ultra vires*, the plaintiffs "were entitled to the return of the premiums paid) I think

" we are bound to decide whether the policies are policies within  
" the terms of the prohibition in the memorandum . . . .  
" namely, whether they are 'upon or in any way relating to  
" human life.' I have already stated the terms of the policy  
" in each of the six cases, and the reading of them seems to me  
" irresistibly to lead to the conclusion that they relate to human  
" life, and if they do, they are plainly void policies."

Holmes, L.J., in delivering judgment to the same effect, said : " The policies I have seen, while differing in some respects, have certain common features. A weekly or yearly premium is payable in respect of each of them to secure payment of a specified sum at the end of a certain number of years to the person insured, provided he is then living, and if he die while the years are unexpired, it is provided that there shall be returned to his representatives a portion of the premiums theretofore paid. The Solicitor-General has argued that these are not policies of assurance on human life in the legal sense of those words, that they are what he calls endowment policies, and that the company in issuing them and in receiving the premiums has been acting as a kind of savings bank. It is said that some risk on the part of the insurer is a necessary element in life assurance, and if this be so, and if the policies I have seen represent the usual transactions of the company, I could not imagine a safer business. No one of ordinary prudence or intelligence would enter into such contracts of assurance. The insured would be in a better position if he lodged the amount paid as premiums in the Post Office Savings Bank, from which he, or his personal representatives, could withdraw the full amount lodged with interest at any time. The persons insured by the policies which have been before the Court may lose, but cannot gain ; and the large business done by the defendant company shows how the thrifty poor may be misled by the glowing language of a clever canvasser. Still, notwithstanding the absence of all risk to the defendant company, I am disposed to regard its policies as assurances upon human life. I find an excellent description of such policies in the defendant company's memorandum of association, 'policies of assurance upon or in any way relating to human life', and if it were necessary for me to define policies of assurance on human life within the meaning of the Life Assurance Companies Act, 1870, I would give as the definition the words I have quoted. . . . The defendant company's memorandum of association in

"stating the objects for which the company is established sets forth several classes of assurance, but excepts 'the issue of ' policies of assurance upon or in any way relating to human ' life', and as I have no doubt that the policies issued to the plaintiffs are of this description, I hold . . . that the issue thereof was *ultra vires*, that the policies are void and of no effect."

**Endowment  
policies as  
policies of life  
assurance.**

The second of the cases referred to is that of *Joseph v. Law Integrity Insurance Company, Limited*, [1912] 2 Ch. 581, and is very similar to the preceding case.

The defendant company was empowered by its memorandum of association to carry on practically all forms of insurance "except the business of life assurance", and it was expressly provided that "nothing herein contained shall empower the company to carry on the business of life assurance, or the granting of annuities within the meaning of the Life Assurance Company's Act, 1870." The company issued, *inter alia*, two classes of policies, under one of which, in consideration of a fixed weekly premium, a certain sum was payable at the end of five years, a certain other sum at the end of ten years, and another sum at the end of fifteen years; and it was further provided, that, in the event of death before the fifth year, all premiums paid should be returned in full. If death occurred after the fifth year and up to the tenth year, all premiums paid since the last payment by the company were to be returned, and similarly for the next period. The second class of policy differed from the first in making the whole sum payable at the end of a fixed period, and providing for the return of a percentage only of the premiums paid in the event of death occurring during that period.

The action was brought by the plaintiff, on behalf of himself and all other shareholders in the company, for a declaration that policies of the description referred to were policies of assurance on human life, and were illegal and *ultra vires* and void, and for an injunction restraining the defendant company from issuing any further policies of assurance on human life, and from carrying on the business of life assurance.

The case came, in the first instance, before the Deputy of the Vice-Chancellor of the County Palatine of Lancaster, who held that the defendant company did not issue policies of assurance upon human life or carry on the business of life assurance within the meaning either of its memorandum of association or of the Assurance Companies Act, 1909, and that

the plaintiff was not entitled either to the declaration or to the injunction he claimed. He accordingly made a declaration negativing the claim, and stating that the policies were not policies of assurance on human life, and were not illegal or *ultra vires*.

From this decision the plaintiff appealed, and on the case coming before the Court of Appeal, the previous decision was reversed and judgment given in favour of the plaintiff. Cozens-Hardy, M.R., said : " We must proceed to consider the question " on the footing that these sums are payable only if the pro- " poser is alive at the critical dates. There is a further obligation " on the part of the company, if death occurs in the interval " between these periods, to return the premiums. The Policy " Form B (the second of those referred to above) is possibly " still plainer. It is quite clear that the obligations on the " part of the company under that policy are alternative obliga- " tions, either to pay the sum named if the proposer is alive, " or to return the amount of the premiums, or a certain pro- " portion of them, if he is dead. I, therefore, think that these " policies are life policies within the meaning of the passage in " Bunyon on Life Assurance which I have read ('The contract " of life assurance may be further defined to be that in which " one party agrees to pay a given sum upon the happening of " a particular event contingent upon the duration of human " life, in consideration of the immediate payment of a smaller " sum or certain equivalent periodical payments by another.' " 4th edition, p. 1.). Their effect is to constitute an agreement " by the defendant company to pay a given sum on the happening " of the particular event contingent on the duration of the life. " That being so, apart altogether from the Act of 1909, it seems " to me reasonably plain that the defendant company is carrying " on the business of life assurance, and that is contrary to and " prohibited by its memorandum of association. I desire on " this point to adopt the judgment of Channell, J., in *Prudential " Assurance Company v. Inland Revenue Commissioners* ([1904] " 2 K.B. 658, *J.I.A.* vol. xli, p. 114), which appears to me to be " extremely clear and forcible, and, if right, is conclusive on the " matter. I think it is perfectly right."

Kennedy, L.J., said : " The question is, looking at the " memorandum of association first of all, is the defendant " company a company which does carry on life assurance " business ? It appears to me that it clearly is in regard to " the two forms of policies which are set out in the case before us.

" It is strenuously argued, as it must be, by the respondents here, that these policies are not assurance policies upon human life. I do not myself think that there could be a better illustration of what in my opinion is the incorrectness of that view, than the fact that in each case a certificate is given that the policy is an investment 'upon the life of'; and if you have these words, and if anything turns upon the exact preposition, it is the preposition 'upon' which occurs in the statute (the Act of 1870) and is also to be found in the document, the validity of which is here impeached both on the ground that it contravenes the memorandum and also on the ground that it contravenes the statute. I think it is quite clear . . . that each of these documents constitutes a policy of assurance upon human life. You may call it if you like an investment, or you may call it a policy, or you may call it a contract. It is a contract which, in its essence, is a contract within the words 'policy of assurance upon human life', and I do not myself wish to add anything to the statement of the essence of a life policy which is contained in the judgment of Channell, J., in *Prudential Assurance Company v. Inland Revenue Commissioners*. I think, judged by any test of what is an assurance upon human life or a policy upon human life, both of these contracts are such . . ."

These cases are of considerable importance at the present time, in view of the efforts being made by the Inland Revenue authorities to exclude from the benefit of the rebate of income tax given by section 54 of the Income Tax Act, 1853, that part of the premium in double endowment assurances which represents the pure endowment portion.

Premiums paid  
by one joint  
tenant at request  
of other.  
Position of  
parties.

The case of *In re McKerrell, McKerrell v. Gowans*, [1912] 2 Ch. 648, deals with the rights of a person assured under a joint life policy who, at the request of the other person assured, pays the whole of the premium. The material facts of the case are as follows: In 1892, the plaintiff and her husband agreed to take out a policy upon their joint lives, and each of them was to pay one-half of the annual premium, and, accordingly, in that year, they effected with the Pelican Life Insurance Company a joint life policy of assurance for £1,000. Pursuant to their agreement, the plaintiff and her husband for eight of the nineteen years between 1892 and 1910 paid one moiety each of the annual premium on

the policy, but in nine other of those years the husband's financial position was such that he was unable to pay his moiety, and on each of those occasions he requested the plaintiff to pay the whole premium, which she did. In 1903 the husband paid the whole premium, and in 1911 the plaintiff and her husband had to borrow the amount of the premium in order to pay it.

By a trust deed in Scottish form, dated 21 October 1910, the plaintiff's husband assigned to the defendant, as trustee for the husband's creditors, all his real and personal property. The policy was not mentioned in the trust deed, and no notice of the deed of assignment was given to the insurance company. The plaintiff's husband died on 26 February 1912, and the amount of the policy moneys which then became payable was £1,327. The plaintiff and her husband had during their lives created certain charges upon the policy moneys, amounting to £932, together with interest, the creation of which charges the defendant did not dispute, but he claimed that, subject to such charges, the whole, or alternatively, one half share of the policy moneys was in the circumstances payable to him as trustee. The plaintiff thereupon took out an originating summons for the determination of the question as to whether the policy moneys, subject to the admitted charges, belonged wholly to her, or to her and the defendant in equal shares, and in the latter event, whether the plaintiff was entitled to a charge on the policy moneys for the premiums paid by her on behalf of her late husband. It appeared that one moiety of the balance of the policy moneys left after payment of the charges on the entirety would not amount to the aggregate sum paid by the plaintiff at her husband's request in respect of his share of the premiums.

Joyce, J., in delivering judgment in favour of the plaintiff, said : "I may be wrong, but I do not feel any doubt about this case. The policy of assurance was taken out by a husband and wife, and the result of the discussion before me has been, and it was practically admitted that it could not be denied . . . that the right to sue upon the covenant in the policy on the death of the husband, who died first, survived to the wife. Hers, therefore, was the legal hand to receive the policy moneys, because whatever else the assignment by the husband may have effected, it did not affect the right of the wife as survivor to sue at law upon the policy. . . . The claim of the trustee under the assignment is in equity only. If the assignee of the husband comes as he does and asks for a share

" of the proceeds of the policy, in the first place he must do  
 " equity. That I think is quite clear. In the next place I do  
 " not see any reason why the wife is not entitled to set off this  
 " claim she has against her husband for the portion of the  
 " premiums paid on the policy by her at his request. I go  
 " further than that, for in my opinion there is enough here to  
 " give her a lien on the husband's portion (if any) of the policy  
 " moneys. I am of opinion and decide that in the circum-  
 " stances of this case she has a lien on the policy moneys, or  
 " rather on the share (if any) of the husband's trustee, to whom  
 " he is said to have assigned his interest in this policy. She  
 " has a lien upon that for the over-payment she has made at  
 " her husband's request in respect of the premiums paid by her,  
 " and she is entitled to set this off before the husband's assignee  
 " gets one penny out of the policy moneys.

" Further, I am not at all satisfied that the assignment  
 " made by the husband transfers or conveys any interest what-  
 " ever, legal or equitable, in this policy or the moneys secured  
 " thereby. Certainly it conveys nothing at law. That is quite  
 " clear. There is no assignment under the Judicature Act,  
 " and before the Act it is clear that there could not have been  
 " a tenancy in common of a legal chose in action, although I  
 " need not decide upon that ground. As at present advised I  
 " do not think the assignment passed anything at all in the  
 " circumstances, although, I agree, it might have passed the  
 " right to all the policy moneys if the husband had survived  
 " the wife, but I am disposed to say that in the result it passed  
 " nothing at all to the trustee. Whether it passed anything or  
 " not, however, it certainly did not pass more than a moiety,  
 " and this lady is entitled to a lien or set off—it does not matter  
 " what you call it. She is entitled to deduct the over-pay-  
 " ments she made at the request of the husband from the share,  
 " if any, coming to the person claiming as his assignee."

*Jurisdiction of  
Court to  
wind up an  
unregistered  
friendly society.*

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The legal position in regard to unregistered friendly societies has not, hitherto, attracted much attention by reason, no doubt, of the fact that as a rule no very large interests have been involved in such societies.

In view, however, of the power given by section 23 of the National Insurance Act, 1911, for unregistered friendly societies to become approved societies, and of the fact that one of the largest approved societies has, it is understood, a constitution of this nature, the recent case of *In re Victoria Society, Knot-*

tingley, [1913] 1 Ch. 167, which deals with the jurisdiction of the Court to wind up such a society under the Companies Acts, may be of interest.

The Victoria Society was an unregistered friendly society, formed in 1832 at Knottingley, in Yorkshire, for the purpose of assisting members in sickness or calamity. The rules provided for certain entrance fees and weekly contributions, and for sickness, disablement, medical and death benefits.

At the annual meeting of the society, held on 21 August 1912, the position of the society, as affected by the National Insurance Act, 1911, was considered, and a resolution was passed by twenty votes to eight that the society should be wound up, and a further resolution was passed that all subscriptions and sick or other payments should cease after 27 August 1912. The number of members of the society at this date was fifty-six, and the funds, which were vested in five trustees, consisted of £800 Consols and £770 in the bank on deposit and current accounts.

An extraordinary meeting of the society was summoned by notice for 29 October 1912, and at this meeting the resolution for winding-up was confirmed by twenty votes to one; but as the members could not agree upon a scheme for the distribution of the assets, a further resolution was passed that the trustees, as the winding-up committee, should be empowered to apply to the Court for power to distribute the assets on the basis decided by the Court. The rules made no provision for the dissolution of the society or the distribution of its assets.

On 9 November 1912, an old member of the society named Pickett, who was permanently disabled, and in receipt of a pension in accordance with the rules, brought an action in the Chancery Division, and applied for an injunction to restrain the officials and trustees of the society from distributing the funds. On 14 November the five trustees of the society presented this petition for a winding-up order.

The case came before Neville, J., who made an order for the winding-up of the society. In doing so, he said: "In my opinion this Court has jurisdiction to make a winding-up order in the case of an unregistered as well as in that of a registered friendly society. There is ample authority for making the order in the case of a registered friendly society, and I cannot see that the absence of registration under the Friendly Societies Acts makes any difference. An un-

" registered friendly society is clearly within the words of " section 267 of the Companies (Consolidation) Act, 1908. I " think that I ought to make the order on this petition, not- " notwithstanding the existence of the action, in order to give " effect to the express wishes of the great majority of the members " of the society."

I am indebted to Mr. John A. Rankin, F.F.A., of the Edinburgh Life Assurance Company, for particulars of a case relating to fraudulent misstatement

Fraudulent misstatement of age on proposal.  
of age in a proposal for life assurance which may be of interest to the readers of these Notes. This case, together with several others relating to errors in age, will be found in a paper read by Mr. Rankin before the Faculty of Actuaries on 4 November last.

The case in question is that of the *Edinburgh Life Assurance Company v. Byrne*, and it came before the Chancery Division in Ireland in March and April 1911. It appears from the statement of claim that the defendant approached the local agent of the plaintiff company, purporting to act on behalf of the proposer who was represented as being a well-to-do farmer, but who was, in reality, an illiterate farm labourer without any means whatever of paying the premium of over £29 for the proposed assurance. Shortly after the policy was effected, it was assigned to the defendant for an alleged consideration of £55. The assured died intestate, leaving practically no assets, and no grant of representation to his estate was taken out.

Prior to the death of the life assured, the age had been admitted on the strength of what purported to be an extract from an entry in a " Family Expositor ", a paraphrase version of the New Testament, and the age was understated in the death certificate to correspond with the policy. In these circumstances the assurance company was not aware of the misstatement of the age, and the claim was paid accordingly to the assignee. Subsequently, however, certain facts were brought to light which proved that the age in the proposal had been understated by ten years, and the company had their suspicions aroused as to the *bond fides* of the transaction. Accordingly they raised the present action, claiming, *inter alia*, the setting aside of the policy on the ground that it was procured by the fraud of the defendant or the assured or both ; and demanding the repayment of the policy moneys with interest and costs.

At the first hearing of the case, it was stated by the counsel

for the plaintiff company that evidence would be given proving that the proposal in question was the result of a systematic course of fraud on the part of the defendant, who, during several years, had effected an enormous number of policies under similar conditions. The defendant's counsel objected to any other matters being gone into in the opening statement except in regard to the one policy which had been challenged, but the company's counsel gave an undertaking that he would state nothing that he could not prove by evidence, and Barton, J., before whom the case came, held that in a case of this nature, the evidence in regard to cases of a like character was clearly admissible, and that, assuming the company alleged, in their statement of claim, a systematic series of frauds, they would then be entitled to give evidence of the system in order to negative the possibility of the misstatements having been made by mere innocent mistake. Originally the statement of claim did not contain a specific allegation of a systematic series of frauds, but before the case was adjourned, the company obtained an order giving them liberty to amend their pleadings to meet this point.

The case was again called but never came to trial, owing to the defendant consenting to judgment on the statement of claim, and judgment was accordingly entered for the company, ordering the defendant to pay back the policy moneys with interest at four per-cent in addition to the costs of the action.

Assured dying  
domiciled outside  
the United  
Kingdom.  
Revenue Act, 1889.  
Estate Duty.

The case of *Haas v. Atlas Assurance Company Limited*, [1913] 29 T.L.R. 307, turns on the effect of section 19 of the Revenue Act, 1889, which provides that

“where a policy of life assurance has been effected “with any insurance company by a person who shall die domi- “ciled elsewhere than in the United Kingdom, the production “of a grant of representation from a court in the United Kingdom “shall not be necessary to establish the right to receive the “money payable in respect of such policy.” The material facts of the case are as follows: Two policies of life assurance were issued by the defendant company on the life of one Wilhelm Haerle, who died on 13 August 1912, domiciled in Switzerland. He left a will duly executed according to Swiss law, and made no disposition of the policies otherwise than by his will. The plaintiff was appointed executor of this will, and was, by Swiss law, the person who had the duty of collecting the policy moneys and the power of giving a good discharge for same. He applied,

accordingly, to the defendants for payment of the policy moneys, but the latter contended that they were not bound to pay until the plaintiff obtained a grant of representation from an English court. They further said that by the terms of the policies the money was payable to the assured's executors, administrators or assigns, which meant English executors, administrators or assigns, and that the plaintiff did not fill any of these capacities.

The case came before Scrutton, J., on 20 February 1913, when he decided in favour of the plaintiff. In the course of his judgment to this effect, he said that : "Until 1884 it was the law of England that an executor could not assert his right until he had proved the will. . . . The practical consequence of that state of the law, which applied equally to foreigners and to Englishmen, was this, that if an insurance company insured the life of a foreigner, his representatives had, on his death, in order to recover the policy money, to go through the probate forms of their own country as well as through those of this country. That, not unnaturally, was apt to dissuade a foreigner from insuring with an English company. Accordingly, it appeared that a number of insurance companies obtained private Acts of Parliament under which they bound themselves to pay without production of an English grant of probate."

His Lordship then discussed the circumstances in which the provisions of the Revenue Act, 1884, relating to the matter were enacted, and also the subsequent amendment of those provisions by the Revenue Act, 1889. He then went on to say, "The plaintiff contended that he had established a right to receive the policy moneys, and that the old law which prevented him from establishing such a right unless on the production of an English grant of representation had been expressly abrogated by section 19. . . . For the defendants it was said that the word 'recover' which was used in section 11 of the Act of 1884 had been omitted from section 19 of the Act of 1889, and that in the latter section the word used was merely 'receive.' It was suggested that that meant that although the plaintiff might have established a right to receive the policy moneys, he could not recover them except upon production of an English grant of representation. His Lordship did not think that Parliament was so subtle as that contention suggested. He could see no difference in this connection between the words 'receive' and 'recover.' That being so, his Lordship thought section 19 of the Act of 1889 entitled the plaintiff to

"succeed, as he had proved that he was the testamentary executor by Swiss law of the testator Haerle, and had proved that by Swiss law he had the right to recover sums due to the estate, which included the sums due under the policies sued on. Further, in his Lordship's view, the plaintiff came within the words 'executors and administrators' in the policies. . . .

"In the absence of the Crown he did not think he ought to decide whether or not English estate duty was payable out of the policy moneys, although he rather thought it was. If estate duty was payable, section 9 of the Finance Act, 1894, gave a charge to the extent of the duty on the moneys, and he thought that if the defendants retained out of the moneys an amount equal to the duty, they would not be going beyond their legal rights."

It was arranged eventually that the defendants should retain £40 out of the policy moneys until it was seen whether the Crown made any claim to estate duty.

#### REVIEW.

*Friendly Society Finance considered in its actuarial aspect. A Course of Lectures delivered at the Institute of Actuaries.* By ALFRED WILLIAM WATSON, F.I.A.

(C. & E. Layton, London.)

STUDENTS of Actuarial Science will welcome the publication in book form of the six lectures which were delivered by Mr. Watson during the session 1911-12, and which were arranged by the Council in view of the prominent part Friendly Societies were destined to take in the administration of Part I. of the National Insurance Act.

At one time it seemed as if the introduction of State Sickness Insurance would retard the Friendly Society movement—if it did not finally lead to its extinction—because the State scheme incorporates the greater portion of each sickness insurance contract up to age 70, and in his first lecture, Mr. Watson states (p. 12) that there has been a steady drift towards neglect of insurance amongst the young men of the industrial population during the past thirty years. In an introductory note, however, he expresses the opinion that recent developments indicate that the scope of the voluntary work of Friendly Societies may be maintained and even extended in the future—thus affording a wider field for actuarial activity. He does not state what these recent developments are, and he may possibly be influenced in his opinion by the facts that (a) many Friendly Societies are permitting those who join their State section to insure death benefits or additional sickness allowances from their voluntary section, and (b) a very large number of

the existing Friendly Society members are regarding the State scheme as an additional contract—comparatively few members desiring amended contracts. This latter feature may be largely due to the general prosperity of the country as indicated by Trade and Unemployment Statistics, and one cannot help thinking that the introduction of the State scheme was well timed, not only on this account, but also in view of the necessity of modifying the old type of society to cope with the altered conditions in recent years. The Workmen's Compensation Acts and Old Age Pension Acts have materially affected the industrial classes, and many Societies are now experiencing an increasing number of claims, and at the same time a diminishing income from voluntary and honorary subscriptions.

Mr. Watson's lectures assume a knowledge of the theory of Sickness Insurance as set forth in the Text Book (Part II), and in Mr. G. F. Hardy's Messenger Prize Essay, *J. I. A.*, xxvii, p. 245. The lectures should be read in conjunction with the Author's papers in Journals XXXV and XLIV, as well as with Mr. Hardy's Essay, and the writings of the late Mr. R. P. Hardy. The first five lectures are a synopsis of practical points which may arise in investigating a sickness experience, valuing a Friendly Society, or advising as to rates of contribution, while the sixth is devoted to the consideration of Sections 72 and 73 of the National Insurance Act. They are worthy of careful study quite apart from their direct bearing on Friendly Societies.

The first lecture contains a clear exposition of the different types of Friendly Society and of the actuarial problems which arise in connection with the large Affiliated Orders. The account of the various relations between District Funds and Lodge Funds of such Societies should prove of great assistance in connection with any affiliation scheme which may be contemplated in the future. In a note (on p. 7) Mr. Watson points out that Friendly Societies measure their solvency by comparing the value of the contributions plus the funds with the value of the benefits; it is to be regretted he did not add a note of warning against relying too much on the "Degree of Solvency" so ascertained. The method does not differentiate between the relative proportions of the assets as constituted by the *actual* realizable assets (the funds in hand) and the *estimated* assets (the value of the contributions). This point is of extreme importance in considering a scheme of reorganization designed not to impair a Society's "Degree of Solvency." For instance, a Society's current expenditure may exceed the current contributions and encroach upon the accumulated fund. If benefits and contributions are reduced under a reorganization scheme, and the reduction in contributions is fixed solely from the point of view that the existing "Degree of Solvency" must be maintained, without considering the effect on the yearly Revenue Account, it might happen that the accumulated fund would be depleted—thus absorbing all realizable assets—at an earlier date than under the former conditions.

The second lecture deals principally with the relation between

varying rates of sickness and mortality, and questions which arise in connection with a Sickness Experience. It indicates the responsibility that is involved in examining a Friendly Society's affairs.

In the third lecture the element of secession is considered along with the valuation of miscellaneous benefits. A practical method is put forth for adjusting the value of the liability in respect of a benefit payable on the death of a member's wife during his lifetime. Frequently minor death benefits are granted, such as an allowance on the death of a member's child or the right of a member's widow, on payment of a nominal subscription, to continue the funeral allowance, which ceased because she survived her husband. Mr. Watson does not specifically deal with such benefits. While the liability involved is comparatively trifling it is not to be neglected, and generally calls for a certain amount of investigation before some practical and satisfactory working formulas are obtained. Account has of course to be taken of possible claims in respect of (1) children who may be born in the future, and (2) widows who may be left by present members whether these members are now bachelors or married men. Mr. Watson's method of treating the liability as a variable function depending upon the age at death of the member might be conveniently extended to the valuations of such benefits. A most interesting account is referred to, and expanded in an Appendix, of a method pursued by the Author in evaluating a particular form of orphan benefit. A student may, however, experience some difficulty in following the processes ; for instance, the particulars in the Tables do not agree with those in the abstract of data owing to the exclusion of age group 16-19, and in place of speaking of the "Average liability per child per year to age 14" (Tables III and IV), and "Average liability for Orphan Benefit per man dying" (Tables V and VI), the meaning might have been clearer by the use of such expressions as "Equivalent average yearly annuity which becomes payable", and "Amount to be set aside at the moment of death of each member to provide the annuities which might then become payable." Further, in such calculations, it is generally safer to apply the final graded values to the statistics from which they are computed in order to test the graduation, as is usual when an ordinary mortality table is constructed.

The fourth lecture contains a number of useful points incidental to a valuation. To these a warning might have been added to see that the particulars submitted for valuation are in harmony with the rules and information in the annual accounts. The question of age for valuation purposes is not dealt with. It is a very important one. Many Societies are only in a position to supply the age last birthday at entry and the year of entry in respect of those who entered in the past, and confusion may be caused by entrants in the future being rated according to age next birthday at entry. Information as to how the valuation age is calculated is called for in the Return to be furnished to the Registrar of Friendly Societies. The value of this lecture would have been enhanced had the

Author dealt with the form of this Return, but perhaps he could not be expected to do so in view of his official position.

The fifth lecture is devoted to the methods of preparing a Profit and Loss Account, and of dealing with deficiencies, as well as to giving a clear exposition of the late Mr. R. P. Hardy's method of valuation by "ages passed through" which Mr. Watson rightly describes as "extremely serviceable."

In the sixth lecture Mr. Watson discusses Sections 72 and 73 of the National Insurance Act. These sections were evidently drawn to give effect to the two propositions (*a*) that it is no longer necessary for a member of a Friendly Society to insure the same benefits from his Society as those for which he had been subscribing in the past, and (*b*) that members of Friendly Societies and employers who subscribe towards Superannuation or Provident Funds for the benefit of their employees, should have some measure of relief in view of their contributions under the State scheme. Section 72 refers to Friendly Societies and Section 73 to Superannuation or Provident Funds. As indicated previously, comparatively few Friendly Society members are accepting reduced contracts and, in consequence, in schemes in connection therewith the fund set free for extended benefits, or for ameliorating the Society's financial position, is comparatively trifling.

In a valuation of a Superannuation Fund under Section 73 it is practically open to the Actuary to avail himself of the full resources of the State scheme. In analysing the section, Mr. Watson displayed his intimate knowledge of Superannuation Funds, and showed various lines on which investigations might be carried out.

Mr. Watson specially refers to the valuable assistance he received from two of his colleagues—at a time of exceptional pressure of official duties—in the revision of the lectures and in the preparation of an index which forms a most useful adjunct to the book. When one realizes that Mr. Watson was called upon to deliver the lectures during a period in which other demands upon his professional knowledge and experience were numerous and pressing, it must be felt that the Institute and actuarial profession generally owe him a special debt of gratitude for the complete way in which he collated the valuable mass of experience and the concise manner in which he dealt with the problems submitted in the course of his lectures.

V. M.

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## *In Memoriam.*

*Ralph Price Hardy.*

As three of the oldest intimate associates of the late Mr. R. P. Hardy, we gladly respond to the request of the Editors, and offer this affectionate tribute to the memory of our friend.

His immediate ancestry left its definite imprint upon his mind. His two uncles were accomplished palaeographists. and successively held the office of Deputy-Keeper of the

Public Records. Now, the art of palæography is clearly related to a taste for Symbolism, and the capacity of Interpretation, or the extraction of Meaning from Results. This mental trait was eminently displayed in the cognate region of the Actuary in our friend's father, Mr. Peter Hardy, the Actuary of the London Assurance; and those who remember the sale-catalogue of Mr. Peter Hardy's Library, were struck with its abounding evidence of a finely-cultivated mind. Thus our friend was happily dowered, through ample ancestral tendency, not merely with the qualities which aid in the construction of the Actuary, but also with the gift of a generous width of literary interests and catholicity of taste. And this transmitted and varied heritage he sedulously matured. To the last he read the Latin poets with ease and delight—Ovid being an especial favourite—and, three months prior to his death, he was found studying the recent book of Saintsbury upon the "History of English Prosody", and, with independent judgment, discussed its doctrines with one of us who happened then to visit him.

Born in 1838, he entered the service of the Eagle Insurance Company in 1854, and received his professional training under that original and sagacious Actuary, Mr. Charles Jellicoe; in 1868 he became chief clerk in the Mutual Life Office, and in the following year he succeeded Mr. Archibald Day as Actuary and Secretary of the London and Provincial Law.

He had an extensive private practice, both in England and abroad, being consulted by State and Municipal Authorities, by the Directors of great Railway Interests and Commercial Enterprises, as well as by Assurance Companies, Friendly Societies, and kindred Institutions; and the feature which particularly impressed his friends was the *maturity* of thought and judgment, which the major portion of that practice demanded. An instance may be cited: In a Public Reference of importance the basis of solution depended upon the detection of that element in British commodities, which, for several generations, had showed the greatest comparative stability of value in exchange, and this involved laborious historical research. But the originality and actuarial resourcefulness of our friend can only be fully known to those who were privileged to receive a copy of his successive Reports upon Friendly Societies and Pension Funds, and particularly to those to whom he showed, and with whom he discussed, his large mass of original investigations and Tables in manuscript—illuminative and suggestive with fresh and striking views and deductions upon a multiplicity of actuarial questions. Although an adept in the use of analysis, his cast of mind in the conception and conduct of these investigations was naturally

of a geometric form, where the concrete conceptions themselves are retained steadily in the mind throughout the entire process of reasoning ; and even when analysis was first employed in the solution of a problem, he invariably pursued also the geometric method and traced the conclusion by this means to its origin in First Principles. Thus he always possessed a firm and serene grasp of all his deductions. This cast of mind was manifest, also, in the studied arrangements of his data, and the construction of his Tables—the precision and symmetry of arrangement economising the strain of apprehension, and liberating, consequently, a larger amount of mental force for survey and estimation of the results. However, apparently, insignificant was the work on which he was engaged, Accuracy and Exhaustiveness were the dominating characteristics. We have referred to the extent of his general culture, and in this he was a type of character which is, unhappily, becoming more rare ; and we revert to it for the purpose of remarking that, by this broadening of the mind, he became the completer actuary ; not narrowly confined within technical boundaries, but competent, by the power of wider vision thus conferred, to survey and assess his professional solutions with ampler and exacter view.

With all his practical keenness in the estimate of men and commercial affairs, his administrative power failed, and the only comment we are disposed to make upon this want of symmetry is Virgil's truism, *Non omnia possumus omnes*.

But sound and energetic as our friend's Head was, no justice to his honoured memory would be in the least degree adequate which omitted an affecting reference to his qualities of Heart. No mere giver of easy advice was he : his was the genuine helpfulness of an unwearying dedication of time, patience and service in aiding struggling young men to usefulness and honour in professional careers.

As a memento of his burial, Mr. Hardy selected the words, *Ex Umbris et Imaginibus in Veritatem*. We cannot find these words in any classical florilegium, and they sound like a passage from Augustine's "De Civitate" or the "De Imitatione" of à Kempis. But they expressed the spirit of his life—the certain assurance that, here, truth would be evolved, to the earnest searcher, out of all mists and speculations ; and they equally left with us the assurance of the same conviction concerning the Unseen.

T. E. YOUNG.  
H. W. MANLY.  
GEORGE KING.

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**JOURNAL**  
 OF THE  
**INSTITUTE OF ACTUARIES.**

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*On the Estimated Age-Distribution of the Indian Population as recorded at the Census of 1911, and the Estimated Rates of Mortality, deduced from a comparison of the Census Returns for 1901 and 1911 ; by THOMAS G. ACKLAND, Fellow of the Institute of Actuaries ; Honorary Fellow of the Faculty of Actuaries.*

[Read before the Institute, 31 March 1913.]

**INTRODUCTORY.**

1. The age-distribution, and the rates of mortality, of the male and female population, recorded by the General Census for India, have been investigated by Mr. G. F. Hardy, in connection with the separate Censuses of 1881, 1891 and 1901. The methods adopted in his investigations, and the resulting tabular figures, are set out in Mr. Hardy's Reports to the Indian Government on these several Censuses, and these are summarized, as regards the 1881 Census, in a Paper submitted by him to the Institute, and published in Vol. XXV of the *Journal*, p. 217 *et seq.*

2. I have recently investigated, at the request of the Indian Government, the figures of the 1911 Census, and have made an official Report upon the results of my investigation, which will be included in the Report volume dealing with that Census. I have thought that it might be interesting to the Members of the Institute to state, in summary form, the main lines adopted in my investigation of the 1911 figures, which have largely followed those adopted by Mr. Hardy in dealing with the figures of 1881, 1891 and 1901, as set out in his Reports, the results of which have not been laid before the Institute, with the exception

of the contribution to the *Journal*, above referred to, which dealt solely with the 1881 Census.

**Fundamental Difficulties.** 3. There are two fundamental difficulties arising in connection with the Indian figures, in respect of

(i) defective and inaccurate Census records as to age, (ii) defective registration of births and deaths, and inaccuracies as to the ages recorded at death. As regards (i), methods have been adopted for adjusting, as far as practicable, the age records, as set out in quinary groups of age, and these methods are stated, later on, in the present Paper. As regards (ii), the omissions from the birth and death registers are, as will be seen in later paragraphs, quite serious, and no means appear to be available for supplying the undoubted deficiencies of the registration records, whilst the inaccuracies of age at death present also a serious difficulty. Under these circumstances, the deduction of mortality tables, by the usual method of dealing with the figures of two successive Censuses (with allowance for migration), and those of the registration records of births and deaths for each of the intervening ten years, appears to be practically impossible of satisfactory application, and special methods have had to be devised for deducing mortality tables which shall have some approach to accuracy, having regard to these exceptional difficulties.

**Summary Census Figures.** 4. The summary figures of the Indian census, taken in March 1911, are set out in Tables I and II

appended to this Paper. In Table I, the enumerated population in 1911, 1901, 1891 and 1881 are given for each Province, State and Agency included in the Census; whilst in Table II the variations during each decennium are shown, with discrimination of sex in the 1901 and 1911 figures.

**Provinces.** 5. The Provinces included in my investigation, which were substantially similar (subject to variations of boundaries, &c.) to those dealt with in Mr. Hardy's earlier investigations, are the following :

- (1) Bengal (including Eastern Bengal and Assam);
- (2) Bombay;
- (3) Burma;
- (4) Madras;
- (5) Punjab (including North-west Frontier Province);
- (6) United Provinces of Agra and Oudh (formerly called the North-west Provinces).

As regards the Province of Bengal, it was thought preferable, in view of the modification of the partition of that Province announced by the King at Delhi, and of the fact that the boundary between the new Provinces of Bengal and Bihar had not been precisely fixed when the figures were under investigation, to prepare a single life Table for Bengal and Eastern Bengal and Assam combined. Mr. Hardy's figures for Bengal in 1901 included Eastern Bengal, but did not include Assam ; but, as the population was not equal to 5 per-cent of that of the combined Provinces, the inclusion of Assam in the figures for 1911 could not affect the age-distribution at all appreciably.

#### DEFECTIVE CENSUS RETURNS, AND ERRORS OF AGE.

*Special Characteristics of Indian Census Returns.* 6. It has long been recognized that the figures recorded in the several Provincial and General Censuses of the Indian population taken in past years, are subject to characteristic peculiarities and anomalies, as compared with figures deduced from Censuses taken in European countries. So far as these peculiarities arise from defective data—that is, from certain male and female members of the population having been omitted from the returns—there has undoubtedly been a progressive improvement, as the organization and administration of the Census operations have become more complete, and as the native population have grown more accustomed to the idea of the Census, with, perhaps, a better appreciation of its true objects. Further anomalies in the Indian figures arise from errors or mis-statements in age, either by under-estimating or over-estimating the true age, or from some preference for particular digits of age. There are also special tendencies affecting the accuracy of the returns of female lives, usually taking the form of an under-estimate of the numbers and ages in early life (about ages 9 to 14), and an over-statement of those in the next following group of ages (about 15 to 19) ; whilst for both sexes there are further anomalies, after middle life, and a decided tendency to over-state the more advanced ages.

*Effect of Famines, etc., on Age-Distribution.* 7. The age-distribution of the figures in the Censuses are also much disturbed by the effects of serious famines, plagues, malaria, &c., arising in the past, and the effect of these disasters upon the birth rate, and upon the death rate, especially in the early and later years of life, will remain in evidence, like permanent scars from old wounds, so long as the populations, in the age-groups originally affected,

are in existence. For instance, a serious famine, reducing the birth rate in a particular Province between 50 and 60 years ago, should still be in evidence in the figures of the present Census, between the ages of 50 and 60 years, although the results are probably much obscured by defective data at these and later ages.

*Errors of Age.* 8. The Census returns for 1911 give the figures, for male and female lives respectively, in respect of each of the infantile ages 0 to 4, and later quinary groups up to age 69 inclusive, the numbers in respect of 70 and over being included in a single group. At the 1901 Census, and at previous enumerations, the quinary groups extended to age 59 only, the data at age 60 and over being returned in a single group. As the returns in quinary groups would give no information as to individual ages, after age 4, separate Schedules were supplied, showing the numbers recorded at each individual age, out of a selected number of persons, usually 100,000 or 200,000 in number, of each sex, in each Province. The numbers in the Census records, and in these specimen Schedules, were then reduced to a similar total of 100,000 of each sex. If the specimen age-distributions were true samples of the actual population, the totals of the quinary age-groups should, of course, agree in both cases. As, however, these totals did not agree (although the data included in the specimen Schedules were fairly representative, and the disparity of age-distribution was not great), it was necessary to distribute the figures in each quinary group, age by age, in proportion to the figures shown at individual ages in the specimen Schedules. This re-distribution is given for all ages in Table III appended in this Paper.

9. A cursory examination of these figures will show marked anomalies, and a closer investigation will show that these are characteristic and peculiar. This is more clearly shown in the two appended diagrams, which show graphically the recorded numbers, for male and female lives respectively, in each Province, and in respect of each age from 0 to 64 inclusive, after which the records are relatively very small. The large numbers recorded at ages which are multiples of 5 and 10 are not unexpected, but are very marked. For male lives, the largest numbers are usually recorded at age 30, closely followed by those at ages 25 and 40, with somewhat smaller numbers at ages 10, 35, 20, &c. For female lives, the favourite age is again 30, closely followed by ages 25, 20, 40 and 35, with somewhat smaller figures at the other

multiples of 5 and 10. The diagrams also show a remarkable preference for even digits of age, as illustrated by the following Table A :

TABLE A.

## MALES.

*Shewing, in each of the six Provinces undermentioned, the numbers, out of a total of 1,000, returned in respect of each Digit of Age; also the mean values for the six Provinces, and the order in which the several Digits were recorded.*

Provinces	DIGIT OF AGE RECORDED IN CENSUS—									
	0	1	2	3	4	5	6	7	8	9
Numbers (per 1,000) recorded in respect of each Digit of Age :										
Bengal ... ...	253 (1)	43 (9)	121 (3)	56 (8)	64 (6)	187 (2)	76 (5)	57 (7)	106 (4)	37 (10)
Bombay ... ...	292 (10)	43 (8)	110 (3)	56 (7)	60 (6)	215 (2)	66 (5)	47 (8)	78 (4)	33 (10)
Burma ... ...	187 (1)	76 (9)	106 (3)	98 (4)	78 (8)	142 (2)	85 (5)	80 (7)	84 (6)	64 (10)
Madras ... ...	264 (11)	48 (9)	113 (3)	64 (7)	73 (6)	171 (2)	89 (5)	48 (8)	90 (4)	40 (10)
Punjab ... ...	279 (10)	44 (9)	110 (3)	55 (7)	67 (6)	198 (2)	78 (5)	49 (8)	84 (4)	36 (10)
United Provinces ...	294 (1)	47 (7)	113 (3)	45 (8)	65 (6)	186 (2)	83 (5)	43 (9)	91 (4)	33 (10)
Totals ... ...	1,569	301	673	374	407	1,099	477	324	533	243
Mean Values ...	262	50	112	62	68	183	79	54	89	41
Order of Record ...	(1)	(9)	(3)	(7)	(6)	(2)	(5)	(8)	(4)	(10)

The even digits are selected in the following order, 2, 8, 6, 4, in each of the Provinces under observation, whilst the odd digits of age are usually selected in the order 3, 7, 1, 9, with slight deviations in Bengal and the United Provinces. In Burma, whilst there is still a preference, first for multiples of 10 and 5, and for the even rather than odd digits, the differences are much less marked, and the figures deduced from the specimen Schedules for this Province are more normal, and presumably more accurate, than those for the other Provinces. Throughout the returns,

there is a decided preference in respect of both sexes (after the multiples of 5 and 10), for the particular ages 8 and 12.

**Adjustment of Returns as to Age.** 10. It is evident from the above that the age-distribution of the population, as taken at the

Census date in the several Provinces, is entirely untrustworthy as a representation of the true ages of the lives, and that any rates of mortality, deduced on the basis of such figures, would be quite unreliable. It is therefore necessary, in the first instance, to make some attempt to adjust the age-distribution, so as to be more in accordance with what may be presumed to be the real facts as to age. Even the figures recorded in quinary groups cannot be considered as properly appertaining to the groups in which they are returned, since the individual ages at which there are the most serious disturbances, namely, those which are multiples of 10 and 5, occur at the beginning of the age groups; and undoubtedly a certain proportion of the excessive numbers returned at these points should be transferred to the previous age-group. The necessary adjustments would be more easily given effect to, if the quinary age-groups were so taken that the multiples of 5 were in the middle of each group, thus: 0, 1, 2, 3-7, 8-12 &c.; and this seems to be a most desirable improvement, which should be given effect to at future Censuses.

11. The method followed by Mr. G. F. Hardy, in his investigation of the figures of previous Censuses, dating from 1881, was to assume that the figures returned, in the specimen Schedules, at the ages which are multiples of 5, should be reduced, by deducting from them any excess over and above the mean value of the numbers at the preceding and following ages, and that one-half of this excess should be transferred to the preceding age-group, and the remaining half retained in the group in which they were returned. A different correction was apparently applied by Mr. Hardy (referred to in paragraph 23 of his Report on the 1901 Census) by adding half the numbers recorded at age 5 (instead of half the excess referred to above) to the group 0 to 4, and deducting the same quantity from the groups 5 to 9. This correction is not given effect to by Mr. Hardy in the numbers for 1901 given in Table C, appended to his Report, but was, no doubt, allowed for in deducing his graduated results. By this means, he obtained corrected figures for the quinary groups, which are, perhaps, as near an approach to accuracy as can be obtained from the very defective data; and, as I do not see that any alternative course would secure greater presumed accuracy, I have followed Mr. Hardy's method in this respect, adopting,

$$(u_{5n} + u_{5n+1} + u_{5n+2} + u_{5n+3} + u_{5n+4}) - \frac{1}{2} [u_{5n} - \frac{1}{2} (u_{5n-1} + u_{5n+1})] + \frac{1}{2} [u_{5n+3} - \frac{1}{2} (u_{5n-4} + u_{5n+6})]$$

$$= \Sigma_0^4 u_{5n+t} - \frac{1}{4} (\Delta^2 u_{5n+5} - \Delta^2 u_{5n}).$$

*Example by Columnar Method—Bengal (Males).*

n	Age $5u + t$	Ungraduated $u_{5n+t}$		$\Delta u_{5n}, \Delta u_{5n+1}$	$\Delta^2 u_{5n}$	$\Delta(\bar{n})$		$-\frac{(6)}{4}$	$-\frac{(6)}{4}$	$(7)$	$(8)$	Corrected Numbers (3) + (7)
		(1)	(2)			(3)	(4)					
0	0	3,065	...	...	...	...	...	...	...	...	...	...
1	1	1,586	13,541	...	...	...	...	-1,499	374	...	...	13,915
2	2	2,961	...	...	...	...	...	...	...	...	...	...
3	3	3,072	...	...	...	...	...	...	...	...	...	...
4	4	2,857	...	+	832	-1,499	...	...	...	...	...	...
5	5	3,689	...	-	667	...	...	...	...	...	...	...
6	6	3,022	...	...	...	...	...	...	...	...	...	...
7	7	3,003	15,599	...	...	...	...	-3,525	882	...	...	16,481
8	8	3,949	...	...	...	...	...	...	...	...	...	...
9	9	1,936	...	+	2,206	-5,024	...	...	...	...	...	...
10	10	4,142	...	-	2,818	...	...	...	...	...	...	...
11	11	1,324	11,975	...	...	...	...	4,624	-1,156	...	...	10,819
12	12	3,782	...	...	...	...	...	...	...	...	...	...
13	13	1,018	...	...	...	...	...	...	...	...	...	...
14	14	1,679	...	+	317	-400	...	...	...	...	...	...
15	15	1,996	...	-	83	...	...	...	...	...	...	...
16	16	1,913	...	...	...	...	...	...	...	...	...	...

Note.—The addition to age group 0-4, and deduction from age group 5-9, of  $\frac{1}{4}(u_4 + u_6)$  or 1,470, in the final column, gives effect to the further correction, referred to in the text, making the final values 15,385 and 15,011 respectively.

also, the correction referred to above, which undoubtedly brings the figures for age-groups 0 to 4 and 5 to 9 into a more natural progression. The process followed is set out symbolically in Table B, and it will be seen that this can also be expressed in a form applicable to columnar summation and differencing.

#### GRADUATION OF MALE AGE-DISTRIBUTION FIGURES.

Computation  
and Graduation  
of Mean  
Census Figures.

12. The figures having been thus obtained in adjusted quinary groups, relative to a total of 100,000 of each sex in each Province, and the mean values in each

age-group at the 1901 and 1911 Censuses computed, and corrected for migration, as explained later on, a curve was passed through each age-group (in respect of male lives) so as to produce the graduated mean numbers, relative to a total number of 100,000 in each Province, which should, as far as practicable, show an approximation to the total figures in each age-group, and, at the same time, produce that smooth progression of the figures at successive ages which would certainly be in evidence, where a large body of facts is dealt with, if the real numbers were recorded at each age. For the purposes of this graduation, a mathematical formula was adopted, which gave the graduated numbers at every age, and at the same time supplied a basis for those at the older ages, where the data were manifestly most defective, and the recorded numbers gave little or no trustworthy indication of the facts. In all the Provinces, except the Punjab, this mathematical expression took the form of a frequency curve (of Type I in Mr. Elderton's Book), and would be represented, where the origin is taken at age 0, by the general formula :

$$y_x = y_0 \omega^{m_1} (\omega - x)^{m_2}$$

where  $m_1$  and  $m_2$  are constants, and  $\omega$  is the age (varying in the six Provinces for male lives from 90 to 95) at which the numbers in the mortality table vanish. An explanation of the methods followed in the graduation of the age-distribution figures for the Bombay Presidency has been prepared, at my suggestion, by Mr. S. J. Gunningham, F.I.A., to whom the work of graduation by this type of formula was entrusted. (See Appendix A.)

13. In Bengal, and Burma, the formula employed was modified, after age 59, and in Madras, prior to age 40, to bring the adjusted numbers into closer agreement with the adjusted figures. (See Appendix C(1).) In the case of the Punjab, the figures were not found to be amenable to treatment by this method, and this was one of the many indications that the

figures in this Province are not complete, or reliable. The figures were ultimately adjusted by the adoption of a formula, based upon the "curve of normal error", which is exceptionally powerful in dealing with grouped figures of the class under consideration, and a smooth curve was thus produced for the Punjab, involving the least possible departures from the adjusted figures in quinary groups. I have included in Appendix B some Notes, prepared by Mr. S. J. Rowland, A.I.A., as to the process of graduation of the Punjab male age-distribution by this formula. The process is also fully illustrated in Mr. G. F. Hardy's Lectures on the construction and graduation of mortality Tables, pp. 91-98.\*

14. In Table IV, the mean figures, deduced from the Census Returns for 1901 and 1911, after correction for age, are given in quinary age-groups, also the graduated figures for each group, as deduced by the mathematical formulæ above referred to, and in Table V, the graduated figures are given for each age, in respect of each Province; all numbers in Tables IV and V corresponding to a total of 100,000 persons.

#### DEFECTIVE REGISTRATION OF BIRTHS AND DEATHS.

*Investigation  
of Rate of  
increase of the  
Population and  
of rate of  
Mortality at  
each age.*

15. The figures of a single Census, even if accurately returned at each age, will not give any trustworthy indication of the mortality arising among the lives, and the usual plan, in such cases, is to take the figures of two successive decennial Censuses, together with the number of births, and the number of deaths at each age, during the intervening ten years, as recorded in the birth and death registers, respectively. Unfortunately, in the case of the Indian Provinces, the registers of births and deaths, although showing great and progressive improvement as compared with previous years, are still very incomplete, and probably also contain many inaccuracies. It appears, from an investigation made officially in India of the variation of the population during the 10 years 1901-1910, by comparison of the relative numbers of registered births and deaths with the movement in population as shown by the Censuses of 1901 and 1911, that the movement, as shown by these alternative methods, varies quite materially in each Province.

*Defective  
Registration.*

This is shown in the following Table C.

\* See, also, an interesting reference to this method in Professor Edgeworth's Presidential Address to the Royal Statistical Society (*Journal*, 1913, pp. 185, 193).

TABLE C.

*Comparison of Variation in Population during the decade 1901–10, as deduced (1) from the Registration of Births and Deaths in each Province; (2) from the Census Returns for 1901–11 of the Population in Registration Areas.*

Province (British Territory only)	VARIATION 1901–10, ACCORDING TO VITAL STATISTICS (EXCESS OF BIRTHS OVER DEATHS +, DEFICIENCY -)			VARIATION 1901–1911, ACCORDING TO CENSUS (EXCESS +, DEFICIENCY -)			DIFFERENCE			Remarks
	Total	Males	Females	Total	Males	Females	Excess or deficit of column 5 over column 2	Excess or deficit of column 5 over column 3	(10)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
Bengal ...	+ 2,047,355 +	927,585 +	1,119,770 +	1,944,935 +	1,124,468 +	820,467 -	102,420 +	196,883 -	299,303	Excludes Angul.
Bombay ...	- 217,469 -	96,683 -	120,783 +	1,110,801 +	661,631 +	449,170 +	1,328,270 +	758,314 +	5169,956	Excludes Aden. Excludes Northern Arakan, Salween, Bhamo, Myitkyina, Ruby Mines, Upper Chindwin and specially administered territories.
Burma ...	+ 274,888 +	91,027 +	183,861 +	1,249,619 +	645,460 +	604,159 +	974,731 +	554,433 +	420,298	{ Khasi and Janaitia Hills, Naga Hills, Lushai Hills, Garo Hills and Chittagong Hill Tracts.
Central Provinces and Berar ...	+ 1,627,508 +	799,781 +	867,727 +	1,944,856 +	1,040,035 +	910,821 +	317,348 +	213,254 +	104,094	{ Excludes Khasi and Janaitia Hills, Naga Hills, Lushai Hills, Garo Hills and Chittagong Hill Tracts.
Eastern Bengal and Assam ...	+ 2,250,288 +	1,061,715 +	1,188,573 +	3,369,643 +	1,800,861 +	1,568,782 +	1119,356 +	739,146 +	380,209	{
Madras ...	+ 2,797,197 +	1,435,021 +	1,362,176 +	3,175,750 +	1,531,626 +	1,614,124 +	378,553 +	96,605 +	281,948	
North-west Frontier Province ...	+ 120,053 +	80,861 +	39,192 +	155,399 +	76,393 +	79,006 +	35,346 -	4,468 +	39,814	
Punjab ...	- 557,447 -	119,652 -	437,795 -	359,881 +	46,672 -	406,553 +	197,566 +	166,324 +	31,242	Excludes Bilooh Trans-Frontier.
United Provinces ...	+ 1,017,726 +	690,985 +	326,741 -	510,233 +	24,755 -	534,988 -	1,527,959 -	666,230 -	861,729	
<b>Total</b> ...	+ 9,360,069 +	4,861,640 +	4,498,451 +	12,080,880 +	6,015,000 +	5,161,088 +	2,720,700 +	2,081,201		

16. It will be seen that in the Province of Bombay, whilst the comparison of the birth and death registration Returns shows a *reduction* of 217,469 of both sexes, the Census figures show an *increase* of 1,110,801, a difference of 1,328,270. In the United Provinces, on the other hand, the registration figures show an *increase* of 1,017,726, whilst the Census figures show a *decrease* of 510,233, a difference of 1,527,959. The combined figures for Bengal, Burma, Central Provinces, Eastern Bengal and Assam, Madras, and North-west Frontier Province, show, from the registration Returns, increases aggregating 9,117,289, whilst the Census figures give a much larger increase of 11,840,202. Finally, in the Punjab, the decrease in the decennium is 557,447 from the registration Returns, and 359,881 from the Census Returns. Over the whole area of registration in India, the comparison of births and deaths gives an increase of 9,360,099, whilst the Census Returns show an increase of 12,080,889, and, assuming that the registration figures are correctly recorded, it seems probable that on the whole a large number of births have escaped registration, whilst in Bengal and the United Provinces a large number of deaths would appear to be unrecorded. The movement of migration between the different Provinces, or to or from India as a whole will in no way account for these large discrepancies. It is evident, therefore, that the registers of births and deaths are, at present, practically useless for deducing rates of mortality, and, even if these discrepancies in the numbers were corrected, it is probable that the death registers would show serious inaccuracies of age, which could not be assumed to be similar, in direction and extent, to the inaccuracies arising in the Census returns.

Scrutiny in selected Registration areas. 17. In view of these comparative results, it seems most desirable that efforts should be concentrated upon limited representative areas in each of the main Provinces, with a view to securing more complete data, in respect of the birth and death rates, and the age-distribution of the deaths.

18. If the suggestion cannot be adopted in its entirety, it is most desirable that a closer supervision should be made of registrations of births, and of the deaths at ages below 15, in representative areas in each Province. The only trustworthy figures relative to births, and deaths at these early ages, have been obtainable from the Reports on the Proclaimed Clans statistics in the United Provinces, (*see §§ (25) to (28), infra*) and the value of these, in the deduction

of complete life tables, can hardly be over-estimated. The record and investigation of these statistics apparently ceased in 1904, and, as explained later on in the present Paper, and in Mr. Hardy's 1901 Report, the data furnished between 1891 and 1904 were so limited as to age as to be practically useless for the purposes desired. It is clear that results based on statistics referring to the period 1876–1890 could not properly be employed in any future investigation of Census Returns ; and it is, therefore, most desirable and, indeed, essential, if complete life tables are to be deduced in future, that some effort should be made to secure trustworthy data as to the births in the several Provinces, and the deaths at the ages of infancy and childhood.

#### INVESTIGATION OF RATES OF INCREASE OF POPULATION, AND RATES OF MORTALITY.

Comparison of Figures at Successive Censuses.

19. Under these circumstances, I have had to deduce, as best I could, the rates of increase in the population, and thence the rates of mortality and expectations of life, from the comparison of successive Census figures for each Province. In his Report on the 1901 Census, Mr. Hardy sought to deduce the rates of mortality which might be assumed to hold, in each Province, and in all India, on the average, irrespective of exceptional periods of stress and strain, arising from famine, plague, &c. ; and he carried this into effect by taking, from the Census figures of 1881, 1891, and 1901, a mean of the results in each quinary age-group, after giving double weight to the figures of 1891. In investigating the 1891 Census figures (the decennium preceding which was free from famine) Mr. Hardy adopted a different method, and limited his investigation to a comparison between the Census figures of 1881 and 1891. In his Report on the 1881 Census, where the figures of the previous Census of 1871–2 (the first General Census of India) were known to be extremely defective, he attempted, as in 1901, to deduce estimated average rates of mortality, after eliminating the effect of famines, plagues, &c.

20. It is evident that where (as in 1881 and 1901) the decennium preceding the Census has been characterized by exceptional and severe general visitations of plague, famine, or malaria, the data of that decennium would not be suitable for deducing mortality Tables which could be generally employed

(for purposes of life assurance premiums. &c.) as representing the *average* rates experienced in the several Provinces, and in India generally, and Mr. Hardy's different treatment in 1891, as compared with that adopted in 1881 and 1901, was, no doubt, fully justified by these considerations.

*Decennium  
1901-1911.* 21. The decennium preceding the 1911 Census has fortunately not been characterized by any general visitations such as those referred to, and, although there were several local famines, and a severe famine in the United Provinces in 1907, and plague has been largely in evidence in Bengal and Bombay, and both plague and malaria in the Punjab and the United Provinces, the period will, I think, taking the country as a whole, compare favourably with previous decenniums. When these visitations have been more widely extended, and prolonged in duration. The rate of increase in the population of India, as a whole, in the decennium 1901 to 1911, is, as will be seen from Table II, considerably greater than in the previous decennium, though much less than in 1881 to 1891; and the improved rate of increase is shown in all the Provinces included in my investigation, excepting Burma (where the figures between 1891 and 1901 are apparently disturbed by variations of boundary), and the Punjab and United Provinces, where plague and malaria have been severely felt, and where the male population has been practically in a stationary condition, and the female population has been reduced. I have, therefore, felt justified, on the whole, in basing my figures on a comparison of the Census Returns of 1901 and 1911, and in this respect have followed the method adopted by Mr. Hardy in 1891. This point must be borne in mind, when comparing my tabular results with those deduced by Mr. Hardy in 1881 and 1901, which were intended to represent the experience of average periods, rather than that of the particular decennium under observation.

*Rates of  
increase or  
decrease in  
Decennium.* 22. The process followed has been to compare the figures in adjusted quinary groups for male lives in each Province, in respect of the 1901 and 1911 Censuses, and to deduce from this comparison rates of increase or decrease in the same age-groups during the decennium. The rates under observation were found to be widely different in different age-groups, and I therefore deemed it advisable (excepting in the Punjab, where a constant rate of increase was assumed at all ages) to deduce rates varying with age, which, whilst showing a curve of smooth progression, should

bring out the relative figures in each age-group as nearly as practicable. A similar course appears to have been adopted by Mr. Hardy, in deducing the rates of increase for the decennium 1881 to 1891, but, in dealing with the preceding and following decenniums (when he sought to deduce *average* rates of mortality) he seems to have employed a constant rate of increase at all ages, separately deduced for each Province.

23. The graduated values of  $\log r_x$ , where  $r_x$  is the rate of decennial increase in the population at age  $x$ , were deduced by frequency curve formulas, adopted so as to be appropriate to the unadjusted rates of increase in each Province. The formulas employed in the several Provinces are set out in Appendix C (3). The graduated rates of increase, thus deduced for male lives, are given for each age, and for each Province, in the appended Table VI (p. 380).

#### PROBABILITY OF LIVING AT EACH AGE.

24. The graduated mean figures, showing the estimated age-distribution of 100,000 male lives, deduced by mathematical formulæ, as explained in paragraphs (12), (13), *supra*, from the figures of the 1901 and 1911 Census, would represent approximately a total population of 100,000 in the middle of the decennium (say, in September 1906), and it was then possible, by multiplying and dividing the mean age-distribution figures by the square root of the graduated rate of decennial increase at each age,  $r_x$ , obtained as above, to deduce the graduated numbers assumed to be living at the same age in March 1911, and in March 1901 respectively, and, from these two sets of numbers to deduce, at each age in 1901, the probability of living 10 years ( $_{10}p_{x+\frac{1}{2}}$ ) up to 1911, and from these by interpolation the probability of living one year ( $p_x$ ) at each age. An alternative process was followed, in the case of some of the Provinces, which appears to secure equally accurate results, with somewhat greater facility. This was by taking the graduated mean age-distribution figures reduced to a total of 100,000, as in the middle of the decennium, deduced as above, and multiplying and dividing them by  $(r_x)^{\frac{1}{6}}$ , where  $r_x$  is the graduated decennial rate of increase at age  $x$ , thus obtaining the estimated population at each age, six months after, and six months before, the mean date. By this process the probability of living one year ( $p_{x+\frac{1}{2}}$ ) at each age was directly deduced, and from these values those of for integral ages were obtained by interpolation.

## RATES OF MORTALITY IN INFANCY AND CHILDHOOD.

*Ages of Infancy and Childhood.* 25. The above methods of deducing the mean population, and thence the probabilities of living at each age, apply, generally speaking, from about age 19 to the end of life. For the ages of infancy and childhood, the Census data were evidently quite unreliable. The deficiencies and inaccuracies in the returns at these early ages are well recognized, and, in each of his investigations into previous Censuses, Mr. Hardy found it impossible to make any use of the recorded figures at these ages. The plan which he adopted, and which I have followed in dealing with the 1911 Census figures, was to employ the data in respect of the "Proclaimed Clans," in the United Provinces of Agra and Oudh, where infanticide was formerly rife, and where a strict legal supervision has been made for some years past into the births and deaths at the earlier ages of life. From the figures available as regards these Proclaimed Clans from 1876 to 1891, over which period alone particulars were available at individual ages, Mr. Hardy deduced a Table of mortality from age 0 to 12, which he graduated by application of a formula of the type

$$l_x = A + Hx + Bc^x + \frac{m}{nx+1}$$

It will be observed that this expression is in the form of Makeham's first modification of Gompertz's formula, with an additional term, which gives effect to the heavy mortality in the early ages, and the rapid fall to age 10, after which this corrective term becomes practically negligible. It will also be seen that the formula is not, as usual, applied to  $\log l_x$ , but to  $l_x$ . This exceptional course appears to have been followed, because there was no object in retaining the law of uniform seniority, the main object being to obtain a smooth graduation; and it was also desired to deduce the values of  $L_x$ , the population living at age  $x$ , corresponding to  $l_x$ , which values cannot correctly be deduced, at the early ages, from the approximate relation  $L_x = \frac{l_x + l_{x+1}}{2}$ , applicable to adult ages, and cannot conveniently be deduced from a mathematical formula for  $\log l_x$ . Having deduced the above equation for  $l_x$ , we have at once

$$L_x = \int_x^{x+1} l_x dx = \left( A + \frac{H}{2} \right) + Hx + \frac{Bc^x(c-1)}{\log_e c} + \frac{m}{n} \log_e \left( 1 + \frac{n}{nx+1} \right)$$

26. The following Table D, extracted from Mr. Hardy's Report on the 1901 Census, gives the ungraduated data, and the graduated values of  $l_x$ , and  $L_x$ , at each age from 0 to 12, as deduced by the above formulae :

TABLE I.

*Mortality Experience of the Prudential Clans (North-west Provinces), 1876-1900.*

## MALES LIVES,

(Extracted from Mr. G. F. Hardy's Report on the 1901 Census.)

Number of Deaths REGISTERED OUT OF 10,000  
"AT RISK" AT EACH AGE

Age (1)	1876-81 (6 years) (2)	1882-90 omitting 1887 and 1889 (6 years) (3)	1876-90 (12 years) (4)	1891-1900 (10 years) (5)	Age (6)	As deduced from column (4) (7)	Graduated numbers (8)	Age $x$ (9)	Numbers living $l_x$ (10)		
0-1	2,333	2,301	2,317	2,155	At Birth	100,000	100,000	0	100,000		
1-2	1,333	1,262	1,297	1,143	Aged 0-1	76,830	76,732	1	70,213		
2-3	786	959	873	682	2	66,662	66,662	2	63,618		
3-4	506	591	549	357	3	61,028	61,325	3	59,313		
4-5	384	449	417	288	4	57,678	57,753	4	56,368		
5-6	291	333	312	188	5	55,273	55,270	5	54,285		
6-7	245	290	267	...	6	53,549	53,485	6	52,758		
7-8	181	211	196	...	7	52,119	52,151	7	51,591		
8-9	150	167	159	...	8	51,098	51,109	8	50,658		
9-10	132	173	153	...	9	50,285	50,257	9	49,877		
10-11	120	133	127	...	10	49,516	49,529	10	49,194		
11-12	137	97	117	...	11	48,881	48,881	11	48,576		
			...	...	12	48,315	48,286	12	48,001		

It will be seen that the graduated values of  $L_x$  in column (8) are remarkably close at all ages to the original population data given in column (7). It will also be observed that the data available for the ten years 1890 to 1900 relate only to ages 0 to 6, and were not employed in deducing the graduated mortality table. The reports on infantile births and deaths in the Proclaimed Clans have been furnished to me for the four years from April 1900 to March 1904, but as the data were only available in respect of children under one year, between one and six years, and over six years, without discrimination of individual ages, they were practically useless for my purpose. Since 1904, these reports have, I believe, been discontinued. I have, therefore, had no alternative but to adopt, for the ages of infancy and childhood, the figures of the Proclaimed Clans from 1876 to 1891, employed and adjusted by Mr. Hardy, with modifications, as explained below, in each Province. This is, of course, far from satisfactory, but the only alternative course appeared to be to omit the figures for the younger ages altogether. For many reasons, it appeared to be the preferable course to deduce estimated rates of mortality at each age from birth throughout the whole of life, but it will, of course, be understood that the figures in respect of ages 0 to 12 cannot be regarded as more than an approximate representation of the course of mortality during the decennium at those ages.

27. In the practical application of the Proclaimed Clans figures, as above, in deducing the estimated mortality table at the early ages in respect of each

Modification  
of Proclaimed  
Clans Figures.

Province, these figures were adopted as a sort of base-line, and such modifications were made in the curve, indicating the rate of mortality from age 0 to 12, as appeared to be necessary, to make a continuous curve throughout life, and a smooth junction with the graduated figures mathematically deduced at higher ages. In the United Province of Agra and Oudh, for ages 0 to 12, and in the Punjab for ages 0 to 6 (with an arbitrary adjustment from 7 to 12) the Proclaimed Clans figures were adopted, without alteration; and in the other Provinces, a constant addition to, or a constant or proportionate deduction from, the number of deaths, was made in order to fit in with the graduated curve at higher ages.

28. In Burma, where the rates of mortality throughout appear somewhat more to approximate to those observed in European countries, it was necessary to make the large deduction

of 25 per-cent from the number of deaths, as given in the mortality table of the Proclaimed Clans, at each age between 0 to 12. In Madras, the deduction was made of  $10\frac{1}{2}$  per-cent from the force of mortality at these ages ; whilst in Bengal, a constant addition of 75, and in Bombay a constant deduction of 100, was applied to the number of deaths shown in the mortality table of the Proclaimed Clans at each age from 0 to 12. After making these several adjustments for the rates of mortality from age 0 to 12, and combining with them the rates of increase as shown in Appendix A, Table 8, a smooth progression was obtained throughout life, by combination of these earlier rates with the rates deduced for ages 19 and onwards by the mathematical formulæ dealt with in paragraphs (12) and (13) *supra* ; the rates of mortality at the intermediate ages 13 to 18 being arbitrarily estimated, so as to make a smooth junction with the graduated rates at ages 12 and 19 in each Province. The modified formulæ for  $l_x$  and  $L_x$  in each Province for ages 0 to 12 are given in Appendix C (2).

#### MIGRATION.

*Migration.* 29. I now turn to the subject of the effect of migration on the Census figures in the several Provinces. Particulars were furnished to me as to the population enumerated in each Province, separately for each sex, and the numbers, out of those so enumerated, who were born in other specified Provinces of India, or outside India. From these statistics, the following results were obtained, showing, for male lives, the mean population during the decennium of each Province brought under investigation, and the number of immigrants included at the Censuses of 1901 and 1911 respectively (that is, those who were enumerated in a particular Province, but born elsewhere), and the number of emigrants included in 1901 and 1911 (that is, those who were born in a particular Province, but enumerated elsewhere); and also, the net number of immigrants and emigrants in 1901 and 1911.

Rates of Male  
Migration in  
the several  
Provinces.

Burma.

30. It will be seen that there was a very large male immigrant population included in the Census figures for Burma, equal in 1901 to about 8 per-cent, and in 1911 to about 9 per-cent, of the mean male population. It is understood that this arises from an influx of industrial workmen, mainly Hindoos and Mohammedans, the indigenous population being almost exclusively engaged in agricultural pursuits. The effect of immigration was eliminated by including,

TABLE E.

*Showing the number of Immigrants and Emigrants, and the balance of net Immigrants or Emigrants, in each Province, as enumerated in 1901 and 1911; with the mean Population during the decennium.*

## MALE LIVES.

PROVINCE	Mean Population 1901-1911	Number of Immigrants 1901	Number of Emigrants 1901	Net Immigrants 1901	Net Emigrants 1901	Number of Immigrants 1911	Number of Emigrants 1911	Net Immigrants 1911	Net Emigrants 1911
Bengal (including Eastern Bengal and Assam) ...	44,339,406	699,868	225,520	474,348	... 155,711	757,285	264,296	492,989	...
Bombay ... ...	13,563,830	477,316	321,605	155,711	... 458,326	578,154	280,602	297,852	...
Burma ... ...	5,762,763	464,781	6,455	... 451,277	312,101	530,623	5,559	525,064	...
Madras ... ...	19,824,988	139,176	451,277	... 170,178	120,379	482,672	120,379	362,193	...
Punjab (including Northwest Frontier Province) ...	14,510,769	413,586	243,408	170,178	571,018	374,708	234,696	140,012	...
United Provinces ...	24,845,106	310,908	881,926	... 571,018	293,393	818,107	...	...	524,714

in my investigation for purposes of age-distribution, the Buddhists only, who comprise about 86 per-cent of the whole population, and are understood to be little affected by migration. In the United Provinces, there was in 1911 a balance of emigrants equal to about 2 per-cent of the mean population, the proportion differing only slightly from that observed in 1901. In Madras, there was also a balance of emigrants, equal in 1911 to rather less than 2 per-cent of the mean population, a rate which was in approximate agreement with that recorded in 1901. Allowance was made for emigration in these two areas (Madras and the United Provinces), as explained later on. In Bengal (including Eastern Bengal and Bengal, Assam), the excess of immigrants was slightly over 1 per-cent, both in 1901 and 1911, and in the Punjab (including Punjab, the North-west Frontier Province), the excess of immigrants was rather more than 1 per-cent in 1901, and rather less in 1911. In these two Provinces (Bengal and the Punjab) the effect of migration on the estimated age-distribution was evidently negligible, and no correction was, therefore, made in the Census Bombay figures. In Bombay there was an excess of male immigrants of rather more than 2 per-cent of the mean population in 1911, whilst in 1901 the rate was rather more than 1 per-cent. No allowance was made by Mr. Hardy, in the adjustment of the 1901 figures, for the net immigrant population in Bombay, and its effect would certainly not have appreciably affected the age-distribution, as then deduced. The average rate of net immigration in Bombay, over the decennium preceding the 1911 Census, would be about  $1\frac{1}{2}$  per-cent of the mean male population, a rate which, again, would not affect the age-distribution at all appreciably. No correction has, therefore, been made for migration in the 1911 Census figures for Bombay.

Adjustment  
for Migration  
in Madras and  
United Provinces 31. As regards the Province of Madras, and the United Provinces, where adjustments have been made in the population figures for emigration, the matter has been dealt with, very much on the lines adopted by Mr. Hardy in his report on previous Censuses. Mr. Hardy investigated in 1891 the mortality in three Provinces (Coorg, Lower Burma and Berar), where the immigrant population was relatively very numerous, and deduced an estimated age-distribution for the immigrants included in these three Provinces. The following Table F, and explanatory Note, extracted from Mr. Hardy's 1891 Report, will show clearly the methods followed in this respect :

*Estimated Ages of Immigrants and Emigrants (Males).*  
(Extracted from Mr. G. F. Hardy's Report on the 1891 Census.)

\* Allowing for additional rate of increase of 6 per-cent her decennium, estimated to obtain in the native population of the three Provinces.

"Columns (2) (3) and (4) show the male population corresponding to a native population of 10,000 in each of the three Provinces, giving a total for the three of 38,628, of whom 30,000 are natives, and 8,628 immigrants. The further columns show the estimated age-distribution of the former number, based upon an average of the Indian Provinces, but allowing for an annual increase of 6 per mille in excess of the rate in India generally. The subtraction of these 30,000 natives leaves us with the 8,628 immigrants, distributed over the various age-groups as shown in column (9). This age-distribution may be assumed to apply, with such approximation as the data will permit, to the migrant population in India generally."

Having thus obtained an assumed age-distribution for the emigrant population, it was easy to apply this to the known numbers of emigrants in Madras and the United Provinces, respectively, and so deduce a mean population Table, corrected for migration, and reduced to a total of 100,000 male lives at all ages.

32. I have followed the same method, with an estimated age-distribution for the migrant population approximating closely to that deduced by Mr. Hardy. The figures of this age-distribution, and their application to the mean population in Madras and the United Provinces (the only Provinces dealt with) are shown in Table G.

It seems most desirable that the full particulars returned in the volume of Census Tables, as to the migrant population, should be supplemented, at future Censuses, by information as to the age-groups in which the emigrants and immigrants are respectively included. This would largely add to the value of the figures given, and would obviate the necessity of deducing an assumed age-distribution for the migrant population by approximate calculations.

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*Showcasing the effect of Migration on the age distribution, in the Province of Madras, and in the United Provinces. (Male Lives).*

UNITED PROVINCES									
Estimated Percentage of Migrant Population according to Age	Age Groups	MADRAS				AGE GROUPS			
		Mean Numbers 1901 & 1911 Censuses corrected for Age (Males)	Net Emigrants per 100,000 of Population	Mean Numbers corrected for Emigration	Corrected Numbers reduced to 100,000	Mean Numbers 1901 & 1911 Censuses corrected for Age (Males)	Net Emigrants per 100,000 of Population	Mean Numbers corrected for Emigration	Corrected Numbers reduced to 100,000
0-4	0	14,998	... 18	14,998	14,739	13,493	13,493	13,207	0-4
5-9	1	13,737	70	13,755	13,517	12,618	12,670	12,402	5-9
10-14	4	11,122	158	11,192	10,999	11,495	11,581	11,336	10-14
15-19	9	10,139	158	10,297	10,119	9,976	10,170	9,954	15-19
20-24	14	7,558	246	7,804	7,669	8,887	303	9,190	20-24
25-29	19	8,776	334	9,110	8,953	9,466	412	9,878	25-29
30-34	15	6,574	261	6,838	6,720	7,209	325	7,534	30-34
35-39	11	6,902	193	7,095	6,972	7,111	238	7,349	35-39
40-44	9	5,237	158	5,395	5,302	5,633	195	5,828	40-44
45-49	7	4,609	123	4,732	4,650	4,558	151	4,709	45-49
50-54	5	3,352	88	3,440	3,381	3,376	108	3,484	50-54
55-59	3	2,079	53	2,732	2,685	2,489	65	2,554	55-59
60-64	2	1,944	35	1,979	1,945	1,796	43	1,839	60-64
65-69	1	943	18	961	914	760	22	782	65-69
70 and over	0	1,430	...	1,430	1,405	1,403	...	1,103	70 and over
	100	100,000	1,758	104,758	100,000	100,000	2,164	102,164	100,000

*Birth-Place and Race.* 33. Before passing from this subject, I may add that the population of India, as enumerated at the Census dates, includes an overwhelming proportion of native lives. The figures for the 1911 Census are not yet fully available, but it appears from the 1901 returns that the numbers born in India, in European countries, and elsewhere, were as follows, respectively :

BIRTH PLACE	TOTALS	MALES	FEMALES
India ... ... ...	292,793,356	149,036,214	143,757,142
European Countries	104,853	87,992	16,591
Elsewhere (or not specified)	1,462,847	827,618	635,499
Native Indians (per 10,000)	294,361,056 9,947	149,951,824 9,939	144,409,232 9,955

The following figures shew the enumeration, by race, in 1901, of Europeans and Eurasians included in the Census :

RACE	TOTALS	MALES	FEMALES
British ... ... ...	154,691	112,687	42,004
Other European (including Armenian) ... ...	14,986	9,909	5,077
Eurasians ... ... ...	87,030	43,829	43,201
Proportion per 10,000 of total Population ...	256,707 9	166,425 11	90,282 6

It will be seen from these figures that the enumerated population on which the present investigation is based, may be considered, for all practical purposes, as consisting entirely of native Indians.

#### FEMALE LIVES.

*Female Lives.* 34. I have, so far, dealt only with the age-distribution and graduation, and the methods followed in deducing the rates of increase, and of mortality, for male lives. As regards female lives, it is evident, for reasons which are fully stated in the

reports on previous Indian Censuses, that the data are extremely defective. The age-distribution appears to be quite untrustworthy, and is certainly affected seriously by inaccuracies in the age returns. It also appears to me probable that some of the anomalies in the figures as returned must arise from omissions of data in certain Provinces, although this cannot be certainly determined. It has, therefore, appeared to me that any elaborate and detailed investigation of the female data, as recorded, would not be worth the trouble taken in making it, and would not be likely to produce results which could be considered as even approximately accurate, or as indicating the true rates of mortality at the several ages throughout life. I have, therefore, followed the plan, adopted at previous Censuses, of taking the adjusted male numbers living as a base line, and deducing therefrom estimated numbers of female lives, having regard to the relative birth-rates, and the proportion of female lives, relatively to male lives, assumed to be in existence at each age. For this purpose, I have compared, in each Province, the male and female population in grouped ages, and thus deduced the number of female lives recorded in each group, corresponding to 10,000 males. Taking, then, the proportion of registered female births to a thousand male births registered in each Province, which varied in the decennium ending 1911 from 902 in the Punjab (including the North-west Frontier Province) to 958 in Madras, with some allowance for unregistered births, and having regard to the ratios indicated in the successive age-groups in each Province, I was able to draw smooth curves, representing, from birth to the end of life, the assumed ratio of female to male lives; and these graduated ratios, given in Table VII appended, for all Provinces (except the Punjab) being applied to the adjusted numbers living at each age for male lives, rates of mortality were deduced for female lives, which are given for the several Provinces in the Tables appended to my Report, and in which the anomalies arising from defective and inaccurate female data may be presumed to be to some extent eliminated. It need hardly be added that the method followed can only be regarded as approximate, and that the resulting mortality Tables for female lives, for each Province, cannot be considered as anything like so trustworthy as those given for male lives.

difference between the ratios at successive ages have been taken out in each Province, and it will be noted that, where these differences are positive in sign, the female mortality is superior to (that is, lighter than) the male mortality, whilst, where the differences are negative, the female mortality is inferior to (that is, heavier than) that of male lives. In the following Table H, the groups of ages are shown, in each Province, in which the estimated female mortality is greater than, equal to, or less than, the male mortality.

TABLE H.

*Comparison of Estimated Female and Male Mortality.*

Province	MORTALITY OF FEMALE LIVES		
	Less than	Equal to	More than
that of Male Lives, in the following Age-groups :			
Bengal ... ... ...	0- 9, 33-end	Ages 10 and 32	11-31
Bombay ... ... ...	0-16, 37-end	Ages 17 and 36	18-35
Burma ... ... ...	0-23, 46-end	Age 45	24-44
Madras ... ... ...	0- 6, 31-end	—	7-30
United Provinces ...	0- 8, 18-end	—	9-17

Having regard to the method by which these ratios were deduced and graduated, too much weight must not be given to those indications in particular Provinces or age-groups, but the general trend, over the whole, as indicating a lighter mortality for female, as compared with male lives, in the early years, and after middle life, with a heavier mortality in the intermediate years, appears to be well marked, and unmistakeable. The ratios deduced for the United Provinces (where the mortality of the decennium has been exceptional) are somewhat abnormal, as indicating a heavier female mortality from the early age of 9 years, with a lighter female mortality from age 19 throughout the remainder of life.

**Punjab.** 36. As regards the Punjab, I made experimental calculations, with a view to deducing the ratio of the female to the male lives, as above, and thence a table of mortality for female lives at all ages. I found, however, that there were such grave irregularities in the ratios between male and female lives, that it was practically impossible to deduce a female mortality

table which could be regarded as even approximately representing the facts, the progression of the rate of mortality, in an experimental table deduced for female lives, being unduly rapid up to about age 12, with an abnormal retardation at the following ages, up to about age 25. These irregularities, no doubt, arise partly from material defects in the data supplied as to female lives, and are also, no doubt, affected by similar defects of less marked character in the figures for male lives, as well as by the heavy visitations to which lives of both sexes in the Punjab has been exposed in the decennium. I was ultimately driven, reluctantly, to the conclusion that no useful purpose could be served by publishing the mortality table for female lives in the Punjab, and have thus had to follow, in this respect, the course adopted by Mr. Hardy in his Report on the 1901 Census.

#### GRADUATED MORTALITY TABLES.

Provincial  
Mortality  
Tables.

37. The rates of mortality given in the following Tables J and K are extracted from the full graduated mortality Tables, for each Province, for males lives, and for each Province (except the Punjab) for female lives. The tables appended to my Report to the Indian Government include, at every age, (1) the numbers living ( $l_x$ ) at each age  $x$ , to a radix of 100,000; (2) the numbers dying ( $d_x$ ) between ages  $x$  and  $(x+1)$ ; (3) the mortality per-cent at each age ( $q_x$ ) deduced from (1) and (2); (4) the numbers living ( $L_x$ ) between ages  $x$  and  $(x+1)$ ; (5) the number living above age  $x$  ( $T_x$ ), deduced by summation of column (4) from the oldest age to age  $x$  inclusive; and (6) the complete expectation of living, deduced by dividing the figures in column (5) by those in column (1), regard being had, at the oldest ages, to the fractional part of the numbers living, omitted from column (1).

TABLE J.

## MALE LIVES.

*Estimated Rates of Mortality ( $q_x$ ).*

Age	Bengal Presidency	Bombay Presidency	Burma	Madras Presidency	Punjab	United Provinces	Age
0	.2986	.2969	.2234	.2739	.2979	.2979	0
1	.0951	.0924	.0637	.0854	.0939	.0939	1
2	.0690	.0659	.0444	.0615	.0677	.0676	2
3	.0511	.0477	.0318	.0450	.0497	.0496	3
4	.0385	.0349	.0232	.0335	.0370	.0370	4
5	.0297	.0260	.0174	.0255	.0281	.0281	5
6	.0237	.0200	.0136	.0201	.0221	.0221	6
7	.0198	.0159	.0110	.0164	.0170	.0181	7
8	.0171	.0132	.0093	.0140	.0146	.0154	8
9	.0153	.0115	.0082	.0124	.0126	.0137	9
10	.0142	.0103	.0075	.0114	.0122	.0126	10
15	.0151	.0095	.0077	.0108	.0177	.0127	15
20	.0182	.0153	.0115	.0115	.0204	.0193	20
25	.0206	.0212	.0155	.0131	.0230	.0252	25
30	.0234	.0263	.0182	.0164	.0262	.0294	30
35	.0274	.0309	.0191	.0212	.0304	.0330	35
40	.0324	.0356	.0204	.0271	.0361	.0365	40
45	.0377	.0406	.0232	.0319	.0418	.0406	45
50	.0438	.0476	.0273	.0372	.0462	.0462	50
55	.0514	.0544	.0325	.0433	.0514	.0541	55
60	.0630	.0646	.0451	.0506	.0580	.0652	60
65	.0831	.0789	.0657	.0610	.0668	.0805	65
70	.1180	.1001	.0936	.0778	.0876	.1027	70
75	.1786	.1346	.1356	.1070	.1415	.1352	75
80	.2832	.1974	.1974	.1638	.2204	.1954	80
85	.4482	.3116	.2858	.2722	.3623	.3304	85
90	.6573	.4872	.4049	.4574	.5839	.5864	90

TABLE K.

## FEMALE LIVES.

*Estimated Rates of Mortality ( $q_x$ ).*

Age	Bengal Presidency	Bombay Presidency	Burma	Madras Presidency	United Provinces	Age
0	.2976	.2958	.2207	.2570	.2975	0
1	.0939	.0910	.0604	.0678	.0934	1
2	.0679	.0645	.0410	.0472	.0672	2
3	.0501	.0464	.0283	.0344	.0493	3
4	.0376	.0336	.0198	.0267	.0366	4
5	.0290	.0248	.0142	.0234	.0279	5
6	.0232	.0188	.0105	.0206	.0220	6
7	.0194	.0149	.0082	.0182	.0180	7
8	.0169	.0122	.0067	.0162	.0155	8
9	.0154	.0106	.0058	.0145	.0139	9
10	.0144	.0096	.0053	.0134	.0128	10
15	.0159	.0094	.0066	.0124	.0129	15
20	.0192	.0156	.0112	.0125	.0189	20
25	.0211	.0218	.0158	.0137	.0241	25
30	.0236	.0268	.0193	.0164	.0280	30
35	.0270	.0309	.0202	.0205	.0311	35
40	.0316	.0348	.0209	.0260	.0349	40
45	.0365	.0393	.0231	.0301	.0394	45
50	.0420	.0450	.0266	.0348	.0450	50
55	.0492	.0523	.0312	.0403	.0527	55
60	.0609	.0621	.0436	.0475	.0634	60
65	.0811	.0766	.0641	.0581	.0786	65
70	.1163	.0980	.0920	.0753	.1002	70
75	.1771	.1329	.1341	.1047	.1336	75
80	.2821	.1960	.1962	.1619	.1941	80
85	.4474	.3108	.2849	.2708	.3295	85
90	.6569	.4867	.4043	.4564	.5860	90

## COMPARISON OF EXPECTATIONS OF LIFE.

Provincial,  
Indian and  
English  
expectations.

38. In the appended Tables L and M, I give the adjusted expectations of life for male and female lives, as deduced from the above mortality tables, in each

Province, and for all India, with the corresponding expectations as deduced by Mr. Hardy in 1881, 1891, and 1901; and I have added the expectations of life, deduced for English male and female lives in 1901 and 1911, the former being taken from English Life Table No. 6, and the latter from tables computed, on the basis of recent population statistics, by the Actuarial Advisory Committee, appointed under the National Insurance Act. These 1911 Life Tables are not published for ages younger than 15, and the expectations, in the appended Tables, at birth, and at age 10, have been based on the mortality shown, up to age 15, by the English Life Table No. 6. In comparing the values of the Indian expectation of life now deduced with those estimated in 1891 and 1901, regard must be given to the fact that the decennium ending 1891 was free from famine or severe visitations, whilst the figures for 1901 were deduced so as largely to eliminate exceptional causes of mortality. The period 1901 to 1911, having been characterized by severe attacks of plague and famine in certain areas, may be considered, generally speaking, as representing an inferior vitality, as compared with that shown by either of the previous tables referred to.

*Male Lives.* 39. For male lives, the expectations of life in Bengal, Madras, and the Punjab are lower than those estimated in 1891 and 1901, at practically all ages, and in Burma are higher at birth, but lower at all ages, than those of 1901. In Bombay, there is an inferior vitality in the last decennium, as compared with previous periods, at ages 0 to 30, and a superior vitality at ages 60 to the end of life, whilst at ages 40 and 50, the expectations of life lie between those of 1891 and 1901. In the United Provinces, the expectations at ages 0 to 30, in the last decennium, are below those of the previous periods, whilst at ages 40 and 50, and 80 and 90, they lie between those of 1891 and 1901, and at 60 and 70 are higher than those of previous periods.

TABLE L.

*Comparative Expectations of Life at decennial ages, as deduced from the results of the 1891, 1901 and 1911 Censuses respectively, in the several Provinces specified, and over the combined area, with corresponding values for England.*

## MALE LIVES.

Age	BENGAL PRESIDENCY			BOMBAY PRESIDENCY			MADRAS PRESIDENCY			Age
	1891	1901	1911	1891	1901	1911	1891	1901	1911	
0	22.78	21.57	21.47	26.12	22.77	22.52	26.92	26.21	25.92	0
10	33.85	32.95	32.54	37.20	34.62	33.33	38.70	36.93	37.78	10
20	27.77	27.50	27.10	30.87	28.39	26.43	32.55	30.43	31.60	20
30	22.51	22.64	22.15	24.67	22.27	21.32	26.57	24.24	25.35	30
40	17.98	18.28	17.56	18.94	16.90	17.23	21.06	18.60	20.06	40
50	13.83	13.93	13.39	13.88	12.48	13.51	15.91	14.05	15.74	50
60	9.89	9.52	9.27	9.59	8.73	9.94	11.06	10.10	11.70	60
70	6.35	5.61	5.40	6.05	5.38	6.55	6.94	6.27	7.68	70
80	3.59	2.86	2.49	3.39	2.81	3.48	3.85	3.35	3.98	80
90	1.69	1.07	.95	1.65	1.07	1.41	1.82	1.56	1.50	90

Age	UNITED PROVINCES			PUNJAB			BURMA			Age
	1891	1901	1911	1891	1901	1911	1891	1901	1911	
0	24.45	25.30	21.21	26.58	23.18	21.23	...	30.29	31.48	0
10	34.10	35.26	31.44	38.07	35.45	31.38	...	39.39	39.88	10
20	27.75	28.43	25.27	31.76	29.59	26.12	...	33.28	32.82	20
30	22.35	22.01	20.89	25.60	24.54	21.60	...	27.68	27.30	30
40	17.74	16.76	17.18	20.22	19.99	17.55	...	22.58	22.04	40
50	13.56	12.64	13.47	15.56	15.43	14.15	...	17.45	16.51	50
60	9.63	8.92	9.84	11.41	10.70	10.63	...	12.18	11.00	60
70	6.15	5.50	6.50	7.60	6.39	6.53	...	7.37	6.66	70
80	3.43	2.96	3.42	4.48	3.28	3.11	...	3.84	3.61	80
90	1.60	1.23	1.11	2.26	1.38	1.13	...	1.75	1.77	90

Age	ALL INDIA			ENGLAND	
	1891	1901	1911	1901	1911
0	24.59	23.63	22.59	44.07	*46.04
10	35.46	34.73	33.36	49.65	*52.35
20	29.24	28.59	27.46	41.04	43.67
30	23.66	22.90	22.45	33.06	35.29
40	18.75	17.91	18.01	25.65	27.27
50	14.28	13.59	13.97	18.89	19.85
60	10.12	9.53	10.00	12.90	13.38
70	6.48	5.80	6.19	8.02	8.25
80	3.65	3.07	3.06	4.40	4.64
90	1.69	1.23	1.15	2.32	2.37

\*Estimated values.

TABLE M.

*Comparative Expectations of Life at decennial ages, as deduced from the results of the 1891, 1901 and 1911 Censuses, respectively, in the several Provinces specified, and over the combined area, with corresponding values for England.*

## FEMALE LIVES.

Age	BENGAL PRESIDENCY			BOMBAY PRESIDENCY			MADRAS PRESIDENCY			Age
	1891	1901	1911	1891	1901	1911	1891	1901	1911	
0	23.73	22.51	21.58	27.07	24.05	22.86	27.99	27.13	27.65	0
10	32.76	32.03	32.44	36.15	33.69	33.50	37.78	36.27	37.62	10
20	27.76	27.55	27.20	30.92	28.52	26.54	32.78	30.65	32.02	20
30	23.52	23.86	22.45	25.69	22.98	21.57	27.90	25.06	26.01	30
40	19.43	19.99	17.91	20.31	17.78	17.60	22.78	19.56	20.73	40
50	15.16	15.14	13.67	15.07	13.37	13.81	17.41	15.03	16.28	50
60	10.65	10.18	9.40	10.24	9.30	10.13	11.89	10.86	12.00	60
70	6.68	5.87	5.43	6.33	5.58	6.62	7.28	6.60	7.79	70
80	3.70	2.95	2.48	3.47	2.92	3.49	3.97	3.51	4.00	80
90	1.59	1.31	.95	1.59	1.20	1.42	1.85	1.77	1.50	90

Age	UNITED PROVINCES			BURMA			Age
	1891	1901	1911	1891	1901	1911	
0	25.25	23.93	21.50	...	32.21	32.61	0
10	32.97	34.90	31.94	...	38.92	40.22	10
20	27.71	28.89	25.88	...	32.98	32.67	20
30	23.31	23.33	21.42	...	28.96	27.21	30
40	19.15	18.38	17.51	...	24.62	22.24	40
50	14.85	13.82	13.69	...	19.00	16.75	50
60	10.36	9.52	9.99	...	13.16	11.15	60
70	6.45	5.74	6.56	...	7.77	6.72	70
80	3.54	3.02	3.43	...	3.96	3.63	80
90	1.65	1.50	1.06	...	1.83	1.77	90

Age	ALL INDIA			ENGLAND	
	1891	1901	1911	1901	1911
0	25.54	23.96	23.31	47.70	*50.02
10	34.40	33.86	33.74	51.98	*55.02
20	29.28	28.64	27.96	43.45	46.36
30	24.69	23.82	22.99	35.43	37.84
40	20.20	19.12	18.49	27.81	29.65
50	15.59	14.50	14.28	20.63	21.87
60	10.87	10.02	10.11	14.08	14.81
70	6.80	5.98	6.22	8.74	9.13
80	3.76	3.12	3.06	4.84	5.10
90	1.75	1.64	1.10	2.68	2.55

\*Estimated values.

**Female Lives.**

40. For female lives, the expectations in the last decennium, in Bengal, are below those of previous periods, at practically all ages, and in Burma are lower than those of 1901, at ages 20 and upwards, but higher at birth, and at age 10; whilst in the United Provinces there are lower expectations up to age 50, and those at higher ages lie substantially between the values of 1891 and 1901. In Bombay, the expectations are lower than in previous periods, up to age 40, and at practically all higher ages lie between the values of 1891 and 1901. In Madras, the expectations lie between those of 1891 and 1901, up to age 50, and are higher than either at practically all older ages.

**All India and England. (Males and Females).**

41. The estimated expectations in 1911 for all India, lie, for male lives, below those of 1891 and 1901 at ages 0 to 30, but are between those of 1891 and 1901 up

to age 70; whilst for female lives, the expectations are lower than those of 1891 and 1901 at practically all ages. The Indian rates for male and female lives are, as might be anticipated, materially below those deduced from English lives, both in 1901 and 1911, at all ages, the Indian expectations at birth being, in 1911, 22.59 years for males, and 23.31 for females, and the English 46.04 years for males, and 50.02 for females, the difference diminishing at higher ages, but being quite marked throughout life. The expectations for female lives in all India are only slightly higher than for male lives, at all ages, the excess being 0.72 years at birth, diminishing to 0.11 at age 60; whilst in England, the superior expectation of female lives is 3.98 years at birth, and 1.43 years at age 60.

**Average Birth and Death Rates.**

42. In Table N, I have attempted to deduce the estimated average death rates and birth rates for male lives in each Province, by methods similar to those adopted by Mr. Hardy in 1901. In columns (2) and (3) are set out the mean population *in registration areas* in each Province, in 1901 and 1911 respectively; and, in column (4) the estimated number in 1911, age 10 and over, deduced from the Census figures for that year, corrected for age, as given in Table IV. After making a suitable correction, in column (5), for migration, the estimated deaths in the population during the decennium (excluding those born in the period) are given in column (6). These deaths may be considered as approximately representing those arising in the decade at age 5 and upwards, and are compared with those registered at 5

TABLE N.  
*Male Lives.*

Showing the number of Registered Deaths at all ages, and the estimated total number of deaths, also the registered Death rate and the estimated Birth and Death rates in each Province as derived from the Population in Registration areas, and the estimated Rates of Mortality, with allowance for Migration.

Province	Average Population under Registration, 1911 Census, Males	Average Population under Registration, 1911 Census, Males	Estimated numbers in last column, aged 10 and over		Deaths in 10 years out of numbers in Col. 2, adjusted for migration during decennium (2) + (5) - (4)	Registered Deaths aged 5 years and upwards, 1901-1911 (2) + (5) - (4)	Registered Deaths at all ages, 1901-1911 (1)	Estimated Deaths at all ages (6) x (8) (7)	Estimated Death rate per 1,000 on mean population (9) 5[(2) + (3)] 10 x (2)	Registered death rate per 1,000 1901-1911 (8) 10 x (2)	Estimated birth rate per 1,000, 1901-1911 (10)
			(2)*	(3)*							
Bengal ...	... 40,273,9,563,	43,198,10,215,	30,068,7,404,	+ 143, + 149,	10,348,2,298,	8,705,2,144,	14,060,3,304,	16,714,3,541,	40·0 35·8	34·9 34·6	46·7 41·0
Bombay ...	... 4,408,	5,054,	5,645,	+ 166,	929,14,692,	671,4,018,	1,118,2,665,	1,148,6,548,	32·7 33·4	25·4 23·0	42·9 41·9
Burma ...	... 18,851,	20,383,	14,692,	- 141,	12,035,	8,865,	3,181,2,884,	4,752,5,241,	43·3 46·0	39·5 38·9	44·3 46·5
Madras ...	... 12,035,	12,158,	8,865,	+ 11,	24,617,	18,211,	6,302,5,331,	9,580,11,325,			
Punjab ...	... 24,617,			- 104,							
United Provinces											
Totals ...	109,737,	115,650,	82,885,	+ 224,		27,076,	22,400,	37,157,	44,917,	39·9	33·9

\* Omitting thousands.

years of age and upwards, set out in column (7). In column (8) are given the deaths registered at all ages, and increasing now the estimated deaths at age 5 and upwards in column (6), in the proportion of the numbers in column (7) to those in column (8), we arrive, in column (9), at the estimated deaths at all ages, from which the death rate per 1,000 of mean population is deduced for each Province in column (10), whilst, in column (11), the average registered death rates are set out for comparison. The defects in registration of deaths, indicated by a comparison of columns (8) and (9) or columns (10) and (11), are on the whole much reduced, as compared with the corresponding figures deduced by Mr. Hardy in 1901, which were as follows :

(Extracted from Mr. G. F. Hardy's 1901 Report).

Province.	Registered Deaths at all Ages	Estimated Deaths at all Ages	Registered Death Rate per 1,000	Estimated Death Rate per 1,000
Bengal ... ...	11,933,000	13,930,000	34·1	38·9
Bombay ... ...	3,507,000	4,421,000	36·0	45·9
Madras ... ...	3,654,000	6,403,000	22·5	38·1
North-west Provinces	8,141,000	10,624,000	33·5	43·4
Punjab ... ...	3,668,000	4,609,000	33·2	40·3
Totals ... ...	30,903,000	39,987,000	32·1	40·7

It will be seen that the proportion of registered to estimated deaths in 1891–1901, deduced by the above process, comes out at about 75 per-cent over the five Provinces combined, as compared with about 83 per-cent brought out for all India in 1901–11, by substantially the same process. An estimate of the birth rate in each Province, deduced from the known rate of increase of the population in the decennium, and the deaths registered, after allowing for immigration, and for the estimated defects in registration shewn in the Tables, is set out in column (12) of Table X.

Comparison of the estimated number of deaths in Registration areas, deduced from the graduated rates of mortality, 1901-1911, with the deaths actually registered in each age-group in each Province

## MALE LIVES.

Bengal Presidency										Bomby Presidency										Burma										Madras Presidency										All India																																																																																																																																											
Estimated Deaths 1901-1911					Estimated Deaths 1901-1911					Estimated Deaths 1901-1911					Estimated Deaths 1901-1911					Estimated Deaths 1901-1911					Estimated Deaths 1901-1911					Estimated Deaths 1901-1911					Estimated Deaths 1901-1911																																																																																																																																																
Age Groups	Total Deaths Registered to 1901-1911 *				Percentage of Registrations to Estimated Deaths				Total Deaths Registered to 1901-1911 *				Percentage of Registrations to Estimated Deaths				Total Deaths Registered to 1901-1911 *				Percentage of Registrations to Estimated Deaths				Total Deaths Registered to 1901-1911 *				Percentage of Registrations to Estimated Deaths				Total Deaths Registered to 1901-1911 *				Percentage of Registrations to Estimated Deaths																																																																																																																																														
	Per 100,000 of Mean Population	On Mean Population Areas *	Per 100,000 of Mean Population	Total Deaths Registered to 1901-1911 *	Per 100,000 of Mean Population	On Mean Population Areas *	Total Deaths Registered to 1901-1911 *	Per 100,000 of Mean Population	On Mean Population Areas *	Per 100,000 of Mean Population	On Mean Population Areas *	Total Deaths Registered to 1901-1911 *	Per 100,000 of Mean Population	On Mean Population Areas *	Total Deaths Registered to 1901-1911 *	Per 100,000 of Mean Population	On Mean Population Areas *	Total Deaths Registered to 1901-1911 *	Per 100,000 of Mean Population	On Mean Population Areas *	Total Deaths Registered to 1901-1911 *	Per 100,000 of Mean Population	On Mean Population Areas *	Total Deaths Registered to 1901-1911 *	Per 100,000 of Mean Population	On Mean Population Areas *	Total Deaths Registered to 1901-1911 *	Per 100,000 of Mean Population	On Mean Population Areas *	Total Deaths Registered to 1901-1911 *	Per 100,000 of Mean Population	On Mean Population Areas *	Total Deaths Registered to 1901-1911 *	Per 100,000 of Mean Population	On Mean Population Areas *																																																																																																																																																
0	16,669	6,957	3,256	46.8	15,597	1,512	665	43.1	9,656	9,656	457	309	67.6	12,755	2,502	1,086	43.4	0	1-4	8,871	3,702	2,098	56.7	7,977	496	5,081	240	138	5,911	43.4	1-4	2,372	1,195	1,708	88.6	887	523	438	2,333	2,191	56.8	5-9	10-14	1,650	689	1,089	166	1,537	807	38	44	115.8	1,224	240	1,022	67.5	10-14	15-19	1,726	720	602	83.6	1,211	120	142	93.4	44	49	1,137	223	148	66.4	15-19	20-29	3,485	1,455	1,309	90.0	3,709	367	91.8	2,556	121	101	83.5	2,294	450	314	69.8	20-29	30-39	3,416	1,426	1,312	92.0	4,081	403	346	85.9	118	105	89.0	2,778	545	351	64.4	30-39	40-49	3,259	1,360	1,113	81.8	3,598	356	306	96.0	2,162	102	85	83.3	2,974	583	373	64.0	40-49	50-59	2,809	1,172	968	82.6	2,804	277	269	97.1	2,022	96	71	74.0	2,573	505	380	75.2	50-59	60-end	3,938	1,643	1,513	92.1	3,275	324	379	117.0	4,180	198	149	75.3	3,787	743	688	92.6	60-end	Totals	48,686	20,319	14,057	69.2	45,596	4,508	3,301	73.3	31,421	1,486	1,119	75.3	38,702	7,592	4,312	57.2	Totals
0	14,321	1,732	1,119	64.6	13,909	3,426	2,636	76.9	7,444	1,833	1,613	88.0	81.0	2,385	104.2	587	576	98.1	237	1,340	120.9	237	196	1,341	110.3	1,608	396	307	77.5	1,608	1,089	742	74.9	1,971	81.7	1,608	396	307	77.5	1,435	1,741	1,445	82.9	15-19	1,995	241	197	81.7	4,422	1,089	749	68.8	3,833	78.3	4,422	1,089	749	68.8	7,329	8,852	5,686	64.2	1-4	5-9	10-14	1,621	196	1,341	110.3	1,608	396	307	77.5	1,288	1,601	1,681	105.0	15-19	20-29	4,047	4,489	3,833	78.3	4,531	1,116	746	60.8	3,791	73.9	4,101	3,239	79.0	30-39	30-39	4,079	4,933	3,791	76.9	3,886	80.5	957	776	81.1	100.4	3,290	3,825	3,029	79.2	40-49	40-49	3,859	4,677	3,76	81.1	3,171	95.3	344	344	781	784	100.4	100.4	2,727	3,192	2,816	88.2	50-59	50-59	2,981	351	344	95.3	1,092	1,030	94.3	130.6	4,433	1,092	1,030	94.3	3,976	4,513	4,429	98.1	60-end	60-end	4,241	513	513	100.4	130.0	4,433	1,092	1,030	94.3	3,976	4,513	4,429	98.1																				

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*Bengal Presidency*  
*Bombay Presidency*  
*Burma*

43. These methods are confessedly only approximate, and are based upon assumptions which may, I think, materially affect the accuracy of the results ; and I have therefore (since the completion of my Report to the Indian Government) made experimental calculations on somewhat different lines. I have ascertained, from the recorded male and female mean population in age-groups in registration areas, the annual deaths which would arise, in each quinary age-group, if the adjusted rates of mortality as given in Table J and K were in operation in each Province. These estimated deaths are compared, in Tables O and P, with those registered at each age-group in each Province during the decennium. It will be seen that the percentage of registered to estimated deaths on this basis varies for male lives (Table O) from 57·2 in Madras to 83·3 in the Punjab, and averages 72·5 for all India ; whilst, for female lives (Table P), the percentage varies from 57·7 in Madras to 86·4 in the United Provinces, and averages 70·3 for all India.

A comparison of the percentages of registered to estimated deaths at quinary age-groups in each Province shows a general progression in the ratios to a maximum at group 10–14 for male lives, or at 20–29 for female lives, after which the ratios fall to about age 45, and rise steadily (except in Burma) at older ages. These variations in the ratios indicate the probability of a progressive overstatement of ages at death in early life (up to ages 10–14 for male lives, and 15–29 for female lives), and at older ages (above 45), with an understatement of age for both sexes at the intermediate ages (15–44 for male lives, and 29–44 for female lives).

44. It would thus appear that the ages at death are mis-stated in a similar way to the ages in the Census returns ; thus, a number of deaths have obviously been transferred from age 0 to age-group 1–4, and from that group to age-group 5–9, and the same tendency is noticeable, though to a smaller extent, as regards age-groups 5–9, and 10–14. The registered deaths in column (7) of Table N are, therefore, doubly over-stated, in comparison with the total registered deaths in column (8), firstly, as being more completely registered (as compared with deaths at ages 0–4), and secondly, as including certain deaths of persons at younger ages ; but, on the other hand, a certain addition should be made to the figures in column (7), in order to make them comparable with those in column (8), as the latter include deaths at infantile ages at the beginning of the

Errors of age,  
as registered.

decennium, which, owing to the heavy rates of mortality at these ages, outweigh the deaths at ages over 5, omitted at the end of the period. I find that, making such assumptions as appear reasonable in these respects, the neglect of this adjustment practically neutralizes the effect of the overstatement above referred to, and the rates in column (10) and (12) of Table N may be regarded as fair approximations, although they are, if anything, probably below the true rates. This conclusion is borne out generally by the total estimated death rates (per 100,000) deduced for each Province in Table O, which are in every case, except Burma, higher than those deduced in column (10) of Table N; the estimated average death rate for the combined Provinces being 39·9 per 1,000 in Table N, and 43·1 in Table O.

45. As regards the average birth rates in each Province, I have deduced, in the following Table Q, estimated rates, based upon the movement of the whole population, as shown by the Census returns of 1901 and 1911, and the estimated deaths, deduced over the whole population on the basis of the graduated rates of mortality given in Table J. After making suitable allowances for migration, the estimated birth rates are deduced in column (7). It will be seen that these are higher in Bengal, Bombay, Madras, Punjab, and the United Provinces, and lower in Burma, than the estimated birth rates deduced on the basis of corrected registration figures, in column (12) of Table N.

TABLE Q.

*Shewing the number of Estimated Deaths and Births and the Birth rate in each Province, for Male Lives, deduced from the mean population 1901-11, and the estimated rates of mortality, with allowance for Migration.*

PROVINCE	Mean of Census Popula- tions, 1901-11	Increase in Popula- tion, 1901-11	Net Immigrants (+) or Emigrants (-)	Estimated Deaths	Estimated Deaths, Corrected for Migration (4)-(3)	Estimated Births (2)+(5)	Estimated Birth Rate per 1,000 (6)÷(1)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Bengal ... ...	44,339,	3,375,	+ 152,	21,587,	21,435,	24,810,	56·0
Bombay ... ...	13,554,	915,	+ 204,	6,180,	5,976,	6,891,	50·8
Burma ... ...	5,763,	841,	+ 202,	1,811,	1,609,	2,450,	42·5
Madras ... ...	19,825,	1,532,	- 143,	7,673,	7,816,	9,348,	47·2
Punjab ... ...	14,511,	849,	+ 13,	6,845,	6,832,	7,681,	52·9
United Provinces	24,845,	44,	- 105,	11,709,	11,814,	11,858,	47·7
Totals ...	122,837,	7,556,	+ 323,	55,805,	55,482,	63,038,	51·3

NOTE.—The thousands are omitted in columns (1) to (6) inclusive.

46. In Table R, I have brought together, for comparison, the estimated birth and death rates in each Province for male lives over the decennium 1901–11, as deduced from Tables N and Q, and N and O respectively; and I have added the average rates deduced by Mr. Hardy in 1891 and 1901, and the weighted averages of the Provincial rates thus deduced. Having regard to the approximate methods adopted in Table N, and the assumption upon which they are based, the results deduced from Table O (for the death rates) and from Table Q (for the birth rates) may be considered as probably nearer the truth than those of Table N.

47. The Life Table for all India, for male lives, is given in Table IX. The numbers living were, in

Life Table for India, 1911 (Males).

the first instance, deduced by weighting the values of  $l_x$  in the Provincial Tables according to the total male population of each Province, enumerated in 1911. I thought that it would then be desirable to see how far Makeham's modifications of Gompertz's law would represent the native Indian male mortality, and applied successfully a graduation based on his *second* modification, with the abnormally high value of  $\log_{10}c = -0.58742$  ( $c = 1.1448$ ), deduced from values of  $\log_{10}l_x$  at ages 30, 45, 60, 75, and 90, which reproduced, with remarkable fidelity, the numbers living at intermediate ages, but was not applicable at ages younger than 30. The closeness of the approximation is shown by the fact that, between the ages of 30 and 90, the maximum difference, at any individual age, between the weighted and the graduated values of  $l_x$  (to a radix of 100,000 at age 0) was –13, or +11, whilst the total of the differences at all ages was equal to –6, and the accumulated differences to –10. The closeness of these results will be less surprising, when it is remembered that the component Provincial Life Tables are themselves graduated by frequency curve formulas; but this indicates that these graduated figures followed a very smooth progression.

The values of  $\text{colog}_{10}(p_x)$ , and of  $\mu_x$ , for all India (males) on the Makeham graduation, were as follows:

$$\text{colog}_{10}(p_x) = -0.0010579 + 0.0003644x + 0.000001686c^x$$

$$\mu_x = -0.0020164 + 0.0008389x + 0.000003628c^x$$

The abnormally high value of  $c$ , of course, indicates an increasingly heavy rate of mortality at the older ages.

TABLE R.

*Comparison of Estimated Average Birth-rates, and Mortality Rates at all ages over 5, as derived for each Province, in each decennium during the period 1881-1911. Male Lives.*

Province	Birth RATES						MORTALITY RATES					
	At all Ages			At all Ages			At Ages over 5			At Ages over 5		
	1881-1891	1891-1901	1901-1911	1881-1891	1891-1901	1901-1911	1881-1891	1891-1901	1901-1911	1881-1891	1891-1901	1901-1911
Bengal ...	... 52.9	43.9	46.7	56.0	55.9	40.0	48.7	28.8	29.6	29.4	29.6	32.0
Bombay ...	... 50.3	43.9	41.0	50.8	36.4	45.9	35.8	45.6	21.2	33.2	27.1	30.0
Burma ...	... ...	... 42.9	42.9	42.5	... 47.2	... 38.0	32.7	31.4	... 33.4	... 27.2	23.1	21.8
Madras ...	... 51.3	44.8	41.9	41.9	47.2	38.4	38.4	38.7	20.5	27.2	24.0	25.9
Punjab ...	... 46.8	47.1	44.3	52.9	37.0	40.3	43.3	47.2	25.3	25.1	30.4	32.8
United Provinces ...	45.1	44.7	46.5	47.7	38.6	43.4	46.0	47.1	24.7	27.7	29.4	33.2
Combined Provinces ...	... 44.6*	44.9	51.3	... 44.6*	... 40.7*	39.9	43.1	... 28.9*	... 28.1	... 29.2	... 28.1	29.2

\* Weighted averages of Mr. Hardy's figures for the several Provinces.

48. The male Life Table for India, at ages younger than 30, represents the weighted Provincial figures, without further graduation. For ages 0 to 12 inclusive, the Provincial figures are deduced, as already explained, from Mr. G. F. Hardy's Proclaimed Clans mortality table, and the values of  $\mu_x$  at these ages were computed by a formula (see Appendix C (2)) obtained directly from Mr. Hardy's expression for the graduated values of  $l_x$ . For ages 13 to 29 inclusive, where the progression of the numbers dying is smooth, the values of  $\mu_x$  were obtained by the usual formula of approximation  $\mu_x = \frac{l_{x-1} - l_{x+1}}{2l_x}$ . It will be observed that the force of mortality at birth is equal to 4.3242, an abnormally high rate, which will be better appreciated if expressed as a rate of mortality (at the point of birth) exceeding 1 per-cent *per diem*.

49. I have set out in Table VIII the Life Table for Life Table for India, 1901 (Males) all India (males), as deduced by Mr. Hardy in 1901,

partly for comparison with the Table deduced ten years later, but chiefly because I consider that Mr. Hardy's 1901 Table (which was deduced from the Census Returns of 1881, 1891, and 1901, with doubled weight for the figures of 1891, the decennium preceding which was free from famine), is probably the best available exponent of the *average* mortality of the native male population, after eliminating, to a large extent, the effect of special and severe visitations. The 1911 Table must rather be regarded as representing the mortality obtaining in a decennium which, whilst by no means free from such abnormal disturbances, was not, on the whole, affected very seriously by them. The values of  $\mu_x$  for the 1901 Table were not given in Mr. Hardy's report, but have, for comparison, been computed, from age 13 to the end of life, by the approximate formula  $\mu_x = \frac{l_{x-1} - l_{x+1}}{2l_x}$ , and for ages 0 to 12 by the formula, deduced from the Proclaimed Clans Table, given in Appendix C (2).

50. It will be seen, by a comparison of the 1901 and 1911 tables for all India, that the male rate of mortality is higher, by the latter than by the earlier table, at all ages up to 48 inclusive, and from 82 to the end of life, but lower than the 1901 rates at the intermediate ages 50 to 81, inclusive; whilst the expectations of life are lower, in 1911, up to age 37 inclusive, and from 80 to the end of life, and higher, at the intermediate ages, 38 to 79 inclusive, than the 1901 expectations.

51. Mr. Hardy deduced a graduation, based on Makeham's *first* modification of Gompertz's law, of the 1901 figures by the special formula

$$\log N'_x = K + ax + bx^2 + mc^x$$

(an expression the third differences of which are in geometrical progression), where  $N'_x$  represented the "population above aged  $x$ ," reduced to a total of 100,000 at all ages, and  $\log_{10} c$  was taken as = .039; but it will be seen that this method and formula can hardly be compared with those adopted by me in 1911, which were based on equidistant values of  $\log_{10} l_x$ , and on the *second* modification of Gompertz's law. Mr. Hardy's graduation as above, was worked out from age 30 to the end of life, but was only applied, in the final Table, from age 55.

52. I have made some experimental graduations of the 1911 Table, taking  $\log T_x$  as a basis, and find that the effect of dealing with this series of a higher order, as it were, than  $l_x$ , is to reduce the rise in the rate of decrement, and therefore the value of  $\log_{10} c$ . Taking decennial values of  $\log_{10} T_x$ , from 10 to 80 inclusive, I find that the *fourth* differences are approximately in G.P., with a common ratio of  $c$ , where  $\log_{10} c = .0455$ , so that

$$\log T_x = K + ax + bx^2 + cx^3 + mc^x,$$

and that a fairly good graduation can be deduced by this formula, although not nearly so exact as that deduced, in Table IX, from values of  $\log_{10} l_x$ , with  $\log_{10} c = .058742$ .

53. The Female Life Table (1911) for all India is

Life Table for  
India 1911)  
Females.

given in Table X, the numbers living being deduced by weighting the value of  $l_x$  in the Provincial tables (omitting the Punjab) according to the total female population of each Province, in 1911. It will be seen, by comparison of Tables IX and X, that the female rates of mortality, as estimated, are lower than the male rates for ages 0-6 and 23-85, and higher than the male rates at the intermediate ages 7-20, and at ages above 85. The expectations of life are higher for female than for male lives at all ages to 82 inclusive, and lower from age 84 to the end of life.

54. Apart from the paper of Mr. Hardy, in Vol. xxv of the *Journal*, on his investigation of the 1881 Census, there appears to be little or nothing in our Transactions that has a direct bearing on the mortality

Other  
investigations of  
Indian mortality.

of the general population of India. I have referred to the paper by Mr. Chatham in the *Transactions* of the 1900 Actuarial Congress (p. 338 *et seq.*), and also to the contribution by Mr. S. C. Thomson, published in the *Transactions* of the 1903 Congress (p. 111 *et seq.*), as well as to the Paper contributed to our own *Journal* by Mr. A. T. Winter (vol. xliii, p. 365, *et seq.*), which usefully brings together the results of previous investigations of Indian mortality. These contributions, whilst giving most useful information as to the mortality of native and Eurasian assured lives (in the British Empire, Standard and Oriental Offices) or of the members of different Funds—such as the Uncovenanted Service Fund, or the Indian Postal Fund—give practically no information as to the mortality of the general native population. The assured lives, or members of the Funds referred to, would be largely, if not entirely, of the class of literate natives; and when it is remembered that, in 1901, less than 10 per-cent of the male population were literate in Indian vernaculars, and only about 7 per 1,000 were literate in English, it is evident that the experience of Assurance Companies and Funds would in no way represent that of the general native population. The above investigations indicate that the native assured lives experience a mortality approximately equal to that of the O<sup>M</sup> Table, with 7 years added to the age, and that the members of the Funds mentioned above experienced a somewhat similar mortality; but it will be evident, from a comparison of the Life Tables deduced from the Census, that the mortality of the general native population is, as would be anticipated, in no way represented by such as addition.

**Mortality of Government Employees in India.** 55. I am engaged upon an investigation for the India Office into the mortality of about 77,000 persons in subordinate Government employ in India, recorded

during a recent period of about 15 years. This investigation, the data for which were collated on lines suggested by the Institute of Actuaries, at the invitation of the Indian Government (See *J.I.A.*, vol. xxxvi, pp. 404-8), has had to be set aside for the larger work connected with the Indian Census; and the tabulation of the data is at present in the preliminary stages only. It is probable, however, that no direct comparison of these data, when tabulated, with the Census tables now deduced, would be of very much service, as it may be anticipated that the lives referred to represent on the whole a select class of native literate persons, whose mortality

experience would differ quite materially from that of the general population.

56. It will be observed that, throughout my investigation, the methods followed have not departed, in any very material respects, from those adopted by Mr. Hardy, although the figures submitted to me have throughout been subject to an independent scrutiny and treatment. Having regard to Mr. Hardy's wide and exceptional experience in matters relating to Indian mortality, and to his unrivalled ability in all questions involving the adjustment and graduation of life tables, it is not perhaps surprising that I have not seen my way to improve upon these methods, or rather to vary them in directions which might or might not be in the nature of improvements. The nature of the investigation was also such that, having regard to the available data, and especially to the known defects in the registration statistics, little or no choice was left as to the fundamental methods to be followed throughout the investigation. It seemed also most desirable that the methods adopted on the present occasion should not, except where absolutely necessary, depart materially from those adopted by Mr. Hardy, in order that the tabular results might conveniently be compared.

57. I have been in constant correspondence, during the course of my investigation, with the Hon. Mr. E. A. Gait, the Census Commissioner for India, who has supplied me promptly with all necessary data, and has most courteously and fully dealt with all points as to which information or explanations were desired. Mr. Gait's intimate acquaintance with the questions involved, and with their treatment in similar investigations made in past years, have been of the greatest assistance to me, during the whole of my investigation.

58. I also desire to express my high appreciation of the able and useful assistance rendered, throughout this work, by Mr. S. J. Gunningham, B.Sc., F.I.A., whose note in Appendix A to this paper will sufficiently testify to the intimacy of his acquaintance with graduations by frequency curve methods. Mr. S. J. Rowland, A.I.A., also gave most useful help in the graduation of the Punjab, and other figures, and in the preparation of a note included in Appendix B; and, in this section of the work, I had the great advantage of voluntary assistance on the part of Mr. A. E. King, F.I.A. Mr. G. M. Reeve, F.I.A., has also rendered able and willing help, especially in the later stages of this lengthy investigation.

*Table Population in the several Provinces, States and Agencies in 1911, 1901, 1891, and 1881.*

Province, State, or Agency	Population 1911			Population 1901			Population 1891	Population 1881
	Total	Males	Females	Total	Males	Females		
<b>India Provinces</b>	<b>315,132,537</b>	<b>161,326,110</b>	<b>153,806,427</b>	<b>294,361,056</b>	<b>149,951,824</b>	<b>144,409,232</b>	<b>28,314,671</b>	<b>253,396,330</b>
1 Ajmer-Merwara	244,267,542	124,873,691	119,393,851	231,605,940	117,653,127	113,952,813	221,240,836	198,882,817
2 Andamans and Nicobars	501,395	266,198	235,197	476,912	251,026	225,886	542,358	460,722
3 Baluchistan	20,459	19,570	6,889	24,649	18,695	5,954	15,609	14,628
4 Bengal	414,412	239,181	175,231	382,106	219,523	162,583	...	...
5 Bombay	52,668,269	26,278,865	26,389,404	50,715,794	25,151,960	25,563,834	49,314,523	46,740,699
6 Burma	19,672,642	10,215,847	9,426,795	18,559,650	9,583,456	8,976,194	18,878,471	16,491,538
7 Central Provinces and Berar	12,115,217	6,183,490	5,931,723	10,490,624	5,342,033	5,148,591	7,722,053	3,736,771
8 Coorg	13,916,308	6,930,392	6,985,916	11,971,452	5,926,357	6,045,095	13,048,972	11,943,363
9 Eastern Bengal and Assam	34,018,527	17,413,910	16,604,617	30,510,344	15,543,023	14,967,321	28,128,968	25,472,141
10 Madras	41,405,404	20,382,955	21,022,449	38,229,654	18,851,329	19,378,325	35,644,428	30,841,154
11 N.-W. Frontier Province	2,196,933	1,182,102	1,014,831	2,041,534	1,035,709	935,825	1,857,519	1,575,943
12 Punjab	19,974,956	10,992,067	8,982,889	20,330,337	10,942,682	9,387,655	19,009,368	17,274,507
13 United Provinces	47,182,044	24,641,831	22,540,213	47,692,277	24,617,076	23,075,201	46,905,512	44,149,959
<b>States and Agencies</b>	<b>70,809,995</b>	<b>36,452,419</b>	<b>34,412,576</b>	<b>62,755,116</b>	<b>32,298,697</b>	<b>30,456,419</b>	<b>66,073,835</b>	<b>55,013,513</b>
15 Baroda State	2,032,798	1,055,935	976,863	1,952,692	1,008,634	944,058	2,415,396	...
16 Bengal States	4,538,161	2,271,673	2,266,488	3,881,448	1,935,974	1,926,474	3,600,886	3,013,235
17 Bombay States	7,411,675	3,765,401	3,646,274	6,908,559	3,512,956	3,395,603	8,081,950	6,937,893
18 Central India Agency	9,356,980	4,801,459	4,555,521	8,497,805	4,361,136	4,136,609	10,136,403	9,261,907
19 Central Provinces States	2,117,002	1,063,630	1,063,372	1,631,140	811,970	819,170	1,712,562	1,387,294
20 E. Bengal and Assam States	575,835	292,486	283,349	457,790	232,127	225,663	137,442	316,707
21 Hyderabad State	13,374,676	6,737,118	6,577,558	11,141,142	5,673,629	5,467,513	11,537,040	9,845,594
22 Kashmir State	3,158,126	1,674,367	1,483,759	2,905,578	1,542,057	1,363,521	2,543,952	...
23 Madras States	4,811,841	2,411,758	2,400,083	4,188,086	2,098,048	2,090,038	3,700,622	3,344,849
24 Mysore States	5,806,193	2,934,621	2,871,572	5,539,399	2,797,024	2,742,375	4,943,604	4,186,188
25 N.-W. Frontier Province (Agencies & Tribal Areas)	1,622,094	804,876	757,218	833,962	53,608	30,354	...	...
26 Punjab States	4,212,794	2,322,908	1,889,886	4,424,398	2,409,809	2,014,589	4,263,280	3,861,683
27 Rajputana Agency	10,530,432	5,515,275	5,015,157	9,853,366	5,171,519	4,681,847	12,171,749	9,334,255
28 Sikkim	87,920	45,059	42,861	59,014	30,795	28,219	30,458	...
29 United Provinces States...	832,036	431,410	400,596	802,097	414,414	387,683	792,491	741,750

Variation of Population in Provinces, States, and Agencies in each Decennium from 1881 to 1911.

Province, State, or Agency	Variation of Population, 1901-1911				Variation, 1891-1901				Variation, 1881-1891			
	Males		Females		Males		Females		Males		Females	
	Actual	Percent	Actual	Percent	Actual	Percent	Actual	Percent	Actual	Percent	Actual	Percent
India ...	+ 20,771,481	+ 6.1	+ 11,274,286	+ 6.1	+ 9,397,195	+ 6.5	+ 7,046,385	+ 4.8	+ 33,418,341	+ 2.5	+ 13,2	+ 11.2
Provinces ...	+ 12,661,602	+ 5.5	+ 7,220,564	+ 5.1	+ 5,441,038	+ 6.0	+ 10,365,104	+ 4.1	+ 22,358,019	+ 4.7	+ 22,358,019	+ 17.7
1. Ajmer-Merwara ...	+ 24,483	+ 5.1	+ 15,172	+ 4.7	+ 9,311	+ 4.7	+ 65,446	- 15.7	+ 81,636	+ 5.9	+ 81,636	+ 6.7
2. Andamans & Nicobars ...	+ 1,810	+ 3.3	+ 875	+ 3.3	+ 935	+ 4.7	+ 9,010	+ 57.9	+ 981	+ 7.1	+ 981	...
3. Baluchistan ...	+ 32,306	+ 85	+ 19,658	+ 8.0	+ 12,618	+ 4.5	+ 825,570	+ 7.8	...	+ 2,573,824	...	...
4. Bengal ...	+ 1,952,175	+ 3.8	+ 1,26,905	+ 3.8	+ 825,570	+ 3.2	+ 1,101,271	+ 3.2	+ 2,383,933	+ 1.7	+ 2,383,933	+ 14.5
5. Bombay ...	+ 1,112,992	+ 6.0	+ 662,391	+ 6.9	+ 450,601	+ 5.0	+ 318,821	-	+ 3,985,282	+ 35.9	+ 3,985,282	+ 106.6
6. Burma ...	+ 1,624,593	+ 15.5	+ 841,461	+ 15.8	+ 783,132	+ 15.2	+ 2,768,571	+ 15.6	+ 1,105,609	+ 8.3	+ 1,105,609	+ 9.3
7. Central Provinces & Berar ...	+ 1,914,856	+ 16.2	+ 1,000,035	+ 16.9	+ 940,821	+ 16.9	+ 1,077,520	-	+ 5.217	+ 4.1	+ 5.217	+ 2.9
8. Coorg ...	+ 5,631	+ 3.1	- 2,979	- 3.0	+ 2,652	+ 3.3	+ 7,552	+ 3.3	+ 2,381,376	+ 8.5	+ 2,656,827	+ 10.4
9. Eastern Bengal & Assam ...	+ 3,508,183	+ 11.5	+ 1,874,887	+ 12.0	+ 1,637,296	+ 10.9	+ 1,646,121	+ 8.5	+ 4,803,274	+ 7.3	+ 4,803,274	+ 15.6
10. Madras ...	+ 3,175,750	+ 8.3	+ 1,531,626	+ 8.1	+ 79,006	+ 6.9	+ 181,015	+ 8.4	+ 281,576	+ 9.9	+ 281,576	+ 17.9
11. N.W. Frontier Province ...	+ 155,399	+ 7.6	+ 76,393	+ 7.3	+ 40,766	+ 0.5	+ 1,320,969	+ 4.3	+ 1,731,771	+ 6.9	+ 1,731,771	+ 10.0
12. Punjab ...	+ 355,381	- 1.7	+ 49,385	+ 1.0	+ 531,988	+ 1.0	+ 786,765	+ 2.3	+ 2,755,553	+ 1.7	+ 2,755,553	+ 6.2
13. United Provinces ...	+ 510,233	- 1.1	+ 24,755	+ 1.0	+ 3,956,157	+ 12.9	+ 13.0	-	+ 318,719	-	+ 5.0	+ 20.1
14. United States and Agencies ...	+ 8,109,879	+ 12.9	+ 4,153,722	+ 12.9	+ 20,624	- 5.1	+ 10.2	...	...	...	...	...
15. Baluchistan State ...	- 32,208	- 7.5	- 11,584	- 5.1	+ 32,805	+ 4.7	+ 3.5	-	+ 162,704	-	+ 19.2	+ 10.7
16. Baroda State ...	+ 80,106	+ 4.1	+ 47,301	+ 4.7	+ 316,639	+ 16.2	+ 310,011	+ 17.6	+ 274,562	+ 7.6	+ 593,651	+ 19.7
17. Bengal States ...	+ 656,713	+ 16.9	+ 503,116	+ 7.3	+ 252,445	+ 7.2	+ 250,671	+ 7.1	+ 1,173,391	+ 14.5	+ 1,114,057	+ 16.5
18. Central India Agency ...	+ 859,175	+ 10.1	+ 410,323	+ 10.1	+ 418,582	+ 24.4,202	+ 29.8	-	+ 1,638,598	+ 16.2	+ 874,496	+ 9.4
19. Central Provinces States ...	+ 485,862	+ 29.8	+ 211,600	+ 29.8	+ 211,600	+ 21.0	+ 21.0	-	+ 81,122	-	+ 4.8	+ 23.4
20. Eastern Bengal and Assam States ...	+ 118,045	+ 25.8	+ 60,359	+ 26.0	+ 57,686	+ 25.6	+ 320,348	+ 23.1	-	+ 179,265	-	56.6
21. Hyderabad State ...	+ 2,233,534	+ 20.0	+ 1,23,189	+ 19.8	+ 1,110,045	+ 20.3	+ 395,898	-	+ 1,691,116	+ 3.4	+ 1,691,116	+ 17.2
22. Kashmir State ...	+ 252,548	+ 87	+ 132,310	+ 86	+ 120,238	+ 8.8	+ 361,626	+ 14.2	...	...	...	...
23. Madras States ...	+ 623,755	+ 14.9	+ 313,710	+ 15.0	+ 310,015	+ 14.8	+ 487,464	+ 13.2	+ 355,773	+ 12.1	+ 355,773	+ 10.6
24. Mysore States ...	+ 265,794	+ 48	+ 137,597	+ 4.9	+ 129,197	+ 4.7	+ 595,795	+ 12.1	+ 757,416	+ 757,416	+ 18.1	
25. N.W. Frontier Province (Agencies & Tribal Areas) ...	+ 1,538,132	+ 1,831.9	+ 811,268	+ 1,513.3	+ 726,864	+ 2,394.6	+ 161,118	...	...	...	...	...
26. Punjab States ...	- 211,601	- 4.8	- 86,901	- 3.6	- 124,703	- 6.2	+ 2,318,383	+ 3.8	+ 401,597	+ 19.0	+ 2,318,383	+ 22.5
27. Rajputana Agency ...	+ 677,666	+ 6.9	+ 343,756	+ 6.6	+ 333,310	+ 7.1	+ 28,556	+ 93.8	+ 2,237,494	+ 9,606	+ 2,237,494	+ 6.8
28. Sikkim ...	+ 28,906	+ 49.0	+ 14,264	+ 46.3	+ 14,642	+ 51.9	+ 12,913	+ 3.3	+ 50,741	+ 1.2	+ 50,741	+ 1.2
29. United Provinces States ...	+ 29,939	+ 37	+ 17,026	+ 4.2	+ 17,026	+ 3.3	+ 12,913	+ 3.3	+ 50,741	+ 1.2	+ 50,741	+ 1.2

TABLE

*Population enumerated at each age, out of a total population of 100,000  
age-group in proportion to the numbers shewn*

Ages <i>x</i>	BENGAL		BOMBAY		BURMA (BUDDHISTS)	
	Males	Females	Males	Females	Males	Females
	(2)	(3)	(4)	(5)	(6)	(7)
0	3,065	3,245	3,300	3,597	2,085	2,187
1	1,586	1,696	1,835	2,054	2,452	2,538
2	2,961	3,160	2,836	3,208	2,949	2,957
3	3,072	3,487	2,928	3,186	3,205	3,241
4	2,857	2,998	2,905	3,143	2,852	2,795
5	3,689	3,747	3,338	3,051	2,909	2,941
6	3,022	3,033	2,559	2,582	3,033	2,961
7	3,003	3,191	2,311	2,378	2,503	2,613
8	3,949	3,363	2,857	2,911	2,904	2,876
9	1,936	2,112	1,549	1,754	2,207	2,049
10	4,142	3,105	3,559	2,799	3,601	3,403
11	1,324	1,379	1,176	1,207	1,898	1,756
12	3,782	2,740	3,370	2,733	2,771	2,492
13	1,048	1,021	1,239	1,081	2,246	2,054
14	1,679	1,643	1,492	1,426	1,847	1,692
15	1,996	1,902	2,765	2,232	2,233	2,245
16	1,913	2,169	1,853	1,692	1,718	1,777
17	933	1,079	964	1,118	1,639	1,897
18	2,717	2,834	2,106	1,984	1,876	2,148
19	771	893	743	887	1,527	1,600
20	3,179	4,125	4,706	4,743	2,906	3,423
21	641	888	506	1,093	1,015	1,137
22	2,073	2,239	2,092	1,967	1,403	1,489
23	705	738	665	848	1,401	1,469
24	975	1,052	844	1,057	1,028	1,123
25	4,732	5,040	5,782	5,618	2,940	3,134
26	1,026	1,003	1,136	966	1,247	1,236
27	926	898	787	826	1,265	1,226
28	1,970	1,869	1,521	1,538	1,251	1,315
29	444	549	379	452	900	1,006
30	5,101	5,167	6,278	6,444	3,714	3,426
31	321	388	282	304	816	818
32	1,822	1,503	1,351	1,237	1,199	1,237
33	385	335	366	314	1,158	970
34	472	407	325	444	781	817
35	3,868	3,176	4,708	4,075	2,535	2,145
36	1,028	899	648	537	993	988
37	416	367	304	296	1,118	890
38	1,025	822	644	648	1,065	1,065
39	277	313	249	319	843	752
40	4,430	4,206	5,226	5,124	2,686	2,615
41	217	248	160	297	600	565
42	747	623	732	560	993	897
43	160	173	152	269	757	817
44	209	202	217	377	622	631
45	2,579	2,132	2,955	2,632	1,735	1,583

## III.

of each sex, obtained by distributing the actual numbers of each quinary at individual ages in the Specimen Schedules.

MADRAS		PUNJAB		UNITED PROVINCES		Ages <i>x</i>
Males (8)	Females (9)	Males (10)	Females (11)	Males (12)	Females (13)	(14)
2,620	2,579	4,018	4,820	3,282	3,428	0
1,724	2,068	1,783	1,908	1,575	1,733	1
2,949	2,858	2,050	2,494	2,185	2,453	2
3,231	3,177	2,562	2,921	2,322	2,657	3
2,804	2,734	2,525	2,890	2,256	2,417	4
3,196	2,933	2,995	3,203	3,255	3,241	5
2,884	2,732	2,826	3,142	2,623	2,731	6
2,400	2,423	2,456	2,782	2,504	2,584	7
3,007	3,193	3,123	2,994	3,193	2,837	8
1,854	1,845	2,085	2,096	1,815	1,866	9
4,000	3,810	3,116	2,922	3,866	3,288	10
1,147	1,145	1,506	1,449	1,572	1,444	11
3,942	3,145	3,697	2,890	3,829	3,042	12
1,398	1,240	1,525	1,273	1,127	1,023	13
1,723	1,577	2,176	1,774	1,848	1,473	14
2,160	1,801	2,239	1,868	2,381	1,881	15
2,519	2,381	2,346	2,182	2,478	2,288	16
773	829	1,041	865	613	596	17
2,595	2,666	2,659	2,491	2,487	2,235	18
717	725	759	590	629	559	19
3,912	5,129	3,949	4,489	4,235	4,685	20
586	720	566	469	615	547	21
1,735	1,793	2,268	2,154	2,040	1,980	22
713	678	655	551	472	524	23
1,228	1,162	998	1,092	1,313	1,552	24
4,118	4,494	4,765	4,887	5,112	5,049	25
1,272	1,292	1,244	1,266	996	964	26
675	711	893	815	635	600	27
1,394	1,384	1,486	1,533	1,844	2,086	28
468	471	333	283	391	423	29
4,964	5,813	4,715	5,583	5,595	6,089	30
408	421	192	229	326	346	31
1,176	1,058	1,948	1,754	1,853	1,667	32
314	343	468	290	279	253	33
592	527	514	449	448	488	34
3,197	3,220	3,587	3,592	3,277	3,065	35
990	752	824	626	1,511	1,421	36
446	352	218	155	243	226	37
856	692	518	534	657	818	38
401	304	179	186	282	344	39
4,525	4,985	4,852	5,535	5,512	5,738	40
395	312	159	161	281	282	41
749	705	548	479	715	639	42
362	267	148	91	121	128	43
395	291	206	155	303	323	44
2,382	2,152	2,946	2,717	2,717	2,485	45

TABLE

*Population enumerated at each age, out of a total population of 100,000  
age-group in proportion to the numbers shew*

Ages <i>x</i>	BENGAL		BOMBAY		BURMA (BUDDHISTS)	
	Males (2)	Females (3)	Males (4)	Females (5)	Males (6)	Females (7)
46	234	252	221	206	674	602
47	211	201	208	177	661	582
48	540	497	401	360	604	612
49	132	158	162	145	409	363
50	3,015	3,083	3,628	3,569	1,970	2,038
51	118	136	149	196	359	354
52	349	358	323	333	578	569
53	88	94	128	195	448	493
54	113	138	122	200	373	384
55	1,108	996	1,283	1,127	973	967
56	190	218	113	117	511	459
57	111	114	70	68	364	390
58	209	242	205	153	340	362
59	69	86	73	90	219	238
60	1,957	2,368	1,919	2,474	1,380	1,662
61	63	112	140	86	318	279
62	150	197	178	192	451	323
63	33	46	117	85	385	298
64	44	72	84	143	251	218
65	440	556	502	558	615	623
66	38	40	61	28	191	141
67	39	57	33	23	273	267
68	66	89	53	85	160	154
69	20	31	30	24	86	98
70	610	775	400	597	681	1,012
71	24	20	12	7	97	106
72	65	63	43	38	144	139
73	14	11	16	65	122	110
74	12	10	10	44	55	63
75	162	178	183	214	203	244
76	11	14	7	8	90	63
77	13	13	7	5	58	41
78	24	27	9	31	62	70
79	7	9	5	2	30	36
80	295	367	243	195	218	361
81	11	11	3	3	19	31
82	17	17	10	9	16	18
83	4	5	3	4	23	28
84	5	6	2	2	8	10
85-89	89	57	58	50	64	93
90-94	70	68	67	52	51	58
95-99	26	19	9	11	33	37
100 and over	31	30	10	21	7	8

III—*continued.*

of each sex, obtained by distributing the actual numbers of each quinary  
at individual ages in the Specimen Schedules.

MADRAS		PUNJAB		UNITED PROVINCES		Ages <i>x</i>
Males (8)	Females (9)	Males (10)	Females (11)	Males (12)	Females (13)	(14)
549	418	234	209	278	272	46
275	231	136	92	149	134	47
588	510	362	365	498	511	48
302	229	104	88	169	210	49
3,302	3,756	3,931	4,031	4,038	4,335	50
302	162	131	107	160	148	51
417	339	343	216	344	321	52
235	128	84	43	84	61	53
282	292	162	89	156	154	54
1,186	1,078	1,390	1,116	1,059	978	55
411	299	169	119	274	271	56
153	133	73	60	80	82	57
306	266	120	116	190	201	58
119	106	68	50	77	87	59
2,276	2,607	1,913	2,667	2,446	2,910	60
148	147	65	68	87	106	61
237	230	112	107	142	155	62
127	104	29	28	28	31	63
160	109	42	24	49	72	64
518	510	1,710	609	485	465	65
127	117	136	29	39	60	66
87	81	114	25	34	35	67
129	130	156	35	74	69	68
78	65	78	19	24	34	69
496	840	805	1,115	670	861	70
23	44	19	19	18	19	71
64	69	30	39	43	48	72
20	41	8	3	5	8	73
61	52	11	9	10	17	74
146	158	114	185	93	108	75
72	33	10	8	11	10	76
17	14	3	3	9	8	77
67	45	6	13	19	18	78
120	29	4	9	7	8	97
212	322	427	372	333	474	80
13	21	5	5	11	11	81
40	44	11	15	16	17	82
67	30	4	1	5	2	83
53	15	4	4	8	8	84
183	153	45	64	47	48	85-89
125	82	122	103	80	92	90-94
10	9	20	19	18	18	95-99
1	3	24	23	20	25	100 and over

TABLE IV.

*Showing Age Distribution of 100,000 persons of each sex for the Censuses  
of 1901 and 1911.*

TABLE IV—*continued.*

Showing Age Distribution of 100,000 persons of each sex for the Censuses of 1901 and 1911.

Province	Ages	MALES				FEMALES			
		1901	1911	Mean 1901-1911	Graduated Numbers	1901	1911	Mean 1901-1911	Graduated Numbers
Madras	0-4	15,071	14,926	14,998	15,286	15,358	14,882	15,120	14,926
	5-9	14,480	12,993	13,737	12,416	14,030	12,818	13,424	12,606
	10-14	11,260	10,984	11,122	11,252	9,781	9,671	9,726	11,315
	15-19	9,910	10,368	10,139	10,218	10,019	10,754	10,386	10,190
	20-24	7,141	7,975	7,558	9,205	8,326	8,912	8,619	9,126
	25-29	8,792	8,759	8,776	8,200	9,621	9,402	9,512	8,105
	30-34	6,754	6,394	6,574	7,193	6,802	6,769	6,785	7,108
	35-39	7,053	6,751	6,902	6,180	6,615	6,368	6,492	6,123
	40-44	5,156	5,318	5,237	5,168	4,923	5,120	5,022	5,153
	45-49	4,578	4,641	4,609	4,227	4,294	4,422	4,358	4,248
	50-54	3,245	3,458	3,352	3,365	3,143	3,288	3,215	3,419
	55-59	2,582	2,827	2,679	2,590	2,592	2,731	2,661	2,673
	60 and over	4,028	4,606	4,317	4,700	4,496	4,863	4,680	5,008
	Total	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000
Punjab	0-4	14,537	14,436	14,486	15,270	15,595	16,635	16,115	
	5-9	12,846	12,647	12,747	12,327	12,851	13,190	13,020	
	10-14	11,815	11,360	11,588	11,396	10,408	9,733	10,071	
	15-19	10,452	10,687	10,569	10,390	10,199	9,976	10,087	
	20-24	7,930	8,615	8,272	9,340	8,307	8,629	8,468	
	25-29	9,152	9,125	9,139	8,286	9,681	9,593	9,637	
	30-34	7,330	7,070	7,204	7,235	7,348	7,168	7,258	
	35-39	6,334	6,209	6,272	6,185	6,548	6,248	6,398	
	40-44	5,058	4,934	4,996	5,141	5,008	5,007	5,008	
	45-49	4,094	4,325	4,210	4,146	4,102	4,171	4,136	
	50-54	3,267	3,357	3,312	3,252	3,105	3,050	3,078	
	55-59	2,425	2,131	2,278	2,493	2,342	2,284	2,313	
	60 and over	4,751	5,104	4,927	4,539	4,506	4,316	4,411	
	Total	100,000	100,000	100,000	100,000	100,000	100,000	100,000	
United Provinces of Agra and Oudh.	0-4	13,739	13,247	13,493	14,849	13,166	14,308	13,737	14,665
	5-9	12,448	12,849	12,648	12,020	13,147	12,455	12,801	11,890
	10-14	11,725	11,265	11,495	11,202	10,211	9,454	9,832	11,072
	15-19	9,666	10,285	9,976	10,491	8,839	9,625	9,232	10,359
	20-24	8,926	8,848	8,887	9,557	9,249	9,599	9,424	9,455
	25-29	9,316	9,617	9,466	8,519	9,539	9,597	9,568	8,464
	30-34	7,386	7,031	7,209	7,422	7,735	7,291	7,513	7,425
	35-39	6,785	7,437	7,111	6,321	6,905	7,286	7,096	6,382
	40-44	5,736	5,530	5,633	5,253	5,680	5,466	5,573	5,351
	45-49	4,582	4,535	4,558	4,249	4,446	4,622	4,534	4,358
	50-54	3,485	3,267	3,376	3,329	3,815	3,324	3,570	3,436
	55-59	2,537	2,440	2,489	2,507	2,673	2,643	2,658	2,606
	60 and over	3,669	3,649	3,659	4,281	4,595	4,330	4,462	4,537
	Total	100,000	100,000	100,000	100,000	100,000	100,000	100,000	100,000

(Not Computed).

TABLE

*Numbers living between ages  $x$  and  $(x+1)$  out of a total*

Age $x$	BENGAL		BOMBAY		BURMA	
	Males (2)	Females (3)	Males (4)	Females (5)	Males (6)	Females (7)
(1)						
0	4,281	4,262	4,034	3,980	3,568	3,456
1	3,676	3,665	3,470	3,429	3,209	3,118
2	3,343	3,337	3,163	3,130	3,005	2,930
3	3,112	3,110	2,953	2,927	2,857	2,796
4	2,945	2,947	2,803	2,782	2,745	2,695
5	2,819	2,824	2,692	2,676	2,656	2,616
6	2,720	2,726	2,606	2,593	2,581	2,551
7	2,638	2,645	2,536	2,527	2,516	2,494
8	2,567	2,576	2,478	2,472	2,458	2,442
9	2,505	2,514	2,427	2,423	2,404	2,395
10	2,449	2,456	2,382	2,380	2,353	2,350
11	2,394	2,402	2,340	2,340	2,305	2,306
12	2,340	2,346	2,307	2,303	2,259	2,264
13	2,286	2,291	2,265	2,267	2,215	2,224
14	2,232	2,235	2,229	2,232	2,172	2,184
15	2,179	2,180	2,194	2,198	2,128	2,146
16	2,126	2,126	2,159	2,163	2,084	2,103
17	2,074	2,072	2,123	2,127	2,040	2,061
18	2,022	2,018	2,086	2,089	1,996	2,018
19	1,971	1,965	2,047	2,050	1,952	1,975
20	1,920	1,913	2,007	2,009	1,908	1,931
21	1,870	1,861	1,964	1,966	1,864	1,888
22	1,820	1,810	1,921	1,922	1,820	1,844
23	1,771	1,760	1,876	1,877	1,777	1,800
24	1,723	1,711	1,830	1,830	1,734	1,756
25	1,675	1,662	1,784	1,783	1,691	1,713
26	1,628	1,615	1,737	1,736	1,648	1,669
27	1,581	1,568	1,690	1,687	1,606	1,625
28	1,535	1,522	1,642	1,639	1,564	1,581
29	1,490	1,476	1,594	1,590	1,522	1,538
30	1,445	1,432	1,546	1,541	1,481	1,494
31	1,401	1,388	1,498	1,493	1,440	1,452
32	1,357	1,345	1,450	1,444	1,399	1,409
33	1,314	1,302	1,402	1,396	1,359	1,367
34	1,272	1,261	1,354	1,348	1,320	1,326
35	1,230	1,219	1,307	1,300	1,280	1,285
36	1,189	1,179	1,260	1,254	1,242	1,244
37	1,149	1,139	1,213	1,207	1,203	1,204
38	1,109	1,101	1,167	1,162	1,165	1,166
39	1,070	1,062	1,121	1,117	1,128	1,127
40	1,031	1,025	1,076	1,073	1,091	1,090
41	993	988	1,032	1,029	1,055	1,053
42	956	951	988	986	1,019	1,017
43	919	916	945	944	984	981
44	883	881	902	903	949	946
45	848	847	861	862	915	913

## V.

*Population of 100,000 of each sex in the following Provinces.*

MADRAS		UNITED PROVINCES.		PUNJAB		Age x (14)
Males (8)	Females (9)	Males (10)	Females (11)	Males (12)	Females (13)	
3,651	3,442	3,583	3,535	3,689		0
3,198	3,085	3,114	3,074	3,203		1
2,955	2,898	2,866	2,831	2,947		2
2,796	2,790	2,700	2,669	2,775		3
2,686	2,711	2,586	2,556	2,656		4
2,600	2,642	2,504	2,476	2,570		5
2,532	2,579	2,442	2,416	2,506		6
2,478	2,518	2,395	2,369	2,456		7
2,427	2,460	2,356	2,331	2,415		8
2,379	2,407	2,323	2,298	2,380		9
2,334	2,356	2,293	2,268	2,349		10
2,292	2,309	2,266	2,240	2,318		11
2,250	2,262	2,240	2,214	2,282		12
2,209	2,217	2,215	2,188	2,244		13
2,167	2,171	2,188	2,162	2,203		14
2,126	2,127	2,161	2,134	2,162		15
2,085	2,082	2,132	2,105	2,120		16
2,043	2,037	2,101	2,074	2,078		17
2,002	1,993	2,067	2,041	2,036		18
1,962	1,951	2,030	2,005	1,994		19
1,921	1,908	1,991	1,968	1,952		20
1,881	1,866	1,951	1,929	1,910		21
1,841	1,825	1,912	1,891	1,868		22
1,801	1,754	1,872	1,853	1,826		23
1,761	1,743	1,831	1,814	1,784		24
1,720	1,701	1,790	1,774	1,742		25
1,680	1,661	1,747	1,734	1,699		26
1,640	1,621	1,704	1,693	1,657		27
1,600	1,581	1,661	1,652	1,615		28
1,560	1,541	1,617	1,611	1,573		29
1,520	1,501	1,573	1,569	1,531		30
1,479	1,461	1,529	1,527	1,489		31
1,438	1,421	1,484	1,485	1,447		32
1,398	1,382	1,440	1,443	1,405		33
1,358	1,343	1,396	1,401	1,363		34
1,317	1,303	1,352	1,360	1,321		35
1,277	1,264	1,308	1,318	1,279		36
1,236	1,224	1,264	1,276	1,237		37
1,195	1,185	1,220	1,235	1,195		38
1,155	1,147	1,177	1,193	1,153		39
1,114	1,107	1,134	1,152	1,111		40
1,074	1,069	1,092	1,111	1,069		41
1,030	1,030	1,050	1,070	1,028		42
994	992	1,009	1,029	987		43
956	955	968	989	946		44
918	919	928	949	906		45

(Not computed.)

TABLE

*Numbers living between ages  $x$  and  $(x+1)$ , out of a total*

Age $x$	BENGAL		BOMBAY		BURMA	
	Males	Females	Males	Females	Males	Females
	(1)	(2)	(3)	(4)	(5)	(6)
46	813	813	820	823	881	879
47	779	780	780	784	848	846
48	746	748	741	746	815	814
49	713	716	703	709	783	782
50	681	685	666	672	752	751
51	649	654	629	637	721	721
52	618	624	594	602	691	691
53	587	594	560	569	661	662
54	557	564	527	536	632	634
55	527	535	494	504	603	606
56	498	507	463	473	575	578
57	469	479	433	443	548	552
58	441	451	404	415	521	525
59	414	424	376	387	495	500
60	387	397	349	360	469	474
61	361	371	323	334	442	448
62	335	346	298	309	414	420
63	310	321	274	285	386	392
64	285	296	251	262	358	364
65	262	272	229	240	331	337
66	239	248	209	219	304	310
67	217	226	189	199	278	284
68	195	204	171	180	253	259
69	174	182	153	162	228	234
70	154	161	137	145	205	211
71	135	141	122	129	182	188
72	117	123	107	114	161	166
73	100	105	94	100	141	146
74	84	89	82	87	122	127
75	70	74	71	76	105	109
76	57	60	60	65	89	92
77	45	48	51	55	74	77
78	35	38	42	46	61	64
79	27	28	35	37	50	52
80	20	21	28	30	40	42
81	14	15	22	24	31	33
82	10	10	17	18	24	25
83	7	7	13	14	18	19
84	4	5	9	10	13	14
85	3	3	6	7	9	10
86	2	2	4	4	7	7
87	1	1	2	2	5	5
88	...	1	1	1	3	3
89	...	...	...	...	2	2
90	...	...	...	...	1	1
91	...	...	...	...	1	1

V—*continued.**population of 100,000 of each sex in the following Provinces.*

MADRAS		UNITED PROVINCES		PUNJAB		Age <i>x</i> (14)
Males (8)	Females (9)	Males (10)	Females (11)	Males (12)	Females (13)	
882	885	888	910	866		46
845	849	849	871	828		47
809	815	811	833	791		48
773	780	773	795	755		49
739	746	736	758	719		50
705	715	700	722	684		51
672	683	665	686	649		52
640	652	631	652	616		53
609	623	597	618	584		54
578	593	564	584	554		55
547	563	532	552	525		56
517	534	500	520	498		57
488	505	470	490	471		58
460	478	441	460	445		59
433	452	412	431	420		60
406	424	385	403	395		61
380	399	358	376	371		62
355	374	332	349	348		63
331	350	308	324	325		64
307	325	284	300	303		65
284	301	261	276	282		66
262	279	239	253	262		67
240	256	218	232	242		68
220	236	198	211	223		69
201	216	179	191	203		70
183	197	162	173	184		71
165	179	145	155	165		72
148	160	129	138	146		73
132	143	114	122	127		74
117	128	100	107	110		75
102	111	87	94	93		76
89	97	75	81	78		77
76	83	64	69	65		78
64	71	54	58	52		79
53	59	44	48	41		80
43	47	36	39	32		81
34	37	29	31	24		82
26	28	22	24	17		83
19	22	17	18	12		84
13	15	12	13	8		85
9	10	8	9	5		86
5	6	5	6	3		87
2	2	3	3	2		88
1	1	1	2	1		89
...	...	...	1	...		90
...	...	...	...	...		91

(Not computed)

(For Table VI, see p. 380).

TABLE

*Shewing the adjusted number of Females ( $k_x$ ) to 10,000 Males living  
the adjusted numbers from*

Age <i>x</i>	BENGAL		BOMBAY		BURMA		MADRAS		UNITED PROVINCES	
	$k_x$	$\Delta$	$k_x$	$\Delta$	$k_x$	$\Delta$	$k_x$	$\Delta$	$k_x$	$\Delta$
0	10,233	+ 15	9,886	+ 15	9,772	+ 34	9,600	+ 224	9,874	+ 5
1	10,248	13	9,901	15	9,806	35	9,824	163	9,879	5
2	10,261	13	9,916	14	9,841	35	9,987	178	9,884	4
3	10,274	12	9,930	14	9,876	35	10,165	113	9,888	4
4	10,286	10	9,944	14	9,911	35	10,278	73	9,892	4
5	10,296	8	9,958	12	9,946	33	10,351	23	9,896	2
6	10,304	6	9,970	12	9,979	31	10,374	- 26	9,898	1
7	10,310	3	9,982	11	10,010	28	10,348	24	9,899	1
8	10,313	2	9,993	10	10,038	26	10,324	23	9,900	- 1
9	10,315	...	10,003	9	10,064	25	10,301	22	9,899	2
10	10,315	- 3	10,012	8	10,089	22	10,279	- 21	9,897	- 2
1	10,312	4	10,020	6	10,111	21	10,258	20	9,895	4
2	10,308	6	10,026	4	10,132	18	10,238	19	9,891	3
3	10,302	6	10,030	4	10,150	16	10,219	18	9,888	2
4	10,296	7	10,034	2	10,166	13	10,201	17	9,886	3
15	10,289	- 9	10,036	1	10,179	12	10,184	- 16	9,883	- 1
6	10,280	9	10,037	...	10,191	10	10,168	14	9,882	- 1
7	10,271	10	10,037	- 1	10,201	8	10,154	14	9,881	+ 1
8	10,261	10	10,036	1	10,209	7	10,140	13	9,882	2
9	10,251	10	10,035	2	10,216	6	10,127	12	9,884	4
20	10,241	- 10	10,033	- 3	10,222	4	10,115	- 10	9,888	4
1	10,231	9	10,030	3	10,226	2	10,105	10	9,892	6
2	10,222	8	10,027	3	10,228	2	10,095	8	9,898	7
3	10,214	7	10,024	5	10,230	- 1	10,087	7	9,905	8
4	10,207	6	10,019	4	10,229	2	10,080	6	9,913	9
25	10,201	- 5	10,015	- 5	10,227	- 3	10,074	- 5	9,922	11
6	10,196	4	10,010	1	10,224	5	10,069	4	9,933	11
7	10,191	3	10,009	6	10,219	7	10,065	3	9,944	12
8	10,188	2	10,003	5	10,212	9	10,062	1	9,956	14
9	10,186	1	9,998	6	10,203	11	10,061	- 1	9,970	14
30	10,185	- 1	9,992	- 5	10,192	- 11	10,060	+ 1	9,984	15
1	10,184	...	9,987	5	10,181	13	10,061	1	9,999	16
2	10,184	+ 1	9,982	3	10,168	13	10,062	3	10,015	17
3	10,185	2	9,979	3	10,155	12	10,065	4	10,032	18
4	10,187	3	9,976	1	10,143	11	10,069	5	10,050	19
35	10,190	3	9,975	...	10,132	- 12	10,074	7	10,069	19
6	10,193	5	9,975	+ 2	10,120	10	10,081	7	10,088	20
7	10,198	6	9,977	3	10,110	10	10,088	9	10,108	19
8	10,204	6	9,980	5	10,100	8	10,097	10	10,127	18
9	10,210	6	9,985	7	10,092	7	10,107	11	10,145	17
40	10,216	8	9,992	7	10,085	- 5	10,118	13	10,162	17
1	10,224	9	9,999	9	10,080	4	10,131	13	10,179	16
2	10,233	10	10,008	10	10,076	2	10,144	15	10,195	14
3	10,243	10	10,018	11	10,074	1	10,159	17	10,209	14
4	10,253	12	10,029	13	10,073	...	10,176	17	10,223	13
45	10,265	12	10,042	14	10,073	+ 1	10,193	19	10,236	13
6	10,277	14	10,056	15	10,074	3	10,212	20	10,249	13
7	10,291	15	10,071	16	10,077	4	10,232	21	10,262	13
8	10,306	16	10,087	17	10,081	5	10,253	23	10,275	13
9	10,322	18	10,104	18	10,086	7	10,276	24	10,288	13

## VII.

at each age, in each of the Provinces specified, also the rise or fall ( $\Delta$ ) in age to age throughout life.

Age <i>x</i>	BENGAL		BOMBAY		BURMA		MADRAS		UNITED PROVINCES	
	$k_x$	$\Delta$	$k_x$	$\Delta$	$k_x$	$\Delta$	$k_x$	$\Delta$	$k_x$	$\Delta$
50	10,340	+ 18	10,122	+ 18	10,093	+ 7	10,300	+ 26	10,301	+ 13
1	10,358	21	10,140	20	10,100	10	10,326	27	10,314	14
2	10,379	21	10,160	20	10,110	10	10,353	28	10,328	14
3	10,400	23	10,180	21	10,120	11	10,381	30	10,342	15
4	10,423	23	10,201	22	10,131	13	10,411	31	10,357	15
55	10,446	24	10,223	22	10,144	13	10,442	32	10,372	16
6	10,470	23	10,245	24	10,157	14	10,474	34	10,388	16
7	10,493	25	10,269	25	10,171	15	10,508	37	10,404	17
8	10,518	24	10,294	25	10,186	16	10,545	36	10,421	19
9	10,542	24	10,319	27	10,202	16	10,581	36	10,440	19
60	10,566	24	10,346	27	10,218	16	10,617	35	10,459	20
1	10,590	24	10,373	28	10,234	16	10,652	35	10,479	20
2	10,614	23	10,401	27	10,250	16	10,687	34	10,499	21
3	10,637	24	10,428	27	10,266	17	10,721	34	10,520	22
4	10,661	24	10,455	26	10,283	17	10,755	34	10,542	21
65	10,685	23	10,481	27	10,300	18	10,789	33	10,563	22
6	10,708	22	10,508	25	10,318	17	10,822	32	10,585	22
7	10,730	22	10,533	25	10,335	18	10,854	33	10,607	22
8	10,752	21	10,558	26	10,353	18	10,887	31	10,629	22
9	10,773	21	10,584	24	10,371	18	10,918	31	10,651	21
70	10,794	21	10,608	25	10,389	19	10,949	31	10,672	22
1	10,815	20	10,633	25	10,408	18	10,980	30	10,694	22
2	10,835	21	10,658	24	10,426	17	11,010	30	10,716	22
3	10,856	20	10,682	23	10,443	18	11,040	29	10,738	22
4	10,876	19	10,705	23	10,461	17	11,069	28	10,760	21
75	10,895	20	10,728	21	10,478	17	11,097	28	10,781	21
6	10,915	19	10,749	21	10,495	17	11,125	28	10,802	20
7	10,934	19	10,770	21	10,512	17	11,153	27	10,822	20
8	10,953	18	10,791	20	10,529	17	11,180	27	10,842	19
9	10,971	18	10,811	19	10,546	16	11,207	26	10,861	18
80	10,989	17	10,830	19	10,562	16	11,233	25	10,879	18
1	11,006	17	10,849	17	10,578	15	11,258	25	1,0897	17
2	11,023	16	10,866	17	10,593	15	11,283	24	10,914	16
3	11,039	16	10,883	15	10,608	14	11,307	24	10,930	16
4	11,055	16	10,898	15	10,622	14	11,331	23	10,946	16
85	11,071	16	10,913	13	10,636	14	11,354	23	10,962	15
6	11,087	16	10,926	13	10,650	12	11,377	22	10,977	15
7	11,103	15	10,939	12	10,662	12	11,399	22	10,992	14
8	11,118	14	10,951	12	10,674	11	11,421	21	11,006	14
9	11,132	14	10,963	12	10,685	10	11,442	20	11,020	13
90	11,146	14	10,975	11	10,695	11	11,462	...	11,033	12
1	11,160	12	10,986	11	10,706	10	...	...	11,045	12
2	11,172	13	10,997	11	10,716	10	...	...	11,057	11
3	11,185	12	11,008	11	10,726	9	...	...	11,068	10
4	11,197	11	11,019	8	10,735	10	...	...	11,078	11
95	11,208	...	11,027	10	10,745	10	...	...	11,089	...
6	...	...	11,037	...	10,755	10	...	...	...	...
7	...	...	...	...	10,765	10	...	...	...	...
8	...	...	...	...	10,775	10	...	...	...	...
9	...	...	...	...	10,785	...	...	...	...	...

TABLE VIII.  
*Life Table. All India. Males (1901).*  
*Deduced from Censuses of 1881, 1891 and 1901.*  
(Extracted from Mr. G. F. Hardy's Report on 1901 Census.)

$x$	$l_x$	$d_x$	$100q_x$	$\mu_x$	$\overset{\circ}{e}_x$	$x$
0	100,000	28,538	28·54	4·2631	23·63	0
1	71,462	6,345	8·88	0·1125	31·98	1
2	65,117	4,155	6·38	·0774	34·06	2
3	60,962	2,853	4·68	·0559	35·34	3
4	58,109	2,028	3·49	·0410	36·05	4
5	56,081	1,497	2·67	·0308	36·34	5
6	54,584	1,152	2·11	·0238	36·32	6
7	53,432	928	1·74	·0192	36·10	7
8	52,504	780	1·49	·0162	35·73	8
9	51,724	690	1·33	·0141	35·26	9
10	51,034	632	1·24	·0128	34·73	10
1	50,402	592	1·17	·0120	34·16	1
2	49,810	568	1·14	·0115	33·56	2
3	49,242	558	1·13	·0114	32·95	3
4	48,684	556	1·14	·0114	32·31	4
15	48,128	562	1·17	·0116	31·68	15
6	47,566	574	1·21	·0119	31·05	6
7	46,992	591	1·26	·0124	30·42	7
8	46,401	610	1·31	·0129	29·80	8
9	45,791	630	1·38	·0135	29·19	9
20	45,161	648	1·43	·0142	28·59	20
1	44,513	666	1·50	·0148	28·00	1
2	43,847	681	1·55	·0153	27·42	2
3	43,166	691	1·60	·0159	26·84	3
4	42,475	699	1·65	·0164	26·28	4
25	41,776	705	1·69	·0168	25·70	25
6	41,071	711	1·73	·0172	25·14	6
7	40,360	721	1·79	·0177	24·56	7
8	39,639	735	1·85	·0184	24·01	8
9	38,904	753	1·94	·0191	23·45	9
30	38,151	772	2·02	·0200	22·90	30
1	37,379	791	2·12	·0209	22·37	1
2	36,588	809	2·21	·0219	21·84	2
3	35,779	825	2·30	·0228	21·33	3
4	34,954	839	2·40	·0238	20·82	4
35	34,115	851	2·49	·0248	20·31	35
6	33,264	861	2·59	·0257	19·82	6
7	32,403	870	2·68	·0267	19·33	7
8	31,533	879	2·79	·0277	18·85	8
9	30,654	888	2·90	·0288	18·38	9
40	29,766	896	3·01	·0300	17·91	40
1	28,870	903	3·13	·0312	17·45	1
2	27,967	911	3·26	·0324	17·00	2
3	27,056	915	3·38	·0337	16·56	3
4	26,141	917	3·51	·0350	16·12	4

TABLE IX.

*Life Table. All India. Male Lives (1901-1911.)*

<i>x</i>	<i>l<sub>x</sub></i>	<i>d<sub>x</sub></i>	100 <i>q<sub>x</sub></i>	<i>μ<sub>x</sub></i>	$\overset{\circ}{e}_x$	<i>x</i>
0	100,000	28,998	29.00	4.3242	22.59	0
1	71,002	6,473	9.12	0.1155	30.72	1
2	64,529	4,241	6.57	0.0797	32.76	2
3	60,288	2,913	4.83	0.0580	34.03	3
4	57,375	2,069	3.60	0.0424	34.73	4
5	55,308	1,523	2.75	0.0319	35.01	5
6	53,785	1,168	2.17	0.0248	34.99	6
7	52,617	933	1.77	0.0200	34.76	7
8	51,684	786	1.52	0.0168	34.38	8
9	50,898	686	1.35	0.0147	33.90	9
10	50,212	626	1.25	0.0134	33.36	10
1	49,586	593	1.20	0.0125	32.77	1
2	48,993	585	1.19	0.0120	32.16	2
3	48,408	590	1.22	0.0121	31.54	3
4	47,818	605	1.26	0.0125	30.93	4
15	47,213	626	1.32	0.0130	30.32	15
6	46,587	651	1.40	0.0137	29.72	6
7	45,936	676	1.47	0.0145	29.13	7
8	45,260	703	1.55	0.0152	28.56	8
9	44,557	724	1.62	0.0160	28.00	9
20	43,833	742	1.69	0.0165	27.46	20
1	43,091	758	1.76	0.0174	26.92	1
2	42,333	773	1.82	0.0181	26.39	2
3	41,560	787	1.89	0.0188	25.87	3
4	40,773	800	1.96	0.0195	25.36	4
25	39,973	817	2.03	0.0202	24.86	25
6	39,156	825	2.10	0.0210	24.37	6
7	38,331	829	2.16	0.0215	23.88	7
8	37,502	833	2.23	0.0222	23.39	8
9	36,669	838	2.30	0.0228	22.92	9

*"Makeham" Graduation.*

30	35,831	843	2.35	0.0234	22.44	30
1	34,988	853	2.44	0.0240	21.97	1
2	34,135	861	2.52	0.0251	21.51	2
3	33,274	868	2.61	0.0260	21.05	3
4	32,406	873	2.69	0.0269	20.60	4
35	31,533	877	2.78	0.0278	20.16	35
6	30,656	880	2.87	0.0287	19.72	6
7	29,776	880	2.96	0.0296	19.29	7
8	28,896	881	3.05	0.0305	18.86	8
9	28,015	879	3.14	0.0314	18.44	9
40	27,136	876	3.23	0.0324	18.02	40
1	26,260	873	3.32	0.0333	17.61	1
2	25,387	868	3.42	0.0343	17.19	2
3	24,519	862	3.52	0.0353	16.78	3
4	23,657	854	3.61	0.0363	16.38	4

TABLE VIII—*continued.*  
*Life Table. All India. Male Lives (1901.)*  
*Deduced from Censuses of 1881, 1891 and 1901.*  
*(Extracted from Mr. G. F. Hardy's Report on 1901 Census.)*

$x$	$l_x$	$d_x$	$100q_x$	$\mu_x$	$\epsilon_x$	$x$
45	25,224	917	3.64	.0364	15.69	45
6	24,397	915	3.76	.0377	15.26	6
7	23,392	911	3.89	.0390	14.84	7
8	22,481	905	4.03	.0404	14.42	8
9	21,576	898	4.16	.0418	14.00	9
50	20,678	890	4.30	.0432	13.59	50
1	19,788	881	4.45	.0447	13.18	1
2	18,907	871	4.61	.0463	12.77	2
3	18,036	859	4.76	.0480	12.36	3
4	17,177	846	4.92	.0496	11.96	4
55	16,331	832	5.09	.0514	11.55	55
6	15,499	820	5.29	.0533	11.14	6
7	14,679	807	5.50	.0554	10.73	7
8	13,872	794	5.72	.0577	10.33	8
9	13,078	781	5.97	.0602	9.93	9
60	12,297	768	6.25	.0630	9.53	60
1	11,529	755	6.55	.0661	9.13	1
2	10,774	741	6.88	.0694	8.73	2
3	10,033	727	7.25	.0732	8.34	3
4	9,306	714	7.67	.0774	7.95	4
65	8,592	699	8.14	.0822	7.57	65
6	7,893	683	8.65	.0876	7.20	6
7	7,210	665	9.22	.0935	6.83	7
8	6,545	647	9.89	.1002	6.48	8
9	5,898	625	10.60	.1078	6.13	9
70	5,273	599	11.36	.1161	5.80	70
1	4,674	569	12.17	.1250	5.48	1
2	4,105	535	13.04	.1345	5.17	2
3	3,570	498	13.95	.1447	4.87	3
4	3,072	460	14.98	.1559	4.58	4
75	2,612	421	16.12	.1686	4.30	75
6	2,191	381	17.39	.1830	4.03	6
7	1,810	340	18.79	.1992	3.77	7
8	1,470	298	20.27	.2170	3.53	8
9	1,172	256	21.86	.2364	3.29	9
80	916	215	23.55	.2571	3.07	80
1	701	177	25.36	.2796	2.87	1
2	524	143	27.29	.3053	2.66	2
3	381	112	29.37	.3347	2.48	3
4	269	85	31.56	.3662	2.30	4
85	184	62	33.88	.3995	2.14	85
6	122	44	36.35	.4344	1.97	6
7	78	30	38.96	.4744	1.79	7
8	48	20	41.74	.5208	1.62	8
9	28	13	44.69	.5893	1.41	9
90	15	8	47.86	.7000	1.23	90
1	7	4	51.29	.8571	1.07	1
2	3	2	55.10	1.0000	.83	2
3	1	1	59.49	1.5000	.50	3

TABLE IX—*continued.*  
*Life Table. All India. Male Lives (1901-1911.)*

<i>x</i>	<i>l<sub>x</sub></i>	<i>d<sub>x</sub></i>	100 <i>q<sub>x</sub></i>	<i>μ<sub>x</sub></i>	$\ddot{e}_x$	<i>x</i>
45	22,803	848	3.72	.0373	15.97	45
6	21,955	839	3.82	.0384	15.57	6
7	21,116	829	3.93	.0395	15.17	7
8	20,287	820	4.04	.0407	14.77	8
9	19,467	809	4.16	.0418	14.37	9
50	18,658	798	4.28	.0431	13.97	50
1	17,860	786	4.40	.0444	13.58	1
2	17,074	775	4.54	.0457	13.18	2
3	16,299	762	4.68	.0472	12.78	3
4	15,537	750	4.83	.0487	12.38	4
55	14,787	738	4.99	.0503	11.99	55
6	14,049	724	5.15	.0520	11.59	6
7	13,325	712	5.34	.0539	11.19	7
8	12,613	699	5.54	.0559	10.79	8
9	11,914	685	5.75	.0581	10.40	9
60	11,229	672	5.98	.0605	10.00	60
1	10,557	659	6.24	.0631	9.61	1
2	9,898	646	6.53	.0659	9.21	2
3	9,252	632	6.83	.0691	8.82	3
4	8,620	618	7.17	.0725	8.43	4
65	8,002	604	7.55	.0764	8.04	65
6	7,398	590	7.98	.0807	7.66	6
7	6,808	574	8.43	.0855	7.28	7
8	6,234	558	8.95	.0909	6.90	8
9	5,676	542	9.55	.0969	6.53	9
70	5,134	522	10.17	.1037	6.17	70
1	4,612	503	10.91	.1113	5.81	1
2	4,109	482	11.73	.1199	5.46	2
3	3,627	458	12.63	.1297	5.12	3
4	3,169	433	13.66	.1407	4.79	4
75	2,736	405	14.80	.1532	4.47	75
6	2,331	375	16.09	.1674	4.16	6
7	1,956	342	17.48	.1836	3.86	7
8	1,614	309	19.14	.2020	3.57	8
9	1,305	273	20.92	.2229	3.30	9
80	1,032	236	22.87	.2467	3.04	80
1	796	200	25.13	.2738	2.79	1
2	596	164	27.52	.3047	2.56	2
3	432	130	30.09	.3400	2.34	3
4	302	100	33.11	.3803	2.13	4
85	202	74	36.63	.4264	1.94	85
6	128	51	39.84	.4789	1.77	6
7	77	33	42.86	.5389	1.62	7
8	44	21	47.73	.6076	1.45	8
9	23	12	52.17	.6860	1.33	9
90	11	6	54.55	.7757	1.23	90
1	5	3	60.00	.8782	1.10	1
2	2	1	50.00	.9955	1.00	2
3	1	1	1.0000	...	.50	3

TABLE X.

Life Table. All India. Female Lives (1901-1911.)

$x$	$l_x$	$d_x$	$100q_x$	$\mu_x$	$\bar{e}_x$	$x$
0	100,000	28,460	28.46	4.3596	23.31	0
1	71,540	6,165	8.62	0.1080	31.49	1
2	65,375	4,027	6.16	.0740	33.42	2
3	61,348	2,766	4.51	.0535	34.58	3
4	58,582	1,974	3.37	.0396	35.19	4
5	56,608	1,485	2.62	.0300	35.40	5
6	55,123	1,169	2.12	.0236	35.34	6
7	53,954	959	1.78	.0193	35.10	7
8	52,995	817	1.54	.0165	34.73	8
9	52,178	728	1.40	.0146	34.26	9
10	51,450	663	1.29	.0134	33.74	10
1	50,787	627	1.24	.0127	33.17	1
2	50,160	615	1.23	.0122	32.58	2
3	49,545	614	1.24	.0124	31.98	3
4	48,931	627	1.28	.0127	31.38	4
15	48,304	646	1.34	.0132	30.78	15
6	47,658	670	1.41	.0138	30.19	6
7	46,988	695	1.48	.0145	29.61	7
8	46,293	722	1.56	.0153	29.05	8
9	45,571	743	1.63	.0161	28.50	9
20	44,828	761	1.70	.0168	27.96	20
1	44,067	774	1.76	.0174	27.44	1
2	43,293	786	1.82	.0180	26.92	2
3	42,507	798	1.88	.0186	26.41	3
4	41,709	808	1.94	.0193	25.90	4
25	40,901	818	2.00	.0199	25.40	25
6	40,083	825	2.06	.0205	24.91	6
7	39,258	832	2.12	.0211	24.43	7
8	38,426	838	2.18	.0217	23.94	8
9	37,588	843	2.24	.0224	23.47	9
30	36,745	848	2.31	.0230	22.99	30
1	35,897	852	2.37	.0237	22.53	1
2	35,045	855	2.44	.0244	22.06	2
3	34,190	858	2.51	.0251	21.60	3
4	33,332	861	2.58	.0258	21.14	4
35	32,471	863	2.66	.0265	20.69	35
6	31,608	865	2.74	.0273	20.24	6
7	30,743	867	2.82	.0282	19.80	7
8	29,876	868	2.91	.0290	19.36	8
9	29,008	869	3.00	.0299	18.92	9
40	28,139	868	3.08	.0309	18.49	40
1	27,271	865	3.17	.0318	18.06	1
2	26,406	861	3.26	.0327	17.64	2
3	25,545	856	3.35	.0336	17.22	3
4	24,689	850	3.44	.0345	16.80	4

TABLE X—*continued.*

Life Table. All India. Female Lives (1901-1911.)

$x$	$l_x$	$d_x$	$100l_x$	$\mu_x$	$\ddot{e}_x$	$x$
45	23,839	842	3.53	.0355	16.38	45
6	22,997	834	3.63	.0364	15.96	6
7	22,163	825	3.72	.0374	15.54	7
8	21,338	816	3.83	.0385	15.12	8
9	20,522	808	3.94	.0396	14.70	9
50	19,714	800	4.06	.0408	14.28	50
1	18,914	791	4.18	.0421	13.87	1
2	18,123	781	4.31	.0434	13.45	2
3	17,342	770	4.44	.0447	13.03	3
4	16,572	759	4.58	.0461	12.62	4
55	15,813	749	4.74	.0477	12.20	55
6	15,064	739	4.91	.0494	11.78	6
7	14,325	729	5.09	.0512	11.36	7
8	13,596	720	5.30	.0533	10.94	8
9	12,876	711	5.52	.0556	10.53	9
60	12,165	703	5.78	.0581	10.11	60
1	11,462	694	6.06	.0609	9.70	1
2	10,768	684	6.35	.0640	9.30	2
3	10,084	674	6.68	.0673	8.89	3
4	9,410	663	7.04	.0710	8.49	4
65	8,747	651	7.44	.0751	8.10	65
6	8,096	638	7.88	.0796	7.71	6
7	7,458	623	8.36	.0845	7.33	7
8	6,835	608	8.89	.0901	6.95	8
9	6,227	590	9.48	.0962	6.58	9
70	5,637	570	10.12	.1029	6.22	70
1	5,067	548	10.82	.1103	5.86	1
2	4,519	525	11.63	.1187	5.51	2
3	3,994	500	12.51	.1283	5.17	3
4	3,494	472	13.50	.1391	4.84	4
75	3,022	441	14.60	.1511	4.52	75
6	2,581	409	15.83	.1647	4.20	6
7	2,172	374	17.22	.1802	3.90	7
8	1,798	340	18.89	.1986	3.61	8
9	1,458	299	20.52	.2191	3.33	9
80	1,159	261	22.49	.2416	3.06	80
1	898	222	24.69	.2689	2.81	1
2	676	184	27.16	.3003	2.57	2
3	492	147	29.90	.3364	2.34	3
4	345	114	32.90	.3783	2.12	4
85	231	84	36.28	.4286	1.93	85
6	147	59	39.93	.4864	1.74	6
7	88	39	44.00	.5568	1.56	7
8	49	24	48.50	.6429	1.40	8
9	25	13	53.50	.7400	1.24	9
90	12	7	59.07	.8333	1.10	90
1	5	3	65.30	1.0000	.96	1
2	2	1	72.39	1.0000	.84	2
3	1	1	80.34	1.0000	.72	3

TABLE VI.

*Showing the adjusted decennial rate of increase at each age, as deduced from the 1901 and 1911 Census figures for each Province.*

*Male Lives.*

Age $x$	Bengal	Bombay	Burma	Madras	United Provinces	Age $x$	Bengal	Bombay	Burma	Madras	United Provinces
0	1.111	1.122	1.107	1.082	.966	45	1.029	1.065	1.146	1.088	1.028
1	1.107	1.116	1.118	1.031	.970	6	1.028	1.067	1.147	1.087	1.031
2	1.104	1.112	1.126	1.081	.975	7	1.027	1.069	1.147	1.086	1.033
3	1.100	1.107	1.132	1.081	.980	8	1.026	1.071	1.148	1.086	1.035
4	1.097	1.103	1.136	1.080	.984	9	1.026	1.074	1.148	1.086	1.037
5	1.094	1.098	1.139	1.080	.989	50	1.027	1.076	1.148	1.086	1.038
6	1.092	1.094	1.142	1.080	.994	1	1.028	1.078	1.148	1.087	1.039
7	1.089	1.090	1.143	1.081	.999	2	1.029	1.080	1.148	1.088	1.040
8	1.087	1.087	1.144	1.081	1.004	3	1.030	1.083	1.148	1.089	1.040
9	1.085	1.083	1.145	1.082	1.009	4	1.031	1.085	1.148	1.090	1.040
10	1.083	1.080	1.145	1.083	1.013	55	1.033	1.087	1.148	1.092	1.039
1	1.082	1.077	1.145	1.084	1.016	6	1.034	1.090	1.148	1.094	1.038
2	1.081	1.074	1.144	1.084	1.018	7	1.035	1.092	1.148	1.097	1.037
3	1.080	1.070	1.144	1.086	1.019	8	1.036	1.094	1.148	1.100	1.035
4	1.079	1.068	1.142	1.087	1.019	9	1.036	1.096	1.148	1.104	1.033
15	1.079	1.065	1.140	1.088	1.018	60	1.036	1.098	1.148	1.108	1.031
6	1.079	1.062	1.138	1.090	1.016	1	1.036	1.100	1.148	1.113	1.028
7	1.080	1.060	1.135	1.092	1.012	2	1.035	1.101	1.148	1.118	1.025
8	1.080	1.057	1.131	1.094	1.009	3	1.033	1.103	1.148	1.123	1.022
9	1.080	1.055	1.127	1.096	1.005	4	1.032	1.104	1.148	1.127	1.020
20	1.081	1.053	1.123	1.098	1.002	65	1.030	1.106	1.148	1.132	1.017
1	1.081	1.051	1.118	1.100	.998	6	1.027	1.107	1.148	1.136	1.014
2	1.081	1.049	1.114	1.102	.995	7	1.025	1.108	1.148	1.140	1.011
3	1.080	1.048	1.109	1.104	.993	8	1.022	1.109	1.148	1.143	1.008
4	1.080	1.046	1.105	1.105	.991	9	1.020	1.110	1.148	1.146	1.005
25	1.078	1.046	1.101	1.106	.990	70	1.018	1.110	1.148	1.150	1.003
6	1.076	1.045	1.099	1.106	.989	1	1.015	1.110	1.148	1.153	1.000
7	1.074	1.044	1.097	1.106	.989	2	1.013	1.110	1.148	1.155	.998
8	1.073	1.044	1.096	1.106	.990	3	1.011	1.110	1.148	1.158	.996
9	1.072	1.044	1.097	1.106	.990	4	1.009	1.110	1.148	1.160	.994
30	1.071	1.044	1.099	1.105	.991	75	1.007	1.110	1.148	1.162	.992
1	1.070	1.044	1.102	1.105	.993	6	1.006	1.109	1.148	1.164	.991
2	1.069	1.043	1.103	1.104	.994	7	1.005	1.108	1.148	1.165	.989
3	1.066	1.046	1.109	1.103	.996	8	1.004	1.106	1.148	1.166	.988
4	1.064	1.047	1.114	1.102	.998	9	1.003	1.104	1.148	1.167	.987
35	1.061	1.048	1.118	1.100	1.000	80	1.002	1.103	1.148	1.168	.986
6	1.057	1.049	1.123	1.099	1.003	1	1.002	1.100	1.148	1.169	.985
7	1.054	1.050	1.128	1.098	1.005	2	1.001	1.098	1.148	1.169	.985
8	1.050	1.052	1.132	1.096	1.008	3	1.001	1.095	1.148	1.169	.984
9	1.046	1.053	1.135	1.095	1.011	4	1.001	1.092	1.148	1.169	.984
40	1.043	1.055	1.138	1.093	1.014	85	1.000	1.088	1.148	1.169	.983
1	1.039	1.057	1.141	1.092	1.017	6	1.000	1.085	1.148	1.169	.983
2	1.036	1.059	1.143	1.091	1.020	7	1.000	1.080	1.148	1.169	.983
3	1.033	1.061	1.144	1.090	1.022	8	1.000	1.076	1.148	1.169	.983
4	1.030	1.063	1.145	1.088	1.025	9	1.000	1.071	1.148	1.169	.982
...	...	...	...	...	...	90	1.000	1.066	1.148	1.169	.982

NOTE.—For the Punjab, a constant decennial rate of increase of 1.0088 was employed at all ages.

## APPENDIX.

(A).—*Notes on the Graduation of the Age-Distribution Tables. Prepared by S. J. GUNNINGHAM, B.Sc., F.I.A.*

THE data were set out in quinquennial groups, as shown in Table IV and the moments first found, about an origin at age  $22\frac{1}{2}$ .

These moments are shown as  $v_1$ , &c.

The calculation of the moments about the mean of the distribution then follows directly from the equations

$$\mu_1 = 0$$

$$\mu_2 = v_2 - d^2$$

$$\mu_3 = v_3 - 3d\mu_2 - d^3$$

$$\mu_4 = v_4 - 4d\mu_3 - 6d^2\mu_2 - d^4$$

giving the values for  $\mu_1$ , &c.

The next step is to decide which frequency curve is most suitable to the distribution under observation; in the Pearsonian system of curves, assistance at this point is given by the use of an expression known as the criterion, namely,

$$\kappa = \frac{\beta_1(\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)}$$

where  $\beta_1 = \frac{\mu_3^2}{\mu_2^3}, \beta_2 = \frac{\mu_4^2}{\mu_2^2}$

In the case before us, this expression is less than zero and equal to  $-256$ , showing that the curve, whose equation, when referred to the mode as origin, is  $y = y_0 \left(1 + \frac{x}{a_1}\right)^{m_1} \left(1 - \frac{x}{a_2}\right)^{m_2}$ , is the most suitable.

The values of  $r$ ,  $a_1$ ,  $a_2$ ,  $m_1$ ,  $m_2$ ,  $y_0$ , are then obtained as shown below:

$$\beta_1 = \mu_3^2 \div \mu_2^3 = \log^{-1}(1.7086004) = \underline{\underline{.5112112}}$$

$$\beta_2 = \mu_4^2 \div \mu_2^2 = \log^{-1}(0.4630668) = \underline{\underline{2.9044701}}$$

$$\kappa = \frac{\beta_1(\beta_2 + 3)^2}{4(4\beta_2 - 3\beta_1)(2\beta_2 - 3\beta_1 - 6)} = -\log^{-1}(1.40855) = -\underline{\underline{.2562}}$$

$$r = \frac{6(\beta_2 - \beta_1 - 1)}{6 + 3\beta_1 - 2\beta_2} = \log^{-1}(0.6854715) = \underline{\underline{4.846982}}$$

$$b = \frac{1}{2} \sqrt{\mu_2} \sqrt{\beta_1(r+2)^2 + 16(r+1)} = \log^{-1}(1.2710562) = \underline{\underline{18.66621}}$$

$$\frac{m_2}{m_1} = \frac{1}{2} \left\{ r - 2 \pm r \sqrt{\frac{\beta_1(r+2)^2}{\beta_1(r+2)^2 + 16(r+1)}} \right\} = \frac{2.517923}{.329059}$$

$$a_2 = m_2 \cdot \frac{b}{r-2} = \log^{-1}(1.2177139) = \underline{\underline{16.50874}}$$

$$a_1 = m_1 \frac{b}{r-2} = \log^{-1}(0.3339453) = \underline{\underline{2.15747}}$$

$$y_0 = \frac{N}{b^{m_1} + m_2 + 1} \quad \frac{\Gamma(r)}{\Gamma(m_2+1)\Gamma(m_1+1)} = \log^{-1}(0.910255) = \underline{\underline{8.13308}}$$

$$\text{Mode} = \text{Mean} - \frac{1}{2} \frac{\mu_3}{\mu_2} \frac{r+2}{r-2} = -2.714071$$

$$\therefore \text{Start of curve at Mode} - a_1 \\ = -2.71407 - 2.15747 = -4.87154 \text{ (i.e., at age } -1.8577)$$

It should be noted that, for the purposes of the numerical work, it is desirable to record the values of  $\log \mu_2$ ,  $\log \mu_3$ , as these reappear two or three times during the work; also the expression  $2\beta_2 - 3\beta_1 - 6$  appears in two places, namely, in  $\kappa$  and in  $r$ , while the expression  $\beta_1(r+2)^2 + 16(r+1)$  appears in  $b$  and in  $m_1$  and  $m_2$ .

A slight check is provided at one or two places, namely,

$$m_1 + m_2 = r - 2$$

$$a_1 + a_2 = b$$

In calculating the values for the ordinates from the curve, I have found it more convenient to refer the curve to its starting point ( $x = -a_1$ ) as origin. It then takes the form

$$y = y_0 x^{m_1} (b - x)^{m_2}$$

and it will be noticed that  $y_0$  in the calculations above refers to this form of the curve.

Writing in the values found for the constants, the curve is for quinary intervals of age

$$y = 8.133 \dots x^{.320} \dots (18.66 \dots - x)^{2.517} \dots$$

and therefore for unit intervals of age

$$y = .016 \dots x^{.320} \dots (93.33 \dots - x)^{2.517} \dots$$

An inspection of this curve shows that it is unsuitable for the young ages ;  $m_1$  having a positive value, it starts at zero and rises to a maximum at age 8. It appeared, however, to suit the adult ages very well, and the values for the quinary groups are shown in Table  $\beta$ .

The only point calling for remark in working out this table relates to column (8). Column (7) gives the values of the ordinates as found from the curve, and from these figures the frequency distribution is given in column (8) by the formula

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} u_0 dx = u_0 + \frac{1}{24} \Delta^2 u_{-1}$$

for all values except the first two. For these this formula is plainly inapplicable ; moreover, the rapid variation in the rise of the curve would render its application unsuitable.

The best method, and one not attended by any very laborious work, is to calculate the area directly from the expression

$$y_0 b^{m_2} x^{m_1+1} \left( \frac{1}{m_1+1} - \frac{m_2}{b} \cdot \frac{x}{m_1+2} + \frac{m_2(m_2-1)}{2b^2} \cdot \frac{x^2}{m_1+3} - \dots \right)$$

(obtained by integrating the expression for the curve between the limits  $x$  and 0), between the limits in this case of

$$(i) \quad x = 37154$$

$$x = 0$$

and of

$$(ii) \quad x = 137154$$

$$x = 0$$

It is not necessary to go any further than the term  $\frac{m_2}{b} \cdot \frac{x}{m_1+2}$  and we obtain

$$(i) \quad \int_0^{37154} = 2,523$$

$$(ii) \quad \int_0^{137154} = 13,275$$

Subtraction gives 10,752, the number for the age group 0 to 4.

The original figures run so unevenly that it is not easy to apply any of the recognized tests, with a view to a comparison of the graduated and ungraduated results ; the following table, however, will show the main results :

Age	NUMBER ABOVE AGE OUT OF TOTAL OF 100,000	
	For Ungraduated	For Graduated
15	61,442	62,665
35	25,105	24,737
55	5,305	5,836

In Table  $\gamma$  is given the age distribution calculated from the formula for unit intervals of age. As far as whole numbers are concerned, the addition of  $\frac{1}{24} \Delta^2 u_{-1}$  has little or no effect throughout the greater part of the table.

As explained in the paper, these figures were adopted from age 19 onwards. In the absence of reliable data for young ages, the age distribution for the earlier ages was obtained by a separate process of which the details are shown in Table  $\delta$ .

Column (1) is the log of the rate of increase per annum of the population during the decennium, found as described in paragraphs (22) (23); column (2) is obtained from this by summing continuously from the bottom upwards; adding these values to those of  $\log L_x$  in column (3) (where  $L_x$  is used in the ordinary meaning of the numbers living between ages  $x$  and  $x+1$  in a stationary population, and has been found by an independent investigation of the Proclaimed Clans data, see paragraph (25)), and reducing these figures, in column (4), to a population of 2,120 at age 20, we arrive at column (5), giving the log of the numbers living in a population which has been increasing at the rate per annum given by column (1) which is the rate of increase taken as applicable to the population with which we are dealing. The numbers living are then given in column (6) joining on at age 19 to those given in Table  $\gamma$ . Finally the column is reduced to a total of 100,000 by multiplying by the factor  $\frac{1}{1.05638}$ .

TABLE *a.**Bombay (Males).*

Central Age	$x$	$L_x$	$\times x$	$\times x^2$	$\times x^3$	$\times x^4$
$2\frac{1}{2}$	-4	14442	- 57768	231072	- 924288	3697152
$7\frac{1}{2}$	-3	12824	- 38472	115416	- 346248	1038744
$12\frac{1}{2}$	-2	11292	- 22584	45168	- 90336	180672
$17\frac{1}{2}$	-1	9494	- 9494	9494	- 9494	9494
$22\frac{1}{2}$	0	8942	- 128318		- 1370366	
$27\frac{1}{2}$	1	9920	9920	9920	9920	9920
$32\frac{1}{2}$	2	7981	15962	31924	63848	127696
$37\frac{1}{2}$	3	6889	20667	62001	186003	558009
$42\frac{1}{2}$	4	5469	21876	87504	350016	1400064
$47\frac{1}{2}$	5	4180	20900	104500	522500	2612500
$52\frac{1}{2}$	6	3262	19572	117432	704592	4227552
$57\frac{1}{2}$	7	2110	14770	103390	723730	5066110
$62\frac{1}{2}$	8	1687	13496	107968	863744	6909952
say 75	$10\frac{1}{2}$	1508	15834	166257	1745698	18329829
			+ 152997		+ 5170051	
		1·00000	+ 24679	+ 11·92046	+ 37·99685	+ 441·67694

$$v_1 = .24679 \quad \therefore d = .24679 \text{ (at age } 23\cdot7340)$$

$$v_2 = 11\cdot92046 \quad \mu_2 = 11\cdot85956 = \log^{-1}(1\cdot0740686)$$

$$v_3 = 37\cdot99685 \quad \mu_3 = 29\cdot20136 = \log^{-1}(1\cdot4654031)$$

$$v_4 = 441\cdot67694 \quad \mu_4 = 408\cdot51305 = \log^{-1}(2\cdot6112060)$$

TABLE  $\beta$ .*Bombay (Males).*

Central Age	$x$	$18.66 - x$	$\cdot 329 \dots$ $\log x$	$2.51 \dots$ $\log (18.66 - x)$	$\text{Log } y, i.e.,$ $(4) + (5)$ $+ \cdot 910255$	$y, i.e.,$ $\text{Antilog } (6)$	Frequency
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
					Area to left of age 0 =		
2 $\frac{1}{2}$	87154	17.79467	1.980351	3.148134	1.038740	10,933	10,752
7 $\frac{1}{2}$	1.	16.	0.089570	.084886	5.084711	12,154	12,097
12 $\frac{1}{2}$	2.	15.	.150747	.017756	4.078753	11,988	11,963
17 $\frac{1}{2}$	3.	14.	.193419	2.946234	4.949938	11,219	11,206
22 $\frac{1}{2}$	4.	13.	.226283	.869705	4.006243	10,145	10,139
27 $\frac{1}{2}$	5.	12.	.252965	.787414	3.950634	8,926	8,924
32 $\frac{1}{2}$	6.	11.	.275440	.698420	3.884115	7,658	7,659
37 $\frac{1}{2}$	7.	10.	.294857	.601541	.806653	5,407	6,410
42 $\frac{1}{2}$	8.	9.	.311948	.495237	.717440	5,217	5,221
47 $\frac{1}{2}$	9.	8.	.327211	.377471	.614937	4,120	4,125
52 $\frac{1}{2}$	10.	7.	.341001	.245479	.496735	3,139	3,145
57 $\frac{1}{2}$	11.	6.	.353576	.095335	.359166	2,286	2,293
62 $\frac{1}{2}$	12.	5.	.365134	1.921248	.196637	1,573	1,580
67 $\frac{1}{2}$	13.	4.	.375826	.714099	.000180	1,000	1,007
72 $\frac{1}{2}$	14.	3.	.385774	.458316	2.754345	568	575
77 $\frac{1}{2}$	15.	2.	.395074	.123827	.429156	269	274
82 $\frac{1}{2}$	16.	1.	.403806	.0639515	1.953576	90	94
87 $\frac{1}{2}$	17.	0.	.412035	1.748679	.070969	12	13
92 $\frac{1}{2}$	18.	...	...	...	...	...	100,000

TABLE γ. *Bombay (Males).*

Central Age	$y$ (Antilog (6))	Central Age	$y$ (Antilog (6))	Central Age	$y$ (Antilog (6))
$\frac{1}{2}$	1889	$35\frac{1}{2}$	1380	$70\frac{1}{2}$	145
$1\frac{1}{2}$	2064	6	1331	1	129
$2\frac{1}{2}$	2187	7	1281	2	114
$3\frac{1}{2}$	2275	8	1233	3	100
$4\frac{1}{2}$	2338	9	1184	4	87
	10753		6409		575
$5\frac{1}{2}$	2382	$40\frac{1}{2}$	1137	$75\frac{1}{2}$	75
6	2412	1	1090	6	64
7	2430	2	1043	7	54
8	2438	3	998	8	45
9	2437	4	953	9	37
	12099		5221		275
$10\frac{1}{2}$	2431	$45\frac{1}{2}$	909	$80\frac{1}{2}$	30
1	2417	6	866	1	23
2	2398	7	824	2	18
3	2374	8	783	3	13
4	2346	9	740	4	10
	11966		4125		94
$15\frac{1}{2}$	2315	$50\frac{1}{2}$	703	$85\frac{1}{2}$	7
6	2281	1	665	6	4
7	2244	2	628	7	2
8	2204	3	591	8	1
9	2163	4	556	9	0
	11207		3143		14
$20\frac{1}{2}$	2120	$55\frac{1}{2}$	522	$90\frac{1}{2}$	...
1	2075	6	489	1	...
2	2029	7	457	2	...
3	1982	8	427	3	...
4	1933	9	397	4	...
	10139		2292		
$25\frac{1}{2}$	1884	$60\frac{1}{2}$	368		
6	1835	1	240		
7	1785	2	315		
8	1735	3	289		
9	1684	4	265		
	8923		1577		
$30\frac{1}{2}$	1633	$65\frac{1}{2}$	242		
1	1582	6	221		
2	1932	7	200		
3	1481	8	181		
4	1431	9	162		
	7659		1006		

TABLE 8.

*Bombay (Males).*

Age	Log (rate of increase per annum) = $\frac{1}{10}$ rate	Log (Factor for stationary population to obtain actual population)	Log $L_x$	(2) + (3)	(4) + $\bar{2} \cdot 67435$ reducing to proportion of 2120 at age 20	Antilog (5), giving numbers living
	(1)	(2)	(3)	(4)	(5)	(6)
0	.00498	.06991	4.88526	4.95517	3.62952	4,261
1	478	6493	.82485	.88978	.56413	3,665
2	459	6015	.78940	.84955	.52390	3,341
3	442	5556	.76420	.81976	.49411	3,120
4	424	5114	.74601	.79715	.47150	2,961
						17,348
5	408	4690	.73268	.77958	.45393	2,844
6	392	4282	.72264	.76546	.43981	2,753
7	376	3890	.71482	.75372	.42807	2,680
8	362	3514	.70848	.74362	.41797	2,618
9	347	3152	.70311	.73463	.40898	2,564
						13,459
10	333	2805	.69837	.72642	.40077	2,516
1	321	2472	.69404	.71876	.39311	2,472
2	308	2151	.68999	.71150	.38585	2,437
3	296	1843	.68606	.70449	.37884	2,392
4	284	1547	.68212	.69759	.37194	2,355
						12,172
5	273	1263	.67807	.69070	.36505	2,318
6	263	0990	.67378	.68368	.35803	2,280
7	252	0727	.66913	.67640	.35075	2,243
8	242	0475	.66400	.66875	.34310	2,203
9	233	0233	.65830	.66063	.33498	2,163
	6991					11,207

(B).—Note on the Graduation of the Punjab figures. Communicated by S. J. ROWLAND, A.I.A.

THE mean census figures for quinary groups, reduced to a total of 100,000, and adjusted for age as already explained, are given in column (2) below, and these are summed from the bottom upwards, and reduced to a total of 1, in column (3). The next step was to deduce, from the table given on pp. 138-9 of Mr. G. F. Hardy's lectures "On the Theory of the Construction of Tables of Mortality", values of  $z$ , corresponding to the values of  $Y_x$  in column (3), where  $z$  is the upper limit in the expression

$$Y_x = \frac{1}{\sqrt{\pi}} \int_{-z}^z e^{-x^2} dx$$

It will be observed that, where the value of  $Y_x$  is  $> \frac{1}{2}$ , the argument with which to enter the table is  $(2Y_x - 1)$ , and that, where  $Y_x$  is  $< \frac{1}{2}$ , the argument is  $(1 - 2Y_x)$ , giving in that case negative values of  $z$ :

Age Group	Numbers Living (from Table IV)	$Y_x = \frac{\Sigma(2)}{100,000}$	VALUES OF $z$			Graduated Values of $Y_x$	Graduated Numbers Living
			Ungraduated	First Graduation	Final Graduation		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0-4	14,486	1.00000	$\infty$	...	...	1.00000	15,270
5-9	12,747	.86514	+.7804	...	+.7248	.84730	12,327
10-14	11,588	.72767	.4283	+.3361	.4211	.72403	11,396
15-19	10,569	.61179	.2008	.1728	.1976	.61007	10,390
20-24	8,272	.50610	.0108	.0123	.0109	.50617	9,340
25-29	9,139	.42338	-.1367	-.1451	-.1519	.41277	8,286
30-34	7,204	.33199	.3068	.2997	.3110	.33001	7,235
35-39	6,272	.25995	.4550	.4514	.4600	.25766	6,185
40-44	4,996	.19723	.6021	.6003	.6061	.19571	5,141
45-49	4,210	.14727	.7401	.7463	.7501	.14490	4,146
50-54	3,312	.10517	.8558	.8893	.8948	.10284	3,252
55-59	2,278	.07205	1.0329	1.0295	1.0419	.07032	2,493
60	4,927	.04927	1.1674	1.1668	1.1960	.04539	4,539
	100,000	...	...	...	...	...	100,000

Examination of the progression of the values of  $z$  in column (4) showed that a first approximation to the values, after age 9, would be obtained by an equation of the second order

$$z_x = .671320 - .03409847x + .00005771x^2$$

and these values are entered in column (5).

On plotting down the differences between columns (4) and (5), it was found that they ran in a fairly regular curve, but with marked inflections at about ages 20, 45, and 55. The differences were then graduated by the graphic method, and trial values of  $z$  and  $Y_x$  obtained. It was then found that the adjusted curve representing the population figures showed an abnormal number of survivors at about age 60, and, as this could hardly be considered truly to represent the facts, this section of the curve was adjusted graphically. The finally graduated values of  $z$  and of  $Y_x$  are given in columns (6) and (7), and the final population figures in column (8).

(c).—SYNOPSIS OF GRADUATION FORMULAS ADOPTED.

(1). *Age Distribution—Male Lives.*

Values of  $Y_x$  = population living at curtate age  $x$  :—

$$\text{Bengal} : \quad Y_x = .710668x^{.007758}(90 - x)^{1.863567};$$

after age 59, a subtractive correction of the form  $(a + bc^x)$  was applied.

*Bombay* :

$$Y_x = .0166468(x + 2.35770)^{.329059}(90.97335 - x)^{2.517923}.$$

$$\text{Burma} : \quad Y_x = .432273x^{.049484}(95 - x)^{1.911777};$$

after age 59, an expression of the form  $(a - bx - km^x)$  was substituted.

$$\text{Madras} : \quad Y_x = .60994x^{.055316}(90 - x)^{1.868926};$$

from age 10 to age 40, a subtractive expression equal to

$$\cdot000064(40 - x) + \cdot0000098(40 - x)^2 - \cdot0000022(40 - x)^3$$

was applied to the *logarithm* of  $Y_x$ , deduced as above.

*United Provinces* :

$$Y_x = .054608(x + 1.33175)^{.278872}(90.88530 - x)^{2.278430};$$

*Punjab* : The graduation employed was

$$Y_x = \frac{1}{\sqrt{\pi}} \int_{-\infty}^z e^{-x^2} dx \text{ (normal curve of error)}$$

where  $Y_x$  represents the population recorded above age  $x$ , relative to a total population of 1 at all ages; and  $z$  is a function of  $x$ .

determined, by examination of the unadjusted data, as of the form  $(a + bx + cx^2)$  for all values of  $x$ .

The above formulæ were employed, generally speaking, from about age 19 to the end of life. For ages under 19, the age distribution was determined by combining the rate of mortality shown amongst the Proclaimed Clans, modified as explained below, with the deduced annual rate of increase of the population at each age.

(2). *Rates of Mortality at ages 0 to 12, deduced from "Proclaimed Clans" data.*

MR. G. F. HARDY's formulæ for the graduation of the rates of mortality were as follows :

$$l_x = 53,675 - 492x + 24,610(\cdot65)^x + \frac{21,715}{(20x+1)}$$

$$L_x = \int_x^{x+1} l_x dx = 53,429 - 492x + 19,997\cdot6(\cdot65)^x + 2,500 \log_{10} \frac{20x+21}{20x+1}$$

These formulæ were employed, in 1911, for the *United Provinces* from age 0 to 12, and in the *Punjab* from age 0 to 6, after which the values were adjusted, so as to make a smooth junction with those already deduced for ages 19 and upwards.

The formulæ given above were modified in the remaining Provinces, as under :

*Bengal :*

$$l_x = 53,675 - 567x + 24,610(\cdot65)^x + \frac{21,715}{20x+1}$$

$$L_x = 53,391\cdot5 - 567x + 19,997\cdot6(\cdot65)^x + 2,500 \log_{10} \frac{20x+21}{20x+1}$$

*Bombay :*

$$l_x = 53,675 - 392x + 24,610(\cdot65)^x + \frac{21,715}{20x+1}$$

$$L_x = 53,479 - 392x + 19,997\cdot6(\cdot65)^x + 2,500 \log_{10} \frac{20x+21}{20x+1}$$

*Burma :*

$$l_x = 65,256\cdot25 - 369x + 18,457\cdot5(\cdot65)^x + \frac{16286\cdot25}{20x+1}$$

$$L_x = 65,071\cdot75 - 369x + 14,998\cdot2(\cdot65)^x + 1,875 \log_{10} \frac{20x+21}{20x+1}$$

*Madras*: The method followed in this Province (of reducing the force of mortality in the Proclaimed Clans Table by  $10\frac{1}{2}$  per-cent at ages 0-12) does not lend itself readily to expressions similar to the above, but the following formulæ give results closely approximating to those finally obtained, which were deduced by a somewhat different method :

$$l_x = 57,016 - 481x + 23,341(\cdot65)^x + \frac{19,643}{20x+1}$$

$$L_x = \int_x^{x+1} l_x dx = 56,775\cdot5 - 481x + 18,964(\cdot65)^x + 2,261\cdot5 \log_{10}\left(\frac{20x+21}{20x+1}\right)$$

*All India—Male Lives*: The numbers living in this table, which were deduced by a weighted combination of the Provincial Tables, are represented by the formulæ :

$$l_x = 54,922 - 523x + 24,000(\cdot65)^x + \frac{21,078}{20x+1}$$

$$L_x = \int_x^{x+1} l_x dx = 54,660\cdot5 - 523x + 19,499(\cdot65)^x + 2,427 \log_{10}\left(\frac{20x+21}{20x+1}\right)$$

$$\mu_x = -\frac{1}{l_x} \frac{d}{dx} l_x = \left( \frac{523 + 10,339(\cdot65)^x + \frac{421,560}{(20x+1)^2}}{l_x} \right)$$

For the 1901 figures, as deduced by Mr. G. F. Hardy by the same methods, the formulæ are :

$$l_x = 55,682 - 509x + 23,535(\cdot65)^x + \frac{20,783}{20x+1}$$

$$L_x = 55,427\cdot5 - 509x + 19,122(\cdot65)^x + 2,393 \log_{10}\left(\frac{20x+21}{20x+1}\right)$$

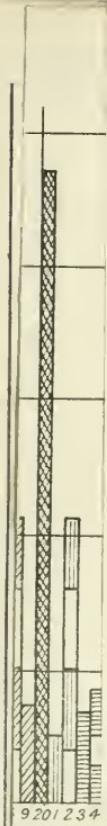
$$\mu_x = \left( \frac{509 + 10,139(\cdot65)^x + \frac{415,660}{(20x+1)^2}}{l_x} \right)$$

*All India—Female Lives (1911)*:

$$l_x = 56,582 - 553x + 22,125(\cdot65)^x + \frac{21,293\cdot6}{20x+1}$$

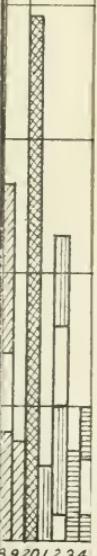
$$L_x = 56,306 - 553x + 17,978(\cdot65)^x + 2,452 \log_{10}\left(\frac{20x+21}{20x+1}\right)$$

$$\mu_x = \left( \frac{553 + 9,531(\cdot65)^x + \frac{425,872}{(20x+1)^2}}{l_x} \right)$$



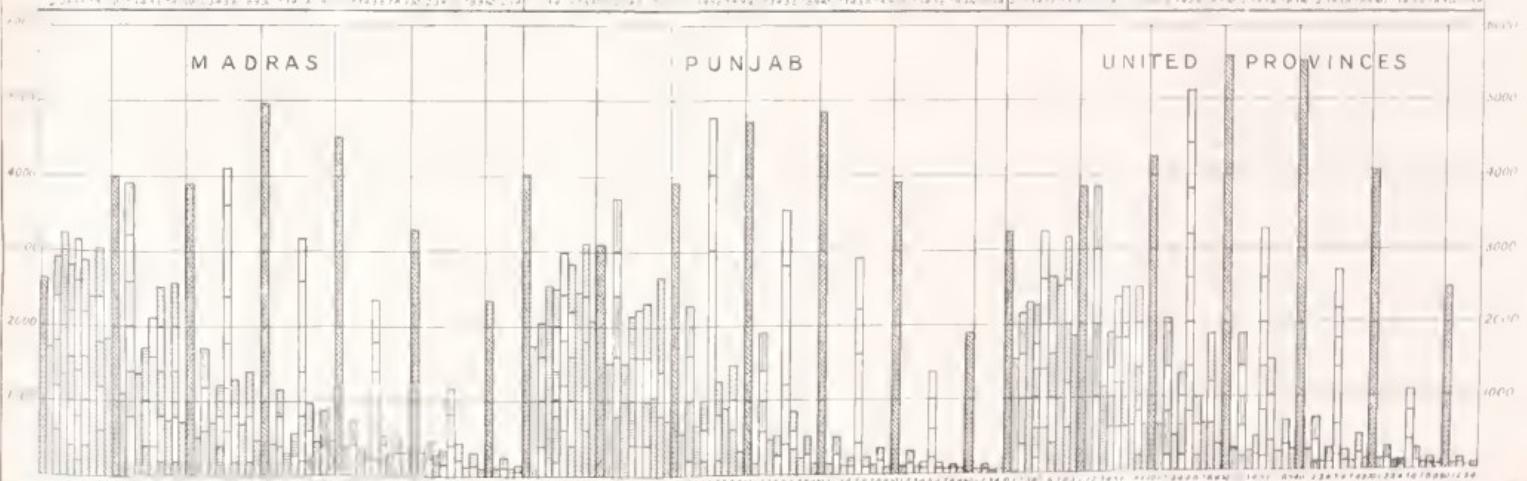
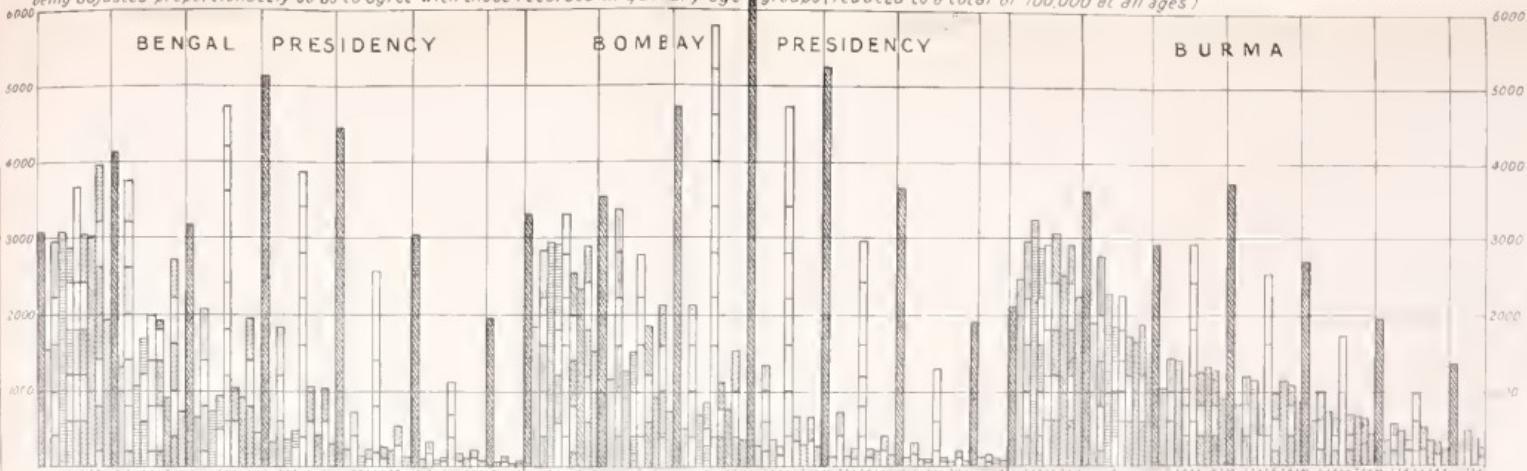
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9 20 1 2 3 4

INDIAN CENSUS 1911. MALE LIVES. Relative Numbers enumerated at each age as deduced from the Spec men Schedules prepared in each Province the numbers being adjusted proportionately so as to agree with those recorded in quinary age groups (reduced to a total of 100,000 at all ages)



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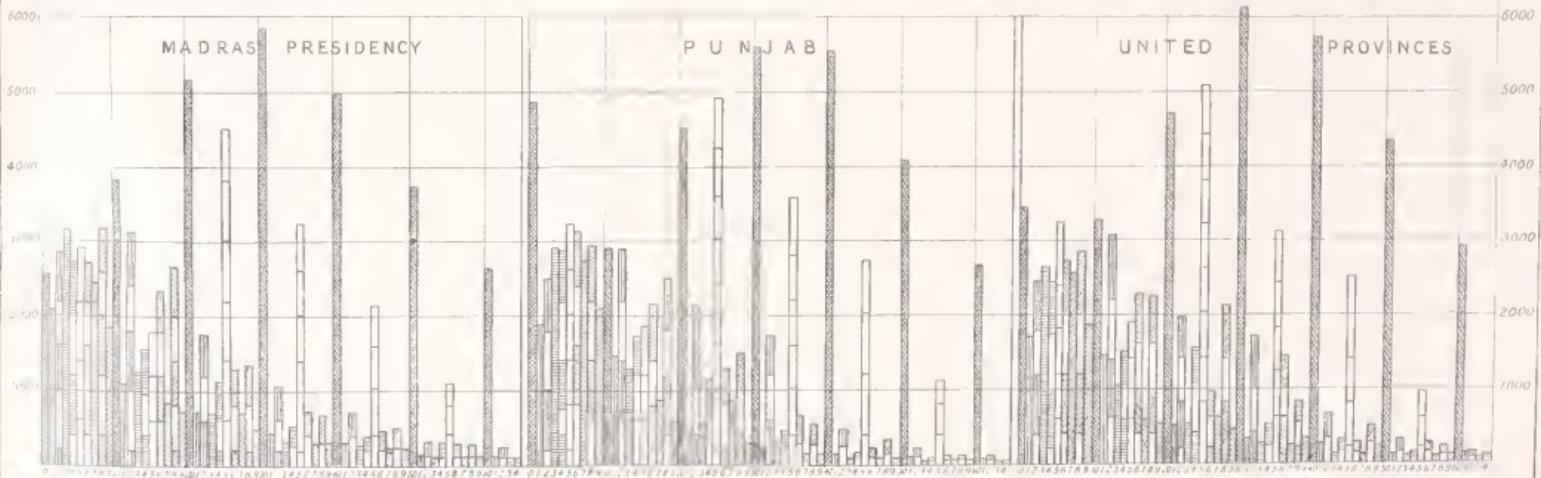
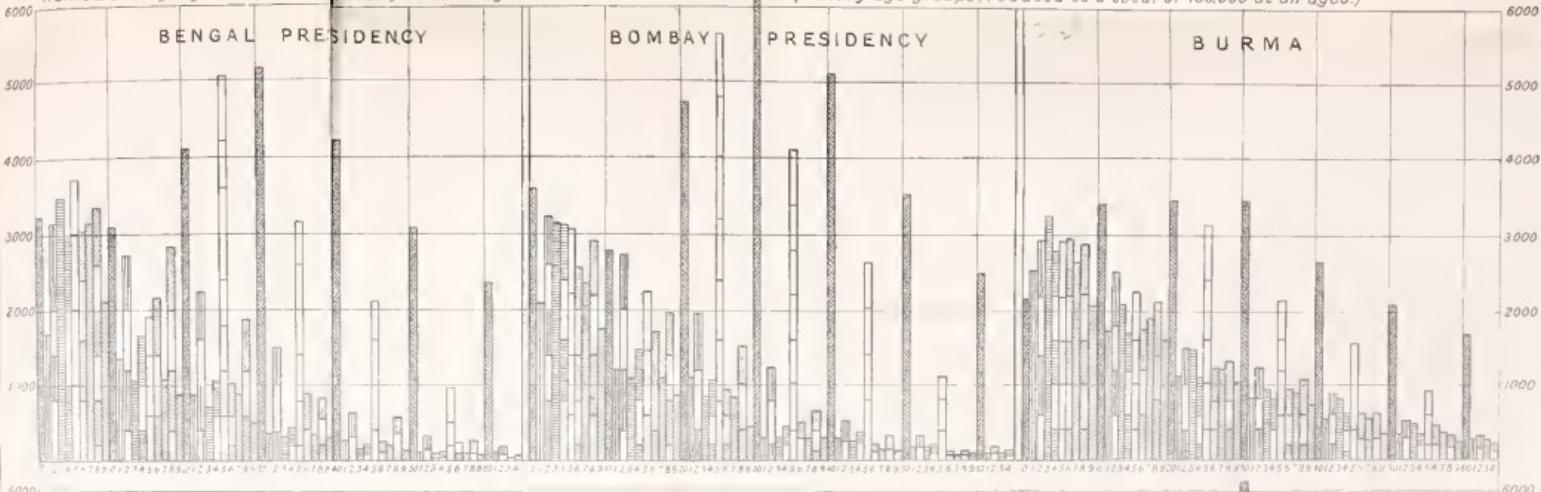
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INDIAN CENSUS 1911, FEMALE LIVES. Relative Numbers enumerated at each age as deduced from the Specimen Schedules prepared in each Province, the numbers being adjusted proportionately so as to agree with those recorded in quinary age groups (reduced to a total of 100,000 at all ages.)



(3) *Decennial rate of increase in Male Population.*

Values of  $\log r_x$ , where  $r_x$  is the rate of increase in the decennium at curtate age  $x$ :—

*Bengal*:

$$\log r_x = \cdot03(e)^{-\frac{(x-25)^2}{500} \log_{e^{10}}} + \cdot015(e)^{-\frac{(x-50)^2}{333.3} \log_{e^{10}}}$$

$$+ \cdot045 - \cdot0016x \text{ (at ages 0 to 25 only)}$$

$$+ \frac{500}{1 + \cdot32(x-25) + 3.9792(x-25)^3} \text{ (at ages 25 to 37 only)}$$

*Bombay*:

$$\log r_x = e^{-(3 + \cdot04x)} \text{ (at ages 0 to 20 only)}$$

$$\cdot02237077 - \cdot000894831x + \cdot0000600851x^2$$

$$- \cdot000000659986x^3 \text{ (age 20 to end of life)}$$

These two curves join at age 20, at which point the differential coefficient of  $\log r$  was made identical for both expressions.

*Burma*:

$$\log r_x = \cdot06 - \cdot02(e)^{-\frac{(x-28)^2}{200} \log_{e^{10}}} - \cdot015805(\cdot73114)^x$$

*Madras*:

$$\log r_x = \cdot040577 - \cdot000907(x-19) + \cdot000028(x-19)^2 \text{ (ages 0 to 20)}$$

$$- \cdot039855 - \cdot0005585(x-37) + \cdot0000016(x-37)^2 \text{ (ages 20 to 60)}$$

$$- \cdot046476 + \cdot0019807(x-60) - \cdot000046(x-60)^2 \text{ (age 60 to end)}$$

*Punjab*:

$$\log r = \cdot00380; \quad r = 1.0088, \text{ at all ages.}$$

*United Provinces*:

$$\log r_x = \cdot025(e)^{-\frac{(x-53)^2}{288}} - \cdot004 + \cdot017(e)^{-\frac{(x-13)^2}{69.106}} - \cdot004 \\ - \cdot00865(\cdot83255)^x$$

## NOTE.

The following paragraphs of this Paper deal with supplementary investigations, made after completion of the Report to the Indian Government:—§§4, 15 (Table C), 16, 25 (formulae), 33, 37 (Tables J and K), 43 (Tables O and P), 44, 45, 46 (Table Q), 47–54, 58.

The following Tables, &c., are also not included in the official Report:—Tables I, II, VIII (values of  $\mu_x$ ), IX (Makeham graduation and values of  $\mu_x$ ), X (values of  $\mu_x$ ); Appendices A and B, and formulæ for All India Tables on page 392 (Appendix C).

## ABSTRACT OF THE DISCUSSION.

Mr. A. HENRY said that the data with which Mr. Ackland had to deal were given in such form that they necessitated treatment by special processes of which but little notice had appeared in their proceedings since Mr. Hardy's paper, over twenty-five years ago. Not only were the registration returns of births and deaths extremely defective, but, in addition, there was gross misstatement of age in both these returns and the returns of the periodical censuses. As a result, the usual methods of constructing a mortality table from census returns and death records had to be abandoned, and recourse had to be made to the censal enumerations alone.

The method originally used by Mr. Hardy in the censuses of 1881–1901, and adopted by Mr. Ackland in the 1911 census, was to obtain graduated values for the population at each age in the middle of the censal period, and by the use of graduated rates of increase of the population at each age, to pass to values of  $p_x$  or to values of  ${}_{10}p_x$ . The problem in India was further complicated by the presence of famine, plague, and other visitations, which, by diminishing the birth rate and elevating the death rate, left effects which could be traced in succeeding age groups for many years. Though the irregularities due to famine were eventually lost sight of in the general mis-statement of the higher ages, yet their presence must obviously have an effect on the shape of the population curve.

It was obviously impracticable to allow for the presence of famine in calculating mortality rates, and the course adopted was to find average rates which might be expected to prevail irrespectively of accidental visitations. This could be done by combining the data for various censuses so as to obtain a normal age distribution which would not be subject to the fluctuation due to the ravages of disease. In the present instance the previous decennium had not experienced any abnormal visitations, and it was accordingly possible to base the figures on a comparison of the 1901 and 1911 censuses.

He would now proceed to examine the various processes adopted, which might roughly be summarized as (1) adjustment of mis-statements of age; (2) construction; and (3) graduation, the last two processes being closely allied. As regards the first of these, specimen schedules taken out from random samples of the population exhibited a general tendency towards stating the age at a decennial or quinquennial figure. As a result, the numbers for individual ages proceeded with extreme irregularity, and could only be dealt with by distributing the excess of numbers recorded at quinquennial ages over the neighbouring age groups. It did not seem that any other assumption than those made would be likely to be much nearer the truth, but, in this connection, he would express a distinct preference for dealing in the subsequent processes, with the total numbers living over a given age and not with the numbers in individual age groups. It seemed to him that by

the former method a better balance of error was secured, and the data were likely to be in a smoother form and more susceptible to adjustment.

As regards the processes of construction and graduation, in the first place the infantile ages were so utterly unreliable that recourse had to be made to certain statistics of the "Proclaimed Clans", which were collected under police supervision. These statistics were, however, very much out of date, and could not be considered as more than an indication of the rates of mortality at present prevailing. These rates were graduated by Mr. Hardy, by a formula of the Makeham type, and in this connection it might be as well to point out that there was an obvious error in the formula as printed on page 3 of the 1901 report.\* He would refer to the infantile data again, when considering the graduation of the data for the older ages. It had been said before at this Institute that India is a continent rather than a country, and accordingly it was not surprising to find that the rates of mortality vary widely in the different Provinces. He would, however, confine his remarks mainly to the Bombay figures, as these were the only ones given *in extenso*. He was, of course, fully aware that where the data were so defective the results must be regarded as indicative rather than absolute, but he proposed to draw the attention of the meeting to certain points which appeared to him to be open to criticism from the theoretical point of view.

In the first place they had a new process introduced, in the graduation of the population by a frequency curve of the Pearsonian type. He was not at all sure that that was a desirable innovation. The use of a frequency curve (in the more limited sense of the term) for a distribution that is not a frequency distribution, was entirely empirical, and its success must be judged purely from a practical point of view. In the present instance the data had been graduated by a curve of Type 1. Now a population curve was a curve beginning with a maximum value and usually decreasing steadily to zero, whereas Type 1 in the present case assumed its most common form, *i.e.*, started at zero, rose to a maximum, and then decreased to zero. The curve was, therefore, of quite a different shape. It was true that it had been used only for ages over 20, and he had no doubt that it might be possible to get a satisfactory graduation of the data by using the right-hand portion of the curve only, but in such event it seemed to him to be essential, from the theoretical point of view, that the moments should agree for the unadjusted and adjusted data for those ages actually used. In the present instance, while the graduation had been used only for ages over 20, the total area and the moments for all ages from 0 upwards were brought into agreement, and this was only done by continuing the curve backwards beyond age 0. It seemed that by this method very much was left in uncertainty for the ages over 20.

But, apart from this, was the method of moments applicable

\* The corrected formula is stated on p. 391 (Appendix C) of Mr. Ackland's paper.

in cases such as these? It had always seemed to him that the ordinary application of the method of moments was only admissible where positive and negative deviations in the age groups were equally likely, and where the errors followed approximately the normal law. This was not so in the present instance. They found a big transference from the ages below 25 into the 25-30 age group, in fact, all the age groups above 25 were swollen by people who had, so to speak, been pushed up from the age groups below. As a result, in taking moments about the mean (between ages 23-24), the values of the moments were all over-stated, and the tendency was to get a curve unduly strung out at the older ages. To obtain more accurate values of the moments, it would be necessary, either to redistribute the unadjusted data so as to remove some of the irregularities referred to, or to ignore the groups affected when calculating the values of the constants. The former would probably be the simpler method, as the latter method would entail numerous difficulties. He had been making some investigations into the question of the calculation of the constants from selected values of the first moment, but had not been able at present to develop it sufficiently to say whether it would work well in the present instance.

On examining the calculation of the moments, he was struck with a somewhat serious flaw. In Table  $\alpha$  the data above age 65 were included in one group, assumed to be centred round age 75. Now the weighted central age was much more nearly  $72\frac{1}{2}$  than 75, and this was a very much more serious factor than the one already mentioned, in the over-statement of the values of the moments. It would be observed that the fourth moment of the final age group represented over 40 per-cent of the total fourth moment. He had roughly distributed the data over age 65 in quinquennial groups, and as a result the total fourth moment was reduced to under 400, and the third moment to about 34. The result of this over-statement of the higher moments was to prolong unduly the graduated curve at the higher ages. This would be at once seen from the comparison on page 383, where, notwithstanding the over-statement of age at the end of the table, the graduated numbers living above age 55 were 5,836, as against 5,305 in the ungraduated; at age 60, 3,543, as against 3,195; and at age 65, 1,963, as against 1,508. In Table IV, there was a similar error in the figures for Madras and the United Provinces, where a similar formula was used. The figures for Bengal, Burma, and the Punjab were quite in accordance with expectations, but in the two former cases the formula was abandoned after age 59 (a subtractive correction being necessary in Bengal), and in the Punjab another method was adopted. He thought, therefore, that in Bombay, Madras, and the United Provinces the vitality at the older ages had been very much over-estimated; it would be seen from the All-India tables that, in spite of a relatively unhealthy period, the values of  $q_x$  in 1901-1911 were lower than those deduced in 1901, at all ages from 50 to 82, a result due purely to graduation. He had made a rough graduation,

which fitted the data fairly well, and he brought out values of  $q_x$  rather higher than Mr. Hardy's.

The population curve was joined on at age 20 to the curve obtained from the "Proclaimed Clans" statistics, suitably modified, a smooth junction being effected. As a result, the total numbers living above age 0 were increased from 100,000 to 105,638, and the numbers at each age were then diminished in a constant ratio to bring the total back to 100,000. This hardly seemed to him to be a defensible proceeding. The effect on the deduced rates of mortality at the older ages was, of course, nil; but the fact that a smooth junction at age 20 entailed an increase in the total population was rather a danger signal. If the original graduated numbers living at the older ages were a correct representation of the proportions living out of 100,000, then the numbers at the young ages must not be allowed to exceed their proper limits, otherwise it followed at once that the rates of mortality at the younger ages were being over-estimated. He thought that this had been the case in the Bombay statistics, where the numbers living at ages 0-20 had been swollen by the deduction of about 5 per-cent from those living at ages from 20 onwards. He felt that the real cause of the trouble was that the frequency curve chosen did not properly represent the statistics for such a low age as 20, and on an examination of the tables he thought that a smooth junction could not be made with the "Proclaimed Clans" statistics until about age 30. If this were so, there would be a very much more clearly marked inflection at ages 20 to 30, and the rates of mortality at these ages would be considerably lower. If the figures were plotted out graphically the point became very apparent. He felt, therefore, that a portion, at any rate, of the increased mortality shown in 1911 at these ages, as compared with Mr. Hardy's figures, was due to graduation alone.

The treatment of the infantile ages called for little comment. The "Proclaimed Clans" statistics were adopted as a base line, and such adjustments were made as to secure a smooth junction with the data for the higher ages. It did not seem that any better method could have been adopted in default of more accurate figures, but the results could be regarded only as an indication of the rate of mortality prevailing.

The Punjab figures were found not to be amenable to graduation by a frequency curve, and this was another indication of the unsuitability of the methods of moments, where systematic errors occurred in the data. Instead, a process was adopted of using the area of the normal curve of error with functional limits. Though this apparently worked well in practice, it was open to the theoretical objection already mentioned, namely, that the normal curve differed entirely in shape from the population curve. He did not know whether Mr. Hardy had considered the applicability of his method to statistics such as these, but in his Lectures he limited its applicability to an asymmetrical series, the terms of which become zero, or nearly so, at each end. He thought it would be better to use only

the right-hand half of the curve, in other words, to use the value of the integral from 0 to  $z$ , instead of from  $-\infty$  to  $z$ . He had done this for the Punjab statistics, and he found that the resulting values of  $z$  ran very nearly by constant first differences, and they could be adjusted practically by inspection.

Where data were subject to such misstatement of age and other errors, as in the present instance, he felt sure that very good results would be obtained, if graduation were made with reference to a standard table. It then became possible to ignore those groups which were systematically affected, and to use ratios for them which could be obtained by observing the tendency of the curve for other ages.

He had taken the Bombay figures for the total population living above each quinquennial age, commencing at age 15, and had compared them with similar graduated figures for the English population 10 years older. Up to the age of 65 the ratio never differed by more than about 3 or 4 per-cent, and was generally much less. It was obvious that small deviations such as these were very susceptible of accurate adjustment, and the results he obtained in this way seemed to him to be very satisfactory, and worthy of considerable confidence. The Punjab figures were also amenable to treatment in the same way, with eight years added. It would be of interest to know what were the special reasons which made the use of a frequency curve unsuitable in this case.

Having obtained a graduated population table, it was now necessary to pass to the values of  $p_x$ , and for this they required to know the rate of increase of the population in each age-group. These rates were also graduated by frequency curve formulas, but it was apparently impossible to get a single curve to fit the data. He had made an endeavour to arrive at the unadjusted rates of increase for Bombay, but had not been able to reproduce rates similar in progression to those given in Table VI. It would be of interest to have some more detailed information of the processes by which these rates of increase were obtained. By multiplying and dividing by  $r_x^{\frac{1}{6}}$ , the population was found 6 months before and 6 months after the mean age, whence the value of  $p_{x+\frac{1}{6}}$  at once resulted. An alternative method would have been to multiply and divide by  $r_x^{\frac{1}{12}}$  and thus obtain the values of  ${}_{10}p_{x+\frac{1}{12}}$ , finally interpolating for the values of  $p_x$ . The only difference in these processes, so far as he could see, was that by the former process the values of  $p_x$  were made to adhere rather more closely to the population curve. By obtaining the values of  ${}_{10}p_x$  and then interpolating, a series of interpolation curves was fitted to the population curves, and thus a certain amount of the rigidity imposed by using a single curve throughout was done away with. The difference was probably slight in practice, and the greater facility of obtaining the results seemed to favour Mr. Ackland's method.

When they came to female lives, they found at once that India is another of those countries where women will not obey the laws of man. The data were quite unreliable, and the only process was

to obtain the ratio of the number of women living at given ages compared with the male figures.

An attempt had been made, by use of the graduated rates of mortality, to give some estimate of the extent of defective registration. It would be of interest to know whether registration was more effective in the towns as compared with the country districts. He did not know whether there were any figures which would show this, but if, as seemed probable, the registration in the towns was somewhat more effective, it might be possible by using selected urban areas to obtain a mortality table in the usual way.

A statistical investigation of this nature must necessarily be a prelude to investigations of the health of the people, and it seemed to him that the first step in this direction, even to a limited extent, would be to get some definite information as regards the incidence of various diseases, and that would help to clear up the seemingly excessive mortality.

Mr. GEORGE KING said that in the short time that had been available since the paper came into their hands it had not been possible for him to give it that study which would enable him to intervene with the best effect in the discussion. Nevertheless, as he was at present engaged in a somewhat similar enquiry, it might not be out of place for him to offer a few remarks. He did not propose to refer to the great mass of statistics which Mr. Ackland had given them. At the right time those statistics would be very useful. The same remarks applied to the very valuable appendices supplied by Mr. Gunningham and Mr. Rowland ; he had not yet had time to study them, but they were well worthy of consideration.

Two points in the paper were of special interest to him : one was the errors in age and their elimination ; and the other, the mortality table constructed from censuses alone. As to the first, he was at present engaged in an enquiry similar to that which Mr. Ackland had been conducting, because he had the honour to be consulted by the Registrar-General over the census of 1911 of England and Wales. In that census he found great misstatements of age. He might say that the Registrar-General in his report on the census would give the population age by age and not only in age groups. That was a great advance on anything they had had before. He had had the population age by age before him, and had been able to deal with it much more effectively than if it had been shown only in groups. It was curious that to a large extent the errors at home were very similar in kind to those which Mr. Ackland had found in India, but he presumed, although he had not had time to make comparisons, that they were not so serious in extent. He found that in England and Wales in giving the age the even digits were in favour at the expense of the odd digits. The digit 0, except at one or two places in the table, was most favoured of all, and next to the digit 0 and not far behind it came digit 8. The other even numbers had a little excess, 5 showed a little excess, but not much, over the other odd digits. Having got the population age by age,

the question was which groups would be the best for the purpose of correcting the errors in the ages, and he found that by taking the groups of digits 4 to 8 and 9 to 13 he got by far the best results, the reason being that by taking those groups they separated 8 and 10, and put three even digits in the group containing 8, and only two in the group containing 10, so that they balanced better in that way. He had tried various groupings, but found that to be decidedly the best. It was necessary to group in fives because errors in age recurred in that way.

In the table on page 321 Mr. Ackland gave a formula for getting graduated quinary groups, but he had not been able to find in the paper any explanation of the rationale of that formula, and one would like to see it clearly stated how it was constructed. Mr. Ackland had, by his formula, brought out the graduated quinary groups. He (the speaker) had given two formulas. On page 169 of Vol. XLIII of the *Journal* he gave a formula from which they obtained graduated quinary groups, as did Mr. Ackland, and from them graduated values at individual ages, but he found that formula did not work with the census figures, because the effect was not to eliminate the errors of age but only to smooth them over. It gave a smooth curve, but with humps, with, as it were, hills and dales. He had to take the more direct formula given on page 115 of Vol. XLIII. From that he got graduated values for quinquennial ages, and then, by osculatory interpolation, a good curve practically free from the minor errors in age due to giving ages in round numbers; but it was not free from the grosser errors, as, for instance, where a woman who was 40 said she was 30. He found that by using this shorter formula, and using the quinary group with the digits 1 and 6 as the central ages, very good results indeed were got. He understood that Mr. Ackland had based his results a good deal on the age distribution derived from sample populations. No doubt that was necessary in the case of India. Of course, that was not now required in England, as they had the age distribution directly from the census, and here it would be very risky to adopt sample populations for age distribution. At the census of 1901 sample populations for special districts were given age by age, the reason being that they could not at that time get the individual ages for the whole census, and that was to help the actuaries in dealing with the distribution of the population according to age. That table would be repeated for the census of 1911 for precisely the same districts. Those districts were selected because they were thought to represent as closely as possible the general population, but it would be found that the age distribution was very different indeed, and if they had the population in groups for the general population and were to try to distribute it by means of the sample populations, it would be found to go far astray.

Probably it would be the same in India, but they must do the best they could with it there. The mortality tables from two censuses seemed very interesting and very important, but they must have recourse to them only as a last resort, and he imagined that

that was Mr. Ackland's own view. There were so many sources of error that it might be very risky indeed. He would like to see the method tested, wherever they had a good check upon it. They had the British censuses of 1891 and 1901, and they had the English Life Table No. 6 based upon those two censuses and the intervening deaths. That table was constructed in the Registrar-General's Office, upon the new and improved methods devised by the late Mr. Waters, and it had been fully tested by himself in his own papers, where he re-constructed it by his own new method, and it had been tested also by Mr. Vyvyan Marr, and, except at the old ages where he ventured to differ from the authorities at the Registrar-General's Office, all the tables re-constructed by the new methods were almost identical with the official table. They had there a trustworthy official table, based upon the two censuses and the intervening deaths, and if Mr. Ackland or some of his skilled associates would construct a table from the same censuses by his method, without the deaths, it would give them some idea how far such a table was really trustworthy.

(The Chair was here vacated by the President, and occupied for the remainder of the Meeting by Mr. G. J. LIDSTONE.)

Mr. A. T. WINTER said that he would confine his remarks to the data underlying the paper—the material of the foundations upon which Mr. Ackland had reared his structure. Mr. Ackland had explained that some of the materials generally considered essential for his purpose were missing, or, rather, totally unreliable; for instance, the registers of births and deaths were valueless, and, in regard to other material, it was so distorted that it had to be straightened out to a very large extent before it could serve any useful purpose. Mr. Ackland had taken them, as in former valuable papers which he had contributed, step by step through his work, and his methods were quite easily followed.

Turning first to Table III, what struck one most was the apparently extraordinary ignorance of the Hindoos in regard to age; or, possibly, amongst certain sections, a disinclination to reveal the truth. They would see, for instance, that the total number returned as living at age 50 exceeded the number living at the age 0 to 1, and the total number returned as living at age 30 was about 66 per-cent in excess of the number living under age 1. The feature of the preference for decennial ages had been referred to, and was very marked in Table III. That feature was, of course, particularly evident in the censuses of all illiterate populations; he believed it was very noticeable in the censuses of Asiatic Russians and in the Negro population of the United States. To a certain extent it was evident in the censuses of our own nation, but it was very much more marked in the case of India than any other censuses on record. It would also be observed that the partiality for decennial ages was shared by the ages 12 and 25. He thought the explanation of the partiality for the age 12 was that 12 was considered by the Indian native as a lucky number.

In Table IV, which gave the adjusted population in quin-

quennial groups, and also graduated numbers, there were some extraordinary incongruities. The first thing that struck one was the relatively small number of females living in the group 10 to 14. According to the tenets of Manu, which were followed by the Brahmins, it was incumbent upon the head of the household to marry the girls before they attained the age of puberty, and it was a disgrace to them among their neighbours if they did not fulfil this obligation ; there was also some special punishment reserved hereafter for the father and mother and eldest son if the daughters of the household were not married. Consequently it was probable that the existence of unmarried daughters was concealed in the returns, or their ages were given in other groups. That was an explanation which had been mentioned in the reports on former censuses. Again, it would be noticed that the age group 25 to 29 for both males and females was disproportionately heavy. As regards the females the following explanation had been suggested in former reports. When a girl married and had children, she generally assumed an age which was higher than her true age, and that age remained practically constant until she had passed the period of child-bearing. Therefore, included in that group were probably a great many that should be in the age groups 15 to 19, 20 to 24, and 30 to 34. On the male side there was also a disproportionately large number in the group 25-29. An explanation suggested was that it included a great number of widowers of uncertain age. In India the proportion of widowers was heavier than usual owing to the system of marriage at early ages, and the widowers were generally desirous of representing themselves as of an eligible age for re-marriage, which might be partly accounted for by the fact that there were generally pecuniary advantages attaching to marriage. It might seem that those explanations were rather far-fetched, but from his own experience in India he thought they could be accepted as fairly satisfactory. He had already mentioned certain particular directions in which over-statement or under-statement of ages probably occurred. There was another one which was, he considered, more serious than those already mentioned. Beyond a certain age there was, he believed, a very decided tendency to over-state age. Mr. Hardy in his Report on the census of 1891 referred to "an indubitable tendency to exaggerate the ages after about age 65." Again Sir Athelstan Baines, a great authority on Indian life who was, he believed, responsible for two of the early censuses, referred to that feature in a paper read before the Statistical Society in 1908. The words used by Sir Athelstan Baines on that occasion were as follows : "There has to be taken into account the very general over-statement of age by those in advanced life, extending often far beyond the period of 10 years." He went on to say in the paper that "amongst the older natives, unless the age can be confirmed by reference to some historic event, or public calamity, a margin of 25 to 30 years can be safely allowed." He (the speaker) was in India at the time the 1901 census was taken, and had an opportunity of speaking to a number of the officials, and also to educated Hindoos, well versed in

the subject regarding the Census, and their general opinion was that certainly at the higher ages the results were altogether unreliable.

In Table L, where the comparative expectations of life of the Indian and English peoples were given, it would be noticed that the difference in expectation of life (1911) at age 60 was only 3·38 for the whole of India's male population, whilst in the Madras Presidency at the age of 60 the expectation of life in 1911 was 11·7, and according to the English census of 1901 it was 12·9, a difference of only 1·2 in the expectation. He thought that must be wrong. From general considerations they would imagine that the expectation of life of a native would be very much less than that of an Englishman. To start with, the stamina of the natives was not nearly as good as that of the English population ; they matured earlier ; senile decay set in earlier ; their poverty was proverbial ; there was no Poor Law to protect the old people ; they had practically no medical assistance which was of any value ; and the old people suffered particularly from the periodical famines and epidemics. In those circumstances he thought they would expect, on general grounds, to find a very great difference between the expectation of the native of India and that of the average Englishman. It appeared also, from the considerations referred to by Mr. Henry, that the feature of over-statement of age at higher ages was slightly exaggerated by the method of graduation adopted.

It must be remembered, in referring to the tables of mortality at the end of the paper, that they included a number of different sections of the community which were subject to very widely-differing rates of mortality, and they might mislead one very much if it were assumed that those rates applied to any particular section with which they were dealing. For instance, the rates of mortality brought out up to the age of 60 were more than twice the rates experienced by native assured lives, so far as they knew them. It was likely also that the difference in mortality amongst the different castes was very marked. That point was considered by the Census Commissioner in Bengal in 1901, and he then made some attempt to differentiate between the mortality of the different castes in Bengal. The method adopted was a very rough one, and the results were perhaps not conclusive, but he thought they showed that the mortality amongst the higher castes was very much lower than amongst the lower castes. and, generally speaking, the higher the caste the lower the mortality.

Mr. Ackland had said nothing about the methods adopted in taking the census. Possibly he considered it outside the scope of the paper. He thought it would be interesting if some information on that subject could be given, because the methods in India must differ very widely from those adopted in England. Here, of course, the head of the family had the printed instructions and forms several days in advance, and so had an opportunity of forming a more or less intelligent view of what was required. In India that was impossible ; only one male in ten and only one female in a hundred could read, even in their own vernacular, so it was no use leaving

forms of instruction for them. When he was in India in 1901 they appointed a host of enumerators, and their duty was to go round to the natives and collect the information from them. To his own knowledge some of the enumerators were men who really knew little more about the subject in hand than the average untutored natives themselves. The officials he knew were of the opinion that a great many of the returns were practically valueless. Then there was a further difficulty in collecting information with regard to females. The purdah system in India rendered many of the females unapproachable to anyone except the near male relatives, and in those circumstances they could know but very little about that portion of the female population of India. The special purpose of the census in India was to show what effect any particular variation in taxation would have, and also the prospects of irrigation and railway schemes, but for mortality purposes he thought the Indian census was a particularly defective instrument. If they could get authentic information regarding, say, 100,000 natives proportionately distributed according to district and conditions of life extending over a period of 30 years, they would be in possession of facts which would, he thought, be very much more serviceable for their purpose than the statistics regarding 300,000,000 people which were given in such an indefinite and irregular way.

He would like to thank Mr. Ackland for his paper, which had been very interesting indeed, and he thought the author had made the most of the material available, but in considering his scientific methods they must not lose sight of the fact that the material he had to work upon was so very unreliable.

The CHAIRMAN (Mr. G. J. LIDSTONE) said it was not until he entered the room that evening that he knew it would be his duty to act as understudy for the President for a short time. Unfortunately he had not been able to prepare for the part by a careful reading of Mr. Ackland's paper, so as to enable him to close the discussion in the way which he was sure the President himself would have done, and which he should himself have tried to do had he known that that duty would have fallen upon him. In the circumstances, he thought it would be but a poor compliment to Mr. Ackland if he endeavoured to make any extempore remarks on the paper at such short notice. He was sure it had been a great pleasure to all of them to have another paper from their old friend, Mr. Ackland, who had laid a heavy debt on them in the past for many valuable contributions and again that evening had given them of his best, and their indebtedness was increased by the fact that Mr. Ackland had given his paper so quickly after the original and valuable work on which it was founded had appeared, so that it came before them in good time to be studied before the results became stale. He would mention that the President had asked him to say that Lord George Hamilton, a former Secretary of State for India, had hoped to be present that evening, but, unfortunately, at the last moment he had to write expressing his regret that he was unable to attend. Had Lord George been with them he would have been able to appreciate

the degree of care with which actuarial investigations entrusted by the great Departments to leading actuaries were made, and he might possibly have noticed also the refreshing degree of independence of thought with which the processes were criticized by other actuaries. They had had a relatively short but a spirited and most interesting discussion, and probably Mr. Ackland, with his usual broad-mindedness, would consider it as a compliment to him that the work had been taken seriously and criticized in the best sense of the word.

He thought Mr. Henry's remarks were an exceedingly valuable example of combined destructive and constructive criticism. Mr. Henry had expressed his disagreement with certain processes which Mr. Ackland had adopted, but had not left it at that, but had stated the processes he would have been inclined to substitute. He hoped the Editors of the *Journal* would see their way to invite Mr. Henry to expand those notes so that they might have an opportunity of seeing in the *Journal*, he would not say the rival, but the alternative, proposals which Mr. Henry suggested.

Mr. T. G. ACKLAND, in replying, said he should like at the outset to express his indebtedness to the gentlemen who had taken part in the discussion for the very interesting remarks they had made, to which he would give due consideration. He felt how much they missed the absence of their old friend, Mr. G. F. Hardy, who had wished very much to be with them and take part in the discussion, but was unable to be present. Sir Athelstan Baines would also have been there, but was detained in Oxford by indisposition. His wide experience in matters relating to Indian Censuses would have added much to their knowledge of the complicated problems involved. He had followed with great interest the remarks of Mr. Henry, and did not in the least demur to his criticisms, whether they were destructive or constructive; and he hoped, with their Chairman, that Mr. Henry might be able to send some notes to the *Journal*, setting out more fully his alternative suggestions. Mr. Henry had applied an alternative method to that adopted for the Punjab figures, and obtained values of  $z$ , of which the first differences were nearly constant. In point of fact, it would be found that the first differences, given in Appendix B to the paper, were themselves nearly constant, and the second differences were practically negligible. The suggestion of dealing with the graduation by reference to a standard table was a valuable one, which had occurred to one in taking up the work, but on the whole he thought the processes adopted were those most likely to give satisfactory results. It would, however, be very interesting to have that suggestion followed out, if practicable.

With regard to the alternative plans of deducing the probabilities of life from the rates of increase, he agreed with Mr. Henry's suggestion that  $r^{\frac{1}{10}}$  was probably preferable to  $r^{\frac{1}{5}}$ , as obtaining the desired results in a more direct form, but he had been puzzled why Mr. Hardy did not adopt what seemed to be a rather obvious course in this respect, but went out of the way, as it seemed, to deduce probabilities of living ten years, which had to be reduced to annual

probabilities by interpolation. He supposed there was some justification for this course, but the explanation had not occurred to him.

Referring to the relative accuracy of registration in town and country districts, mentioned by Mr. Henry, he thought that was mentioned in some of the official papers which had come under his notice. A large amount of trouble was taken in arriving at the registration results, and the marvel was that, notwithstanding the admirable administrative machinery employed, the results were so far from those deduced by the census comparisons.

Mr. King had given them some very interesting matter to think over, particularly as regarded his methods of dealing with some analogous problems, but in respect of which the accuracy of the data was much more marked than in the Indian figures. Mr. King's method of quinary grouping was very interesting and suggestive. Mr. King had asked for the rationale of the method employed in Table B, in which the adjustment of initial errors of ages was dealt with. He had believed that the rationale was sufficiently shown in paragraph 11 of the paper, where he stated the method followed by Mr. Hardy in this respect, but thought that it was based more on practical than on theoretical considerations. With regard to samples of the population at individual ages, Mr. King had made some reference to the importance of their being representative of the general population, in which he entirely agreed; he understood from the authorities that very great care was taken that that should be so. On a former occasion (he thought in 1881) sample schedules were supplied to Mr. Hardy, which were taken from selected literate people resident in the city of Madras, which the authorities appeared to consider would be appropriate for Mr. Hardy's purpose; but he had to say that they were useless for his purpose, because they were based on a particular literate class, and were not representative of the whole population. It would be interesting to follow out Mr. King's suggestion of getting out an English mortality table by taking the numbers recorded at two successive censuses, without regard to the records of deaths and births in the intermediate period, but he was afraid that that would not throw very much light upon the Indian problem.

Mr. Winter had given them the fruits of his knowledge on the subject generally, and his experience on the spot some years ago was very useful, and his references as to the methods adopted in the census were especially interesting. He (Mr. Ackland) had not dealt with that subject in the paper, because he thought it lay outside the main objects of the paper, and would greatly extend its bulk; and those methods would, moreover, be very fully dealt with in the Report of the Chief Commissioner, which he understood would be published shortly. Generally speaking, the methods adopted were devised with extraordinary care and minuteness, and he should say that the administration of the machinery was altogether admirable. He had had the pleasure of listening to a course of lectures given recently by the Census Commissioner for Burma at the London School of Economics, when

the Commissioner explained the methods followed in that Province, which were those followed generally throughout India. There was a most elaborate system of instruction given to the enumerators, who were mostly, he thought, Police and Postal Officers in the Government employ, and there was a "dress rehearsal", as it were, of the whole proceedings on a given date, so that the enumerators would be well equipped for their duties. The difficulty of illiteracy was got over by the enumerator taking down the particulars when he called at the house, in his own handwriting, because it was impossible in the majority of cases to get the recording done by the householder, and it was hoped that, having regard to the fact that each enumerator did not take more than about 50 houses, and that the enumerator was well acquainted with the whole of the inhabitants of those houses, the results could not be very far wrong as to numbers, and would have some approach to accuracy in the matter of age. They knew that they were, in fact, very far wrong as to age, but he thought that the machinery laid down was well devised to secure a much more accurate result.

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[We have pleasure in publishing the following communications with reference to the graduation of the statistics.—ED. *J.I.A.*.]

NOTE BY MR. A. HENRY ON THE GRADUATION REFERRED TO IN HIS REMARKS.

The figures representing the population at each age were summed from the bottom upwards, to obtain the numbers living at ages over the various quinquennial ages.

The standard chosen was the population of England and Wales, which had been graduated by a mathematical formula, and therefore, fulfilled the requisite conditions of smoothness.

The figures above mentioned were accordingly compared with similar figures for England and Wales at an age 8 years older, the interval of 8 years being chosen as the resulting ratios were fairly uniform in value.

The figures were brought to a common radix at age 15, as it was thought that the Indian data would show a balance of errors about that age. As in any case it was not intended to use the graduation to obtain rates of mortality for such a young age, the use of a common radix at age 15 would not appreciably affect the rates at older ages.

The Bombay figures are as follows :

Age <i>x</i>	NUMBERS LIVING OVER AGE <i>x</i>		RATIO OF CORRESPONDING FIGURES OF STANDARD TABLE AT AGE ( <i>x</i> + 8)	
	(ungrad.)	(grad.)	(ungrad.)	(grad.)
15	61,442	61,442	1·	1·
20	51,948	51,187	1·012	.996688
25	43,006	41,437	1·025	.987424
30	33,086	32,560	.989	.973216
35	25,105	24,778	.968	.955072
40	18,216	18,193	.935	.934000
45	12,747	12,806	.907	.911008
50	8,567	8,554	.888	.887104
55	5,305	5,337	.858	.863296
60	3,195	3,033	.886	.840592
65	1,508	1,509	.820	.820000
70	...	618·7	...	.802528
75	...	190·2	...	.789184
80	...	37·5	...	.780976
85	...	3·9	...	.778912
90	...	·2	...	.784000

The data at ages 20–35 are greatly affected by misstatement of age, and the ungraduated values of the ratio at these ages must be ignored in obtaining graduated values.

In the circumstances the following curve was considered to give a sufficiently good fit.

$$y = 1 - \cdot0001392(x - 15)^2 + \cdot000001344(x - 15)^3$$

the form of the curve being determined from a general consideration of the flow of the ungraduated values.

The populations so found were combined with Mr. Ackland's rates of increase, to obtain values of  $p_x$  for quinquennial ages. The following table gives the resulting values of  $q_x$ , together with approximate values of the complete expectation of life.

Age	RATE OF MORTALITY			EXPECTATION OF LIFE		
	Hardy, 1901	Ackland, 1911	Henry, 1911	Hardy, 1901	Ackland, 1911	Henry, 1911
25	·0155	·0212	·0151	25·26	23·8	24·17
30	·0190	·0263	·0226	22·27	21·32	21·23
35	·0240	·0309	·0289	19·46	19·2	18·76
40	·0305	·0356	·0346	16·90	17·23	16·55
45	·0385	·0406	·0409	14·57	15·3	14·47
50	·0480	·0476	·0478	12·48	13·51	12·47
55	·0590	·0544	·0575	10·55	11·7	10·49
60	·0726	·0646	·0721	8·73	9·94	8·59
65	·0929	·0789	·0953	6·99	8·2	6·77
70	·1252	·1001	·1317	5·38	6·55	5·16
75	·1774	·1346	·1871	3·96	4·9	3·83
80	·2573	·1974	·2676	2·81	3·48	2·75
85	·3750	·3116	·3774	1·91	2·3	1·93
90	·5400	·4872	·5149	1·07	1·41	1·35

The value of  $q_x$  at age 25 is probably too low, but possibly the shape of the new population curve might cause some modification in the ratio of increase at that age. In any case the values could be made to join on smoothly to those obtained from the "Proclaimed Clans" statistics.

Grateful thanks are due to Mr. V. P. A. Derrick, F.I.A., for kind assistance in checking the results.

#### FURTHER OBSERVATIONS BY MR. ACKLAND.

I have read with great interest Mr. Henry's able and valuable contribution to the discussion on my Paper, and I am glad to avail myself of the opportunity, given to me by the Editors, of adding any remarks arising on the discussion and on Mr. Henry's Addendum.

I must leave it to those who have a much wider acquaintance than myself with the theory and practice of frequency-curve methods to discuss Mr. Henry's remarks as to their appropriateness to data such as those dealt with in my Paper, and as to the treatment of the "moments"; but we are, I think, in agreement that there can be no practical objection to the adoption of such portion of a frequency curve as seems to conform with the general trend of the figures under graduation.

Mr. Henry's practical criticisms on the graduation of the Bombay figures (the data for which have been alone available to him in full detail) seem to resolve themselves into two points, first that, in the practical application of the method followed, the rate of mortality after middle life (from about 50 to 80) has been largely understated, and secondly, that the mortality between ages 20 and 30 has been over-stated, by the junction made with the Proclaimed Clans figures at about ages 18 or 19, rather than at about age 30.

On the first point, I would remark that Mr. Henry's comparative figures for Bombay, taken from Appendix A, page 383, and for older ages from Table  $\gamma$ , do not represent, so far as the graduated numbers are concerned, the final figures employed, which, as clearly explained in the text, were reduced by somewhat more than 5 per-cent. If this reduction be given effect to, the excess of the graduated numbers above the ungraduated at ages 55, 60, and 65 will be found to be quite materially reduced, whilst, at older ages, the graduated figures are less than the ungraduated, and increasingly so with advancing age. This is shown in the Bombay section of the appended Table, which compares the final graduated figures with the ungraduated, in quinary age-groups from 45 to 84 inclusive, and it will be seen that the final graduated figures at ages 55, 60, and 65, are reduced to 5,519, 3,349, and 1,854 respectively, as compared with 5,305, 3,195, and 1,508 in the ungraduated Data, thus bringing the figures into much closer agreement than those quoted by Mr. Henry.

As regards the All India Table for Males, Mr. Henry makes

the surprising statement that the lower value of the rates of mortality, as compared with those deduced by Mr. Hardy in 1901, at all ages from 50 to 82, is a "result due purely to graduation." This remarkable conclusion is rather calculated to "make one's flesh creep", since the practical usefulness of the results would be materially discounted, if it could be shown that the graduation were faulty to such an extent. Mr. Henry, however, whilst adducing reasons for thinking that the methods adopted would have the effect of prolonging the population curve at the older ages, has not, I think, been in a position to verify this theory by a reference to the recorded numbers for all India. I have looked into the figures of each of the Provinces, and have set out, in the appended Table, the ungraduated age distribution (corrected for excess at quinary ages only) in age-groups from 45 to 84, with the graduated results for comparison; also the ratios of the numbers in each age-group, to the total numbers, in that and succeeding groups, to the end of life. These ratios are somewhat analogous to expectations of life, and a comparison of the graduated and ungraduated ratios applies a definite and, I think, conclusive test, as to the effect of the graduation on the mortality in different age-groups.

It will be seen from the Table that, whilst in Bombay, Madras, and the United Provinces, the graduation has had the effect of reducing the mortality from age 45 to age 64, the effect has been in the contrary direction at all later ages; also that, in Bengal, Burma, and the Punjab, the graduated figures show increased mortality in all age-groups from 45 to the end of life, excepting at ages 45 to 49 in Bengal, and 65 to 69 in the Punjab, where the mortality is slightly reduced. It is clear that no combination of these provincial results (in whatever proportions) could have the effect of under-estimating the mortality from age 50 to 82, in the Table for all India, and that, even over the age period 45 to 64, where the mortality is reduced in three provinces, the effect must be largely counteracted by the increase in the remaining three provinces. I submit, therefore, that Mr. Henry's confident statement in this respect is entirely devoid of foundation, there being no evidence whatever of the mortality being under-estimated, over any considerable tract of ages, in the Table for all India.

It is also, I think, quite unsuitable to take Mr. Hardy's graduated results for 1901, as a basis for comparison with those now deduced for the later decennium. Mr. Hardy's 1901 figures were, it will be remembered, deduced from the age distributions as in 1881, 1891 and 1901, the figures of 1891 being given double weight. The population of India were subject to serious famines in the decennium ending 1881 (during which the population increased by less than 7 per-cent), and to both famine and plague in that ending 1901 (during which it increased by 2·5 per-cent only), and their effect must have been seriously felt in the age-distribution figures at these dates, and, to a less marked extent, in the mean figures deduced by Mr. Hardy from the three successive censuses. The results, therefore, whilst giving a most valuable indication of the average mortality

in India generally, cannot properly be compared with those deduced in 1891, and in 1911, when censuses were taken at the close of periods comparatively free from general attacks of famine or plague, and the population had increased by 13 per-cent and 7 per-cent respectively.

Having regard to these considerations, it is not at all surprising that the rates of mortality now deduced in 1911 should be more favourable than those deduced by Mr. Hardy in 1901, in respect of lives between 50 and 80 years of age; and it will be seen, by the appended Table, showing the rates of mortality and expectations of life for all India, as deduced in 1891, 1901 and 1911, that, at these ages, the 1911 figures lie between those of 1891 and 1901, excepting only over ages 65 to 69, when the 1911 rates of mortality are slightly more favourable than those of either of the preceding decenniums. This, again, confirms the view that whilst, in the all India Table, the mortality may be slightly under-estimated between ages 65 and 70, it is certainly not so under-estimated over any larger age-period.

It is also to be borne in mind, in considering Mr. Henry's criticisms of the Bombay figures, and of the all-India Table generally, that the figures for the Punjab were got out by an entirely different process of graduation, but show substantially very similar results to those deduced in other Provinces, where the frequency-curve method was followed, without special modification at the older ages. This will be seen on reference to the rates of mortality and the expectations for 1911, given in Tables K and L of my Paper, for the Punjab and the United Provinces, which were exposed to similar conditions during the preceding decennium, and show (on the graduated figures) generally similar results. I think there is also some weight in the fact that a graduation by Makeham's formula, based upon a high value of  $\log c$ , brings out rates of mortality, and expectations of life, which, as stated in the Paper, cannot be discriminated, at any age between ages 30 and 90, from the results brought out by the frequency-curve methods, and this seems to me again to demonstrate that there is no such under-statement of the mortality, over a large tract of ages, as that suggested by Mr. Henry.

Whilst, therefore, I concur that there is considerable weight in Mr. Henry's criticisms, from the standpoint of theory, I think that he will agree, on further consideration, that in practice any tendency to error has been very largely counteracted, and I have no doubt that this result has been attained by the technical and practical skill of the assistants whose services I had the good fortune to secure.

As regards the earlier years of life, I quite agree with Mr. Henry that the junction between the Proclaimed Clans figures (ending at age 12) and the normal data, from about age 19 onwards, is not altogether satisfactory, and is open to theoretical objections, and probably also somewhat over-estimates the rates of mortality in early life. It appeared to me, however, that it was desirable to make a junction with the facts at the earliest practicable point,

so that only six or seven ages (12 to 19) were left to arbitrary adjustment, whilst, if the course suggested had been adopted of deferring the junction until age 30, the larger period of life from 12 to 30 would have had to be deduced arbitrarily, and apart from the actual experience.

I have been much interested in Mr. Henry's Addendum, in which he gives the results of an attempted graduation of the Bombay figures, by Mr. Lidstone's method of reference to a standard Table. The lines followed have evidently been carefully thought out, and, so far as coincidence with the ungraduated data is concerned, the results may be considered as quite satisfactory, from about age 35 to age 60. I have, however, grave doubts whether, in dealing with data so defective and inaccurate as those in question, the aim of the graduator should be to reproduce the (erroneous) ungraduated figures, and whether a Table deduced on such lines can be considered as in any sense representing the true rates, presumed to underlie the figures recorded. It is also to be remarked that the reference to the standard Table is rather remote, up to about middle life, as the graduated rates deduced represent an addition, between ages 25 and 40, of about 20 years to the age, the 8 years adopted as a standard being approximately preserved at higher ages only.

It would be interesting to have a Table deduced for all India on similar lines, but with, I would suggest, a less close adherence to the numbers actually recorded, where these are known to be affected by errors in a particular direction.

The graduation of data, such as those here dealt with, which are affected by both systematic and accidental errors, is a difficult task, and the results can, I suppose, at best only be taken as an approximate indication of the actual mortality prevailing; and I submit that, with due allowance for these considerations, my graduated Tables may be considered as representing the facts with a reasonable approach to accuracy.

*Bengal*

Age groups	UNGRADUATED			GRADUATED			UNGRADUATED			GRADUATED		
	Numbers recorded in age group	Ratio of col. (2) to col. (1)	Numbers recorded in group	Numbers recorded in age group and at higher ages	Ratio of col. (5) to col. (4)	Difference between columns (5) and (3)	Numbers recorded in age group	Ratio of col. (2) to col. (1)	Numbers recorded in group	Numbers recorded in age group and at higher ages	Ratio of col. (5) to col. (4)	Difference between columns (5) and (3)
45-49	4,041	12,535	3:10	3,899	12,990	3:33	4,180	12,747	3:05	3,905	12,400	3:18
50-54	2,831	8,494	3:00	3,092	9,091	2:94	-'06	3,262	8,567	2:63	8,495	2:85
55-59	2,186	5,663	2:59	2,349	5,499	2:55	-'04	2,110	5,305	2:52	5,519	2:54
60-64	1,567	3,477	2:22	1,678	3,650	2:18	-'04	1,687	3,195	1:89	3,319	2:24
65-69	711	1,910	2:69	1,087	1,972	1:81	-'88	632	1,508	2:39	951	1:95
70-74	521	1,199	2:30	590	885	1:50	-'80	365	876	2:40	903	1:67
75-79	287	678	2:36	234	295	1:26	-'10	236	511	2:17	364	1:39
80-	391	391	...	61	61	...	...	275	275	102	102	...

Age groups	UNGRADUATED			GRADUATED			UNGRADUATED			GRADUATED			UNGRADUATED			GRADUATED		
	Numbers recorded in age group	Ratio of col. (2) to col. (1)	Numbers recorded in group	Numbers recorded in age group and at higher ages	Ratio of col. (5) to col. (4)	Difference between columns (5) and (3)	Numbers recorded in age group	Ratio of col. (2) to col. (1)	Numbers recorded in group	Numbers recorded in age group and at higher ages	Ratio of col. (5) to col. (4)	Difference between columns (5) and (3)	Numbers recorded in age group	Ratio of col. (2) to col. (1)	Numbers recorded in group	Numbers recorded in age group and at higher ages	Ratio of col. (5) to col. (4)	Difference between columns (5) and (3)
45-49	4,274	15,685	3:67	4,242	15,248	3:59	-'08	4,609	14,954	3:25	4,227	14,882	3:52	+	'27	45-49		
50-54	3,269	11,411	3:49	3,457	11,066	3:18	-'31	3,351	10,345	3:09	3,365	10,655	3:17	+	'08	50-54		
55-59	2,654	8,142	3:07	2,742	7,549	2:75	-'32	2,677	6,994	2:61	2,590	7,290	2:81	+	'20	55-59		
60-64	2,215	5,488	2:48	2,069	4,807	2:32	-'16	1,927	4,317	2:24	1,905	4,700	2:47	+	'25	60-64		
65-69	1,431	3,273	2:29	1,394	2,738	1:95	-'34	964	2,390	2:48	1,313	2,795	2:13	-	'35	65-69		
70-74	953	1,842	1:93	811	1,344	1:66	-'27	566	1,426	2:52	829	1,482	1:79	-	'73	70-74		
75-79	523	889	1:70	379	533	1:41	-'29	383	860	2:25	418	653	1:46	-	'79	75-79		
80-	366	366	...	154	154	...	...	477	477	...	205	205	...	...	...	80-		

Age groups	Burma			Madras			Punjab						
	Numbers recorded in age group	Ratio of col. (2) to col. (1)	Numbers recorded in group	Numbers recorded in age group and at higher ages	Ratio of col. (5) to col. (4)	Difference between columns (5) and (3)	Numbers recorded in age group	Ratio of col. (2) to col. (1)	Numbers recorded in group	Numbers recorded in age group and at higher ages	Ratio of col. (5) to col. (4)	Difference between columns (5) and (3)	
45-49	4,274	15,685	3:67	4,242	15,248	3:59	-'08	4,609	14,954	3:25	4,227	14,882	3:52
50-54	3,269	11,411	3:49	3,457	11,066	3:18	-'31	3,351	10,345	3:09	3,365	10,655	3:17
55-59	2,654	8,142	3:07	2,742	7,549	2:75	-'32	2,677	6,994	2:61	2,590	7,290	2:81
60-64	2,215	5,488	2:48	2,069	4,807	2:32	-'16	1,927	4,317	2:24	1,905	4,700	2:47
65-69	1,431	3,273	2:29	1,394	2,738	1:95	-'34	964	2,390	2:48	1,313	2,795	2:13
70-74	953	1,842	1:93	811	1,344	1:66	-'27	566	1,426	2:52	829	1,482	1:79
75-79	523	889	1:70	379	533	1:41	-'29	383	860	2:25	418	653	1:46
80-	366	366	...	154	154	...	...	477	477	...	205	205	...

*All India—MALES.*

Age	RATE OF MORTALITY			EXPECTATION OF LIFE			Age
	Hardy, 1891	Hardy, 1901	Ackland, 1911	Hardy, 1891	Hardy, 1901	Ackland, 1911	
15	.0113	.0117	.0132	32.34	31.68	30.32	15
20	.0139	.0143	.0169	29.24	28.59	27.46	20
25	.0169	.0169	.0203	26.35	25.70	24.86	25
30	.0204	.0202	.0237	23.66	22.90	22.45	30
35	.0243	.0249	.0277	21.13	20.31	20.16	35
40	.0285	.0301	.0324	18.75	17.91	18.01	40
45	.0337	.0364	.0372	16.47	15.69	15.97	45
50	.0398	.0430	.0428	14.28	13.59	13.97	50
55	.0478	.0509	.0498	12.16	11.55	11.99	55
60	.0593	.0625	.0606	10.12	9.53	10.00	60
65	.0764	.0814	.0757	8.22	7.57	8.06	65
70	.1016	.1136	.1017	6.48	5.80	6.19	70
75	.1393	.1612	.1471	4.95	4.30	4.50	75
80	.1958	.2355	.2255	3.65	3.07	3.06	80
85	.2803	.3388	.3635	2.59	2.14	1.93	85
90	.4062	.4786	.5720	1.69	1.23	1.15	90

*The Medico-Actuarial Investigation of the Mortality of American and Canadian Life Assurance Companies.*

[Communicated by Mr. ARTHUR HUNTER, A.I.A., F.F.A., Chairman of the Central Bureau.]

THE principal subjects of Volume II.\* of the Report of the Joint Committee of the Medico-Actuarial Mortality Investigation are :

1. Influence of build on mortality among men.
2. Mortality among women.
3. Causes of death among men and among women.

The results of these three investigations will be dealt with briefly.

The expected deaths were calculated by the M. A. Select Table which was based on the records of 500,375 policies on standard lives issued by the forty-three companies in the investigation. (See Vol. I).

\* An Abstract by Mr. Hunter of the contents of Vol. I of the Report has already appeared in this Journal—Vol. xlvi, pp. 384-394.

## INFLUENCE OF BUILD ON MORTALITY AMONG MEN.

This investigation was based on the records of 744,672 policies issued during the years 1885 to 1908 in the United States and Canada on standard lives, the history being traced to the anniversaries of the policies in 1909. The data were divided into "5-pound" groups. The "average" group is the one in which the average weight was located; the "+5-pound" group is the next higher group. The overweights and underweights were thus classified in 6 underweight and 8 overweight groups and the following Table A (which excludes the extreme weight groups because of insufficient data) shows the influence of build on the mortality according to ages at entry:

TABLE A.

*Graduated Percentages of Actual Deaths to Expected by M. A. Table.*

Ages at Entry	VARIATION FROM AVERAGE WEIGHT IN POUNDS												
	-35 to -45	-25 to -30	-15 to -20	-10	-5	Aver- age	+5	+10	+15 to +20	+25 to +30	+35 to +45	+50 to +60	+65 to +80
20-24	135	127	115.5	107	105.5	104	102	99	97	102	104	110	125
25-29	122	116	108.5	102	101	100	99	97.5	96.5	104	108	116	132
30-34	112.5	108	102.5	98	97.5	97	96.5	96	97	109	118.5	131	149
35-39	105	101	97.5	94.5	95	95	96	96.5	101	112.5	133	151	172
40-44	99	95.5	93	91.5	93	94	96.5	97	108	115	141	157	181
45-49	93.5	91	89.5	89.5	91.5	93.5	97.5	100	112	116.5	139	155	178
50-53	88.5	88	87	88.5	90.5	94.5	99	102	112.5	116.5	132	150.5	172
54-56	86	86	86	88	90.5	95.5	99.5	102.5	112	116	122	142	162
57-59	86	86	86	88	90.5	95.5	99.5	102	111.5	114.5	117.5	134	153
60-62	86	86	86	88	90	95	98.5	101	110.5	112.5	114	130	148

The significant features of the above table are: (a) the lowest mortality is among the insured distinctly above the average weight at the young ages at entry and markedly below the average weight at the older ages of entry; (b) the slight influence of underweight on mortality, except at the youngest ages at entry; (c) the marked effect of overweight at the middle ages of entry; and (d) the tendency to a decreasing mortality among the overweights with the increase in the age at entry.

In studying the percentages of actual to expected deaths, the effect of medical selection must be considered. As the companies were aware that underweight at the young ages of

entry and overweight at the older ages of entry were serious impairments, special care in selection was exercised for these types of risks. It may be said, in general, that the greater the departure from the average weight, the more strict was the selection by the companies.

In the report there is a table giving the relative mortality according to the *percentage* variation from the average weight, but the foregoing table showing the effect of the *number of pounds* departure indicates briefly the main results of the investigation.

In order to determine whether the height of the insured had any effect on the mortality, the statistics were divided into three groups of height; namely, 5' 3"-5' 6", 5' 7"-5' 10" and 5' 11"-6' 2". These divisions were selected, as the average height of the men was found to be 5' 8 $\frac{1}{2}$ ". A synopsis of the percentages of actual to expected deaths appears in the following table:

TABLE B.—MEN.

*Effect of Height on Mortality. All Policy Years Combined.*  
*Ratio of Actual to Expected Deaths.*

Ages at Entry	VARIATION FROM AVERAGE WEIGHT									
	-25 to -45 Pounds			-10 to -20 Pounds			-5 to +5 Pounds			
	5' 3" to 5' 6"	5' 7" to 5' 10"	5' 11" to 6' 2"	5' 3" to 5' 6"	5' 7" to 5' 10"	5' 11" to 6' 2"	5' 3" to 5' 6"	5' 7" to 5' 10"	5' 11" to 6' 2"	
20-29	107%	117%	129%	96%	111%	119%	86%	101%	110%	
30-39	100	105	108	96	103	102	106	97	99	
40-49	88	95	95	81	92	94	107	88	106	
50-59	89	91	81	99	88	*84	100	92	*89	
+10 to +20 Pounds										
20-29	*84%	92%	109% *103%	103%	*133%		*98%	114%	122%	
30-39	88	93	96	124	128	127	161	146	151	
40-49	116	103	115	129	136	147	175	155	185	
50-59	127	115	*101	116	127	117	164	150	142	

\* Groups with less than 100 Deaths.

The foregoing percentages indicate that tall men have not been so desirable risks as short men at the younger ages of

entry, while at the older ages, the short and medium-sized men have not been so good risks as the tall men. It was not deemed advisable to prepare a graded table of mortality ratios showing the effect of height on longevity.

In order to determine whether there was any connection between the plans of insurance selected by the applicants and the mortality experienced, the policies were divided into four groups by plan, namely : Ordinary Life, Limited Payment Life, Endowment Insurance, and Temporary Insurance ; the last mentioned plan was omitted from the comparisons because of the small amount of insurance on that plan.

These tables indicated that at the younger ages of entry the percentage of endowment insurance was greater among the policyholders markedly *underweight* than among those of average weight, and this was probably due to the action of the Companies. It was also shown that the percentage of Endowment insurances at the older ages of entry among *overweights* was twice as great as among those of average weight. The Companies evidently sought to protect themselves against adverse selection by limiting to Endowment insurance many applicants who were distinctly *underweight* at the young ages or markedly *overweight* at the old ages of entry.

#### MORTALITY AMONG WOMEN.

The insured women were divided into four classes :

- (a) Spinsters,
- (b) Married women, beneficiary husband,
- (c) Married women, beneficiary other than husband,
- (d) Widows and divorced.

For the four classes combined the following was the experience :

*All Policy Years Combined.*

Ages at Entry	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
15-29	3,975	3,481.34	114%
30-39	4,438	4,258.40	104
40-49	3,431	3,498.22	98
50-59	3,040	3,034.41	100
60 and over	623	699.04	89
Total .....	15,507	14,971.41	104%

The mortality among women is higher than among men at the younger ages at entry, and lower at ages at entry 60 and above.

Tables are given in the report which show a distinctly higher mortality in the first five policy years than in the sixth and succeeding policy years, the force of adverse selection being felt by the companies most in the first policy year where the mortality was on the average 13 per-cent higher than the M. A. Table.

The following table shows the mortality according to conjugal condition at the date of issue of the insurance :

CLASS	Number Entering	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Spinsters ... ... ...	149,519	3,557	4,412	81%
Married women, beneficiary husband ... ...	94,813	3,235	2,566	126
Married women, beneficiary other than husband ...	82,973	4,403	3,869	114
Widows and divorced ...	65,727	4,312	4,124	105
Total ... ...	393,032	15,507	14,971	104%

Tables are given in which the mortality is analysed by ages at entry, and also by policy years.

The question was raised whether the difference between the mortality among spinsters and among married women was accounted for by the more frequent selection of Endowment insurance by spinsters. The mortality was accordingly prepared by plans of insurance :

- (a) Ordinary Life.
- (b) Limited Payment Life.
- (c) Endowment Insurance.

Combining the four classes of women, it was shown that the mortality according to the foregoing plans of insurance was 121 per-cent, 100 per-cent, and 89 per-cent respectively. Investigations showed that the mortality among married women was much higher than among spinsters insured on the same plan, and also that the relative mortality among the four classes of women under each plan of insurance differed little from the aggregate experience.

An interesting table is given of the distribution by plan of insurance, as follows :

	Ordinary Life	Limited Payment Life	Endowment Insurance
Spinsters	13%	33%	54%
Married women	36	38	26
Widows and divorced	43	32	25

The Committee states that it does not follow from the high mortality among married women that the companies should not insure them, or that it is impossible to select married women so as to obtain as favourable an experience as among men. Reference is made to one company who had a favourable mortality in a group of married women who paid the premiums out of their own incomes and whose children were the beneficiaries.

A warning is given that it should not be deduced from the experience of the forty-three companies that the mortality among spinsters in the general population would be much better than among married women or widows.

#### INFLUENCE OF BUILD ON MORTALITY AMONG WOMEN.

An investigation into the influence of build on the mortality of women did not give satisfactory results. This was due to combining four classes which showed such a substantial difference in mortality. As the ratio of actual to expected deaths was 81 per-cent among spinsters and 131 per-cent among married women, it follows that spinsters markedly overweight might have a lower mortality than married women of average weight. It was not thought advisable to make an investigation into each of the classes, partly because the amount of data would hardly justify the investigation, especially if divided according to plan of insurance.

#### CAUSES OF DEATH.

Tables of the proportion of deaths due to various causes in a group of standard lives were prepared for comparison with similar tables based on the lives of those engaged in hazardous occupations, or showing medical impairments. The proportions by cause of death were shown in two ways :

- (a) Percentage from each cause to the total number of deaths from all causes.
- (b) Ratio for each cause per 10,000 exposed to risk.

For the most common causes of death the statistics were given according to policy years.

Interesting tables are given comparing deaths by cause in three groups :

- (1) Those who were 50 pounds and more overweight (overweights).
- (2) Standard lives accepted at regular rates of premium. (Only a small proportion showed marked variation from average weight).
- (3) Those who were 25 pounds or more underweight (underweights).

Each of these was subdivided into three groups of ages at entry. At ages at entry 30 to 44, the death rate from tuberculosis of the lungs was nine times as heavy among the underweights as among the overweights. On the other hand, at the same ages at entry the death rate from diabetes was 11 times as heavy among the overweights as among the underweights.

Tables of causes of death are given for women and also a comparison of the causes of death among men and women. One of the noteworthy points brought out in the last mentioned comparison is the much higher death rate from typhoid fever and from appendicitis among men than among women.

#### MORTALITY AMONG NEGROES.

The negroes were divided into two classes :

- (1) Ministers, teachers, and other professional men.
- (2) All other negroes (men).

The result was as follows :

		Actual Deaths	Expected Deaths	Ratio of Actual to Expected
Professional men	...	202	147·4	137%
Other men	...	792	540·0	147

As a higher mortality was anticipated among negroes than among white men, an equally stringent selection of risks was

made. The higher mortality is, therefore, probably due to racial differences.

Investigations were also made into the mortality among Chinese and Japanese resident in the United States and Canada, but the numbers were too few to justify any satisfactory conclusions.

With regard to the North American Indians insured by the Companies, the expected deaths were 37·9, the actual 47, a ratio of 124 per-cent.

It is expected that Volume III., containing the report on the occupations, will be issued in the autumn.

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#### LEGAL NOTES.

By ARTHUR RHYS BARRAND, F.I.A., *Barrister-at-Law.*

Substituted policy.  
Rights of parties after substitution. THE case of *In re Shrager, March (Trustee) v. The North British and Mercantile Assurance Company*, deals with the question of the title to a policy which had been substituted for another after the holder of the original policy had been adjudicated bankrupt. A brief report of the case will be found in the *Law Journal* for 8 March 1913, and also in the *Law Times* for 15 February 1913, but it does not appear to have been reported in any of the ordinary Law Reports.

In this case the assured assigned a policy of life assurance for Rs. 60,000 to his wife by a voluntary assignment dated 26 April 1905. This assignment contained a proviso that the policy should revert to the assured in the event of his surviving his wife. On 19 November 1905, he was adjudicated bankrupt. The premium due in June 1906 was not paid, whereupon the policy lapsed, but certain rights remained to the owner of the policy for a period of eleven months following such lapsing. On 20 December 1906, the wife, in the exercise of her rights under the assignment, commuted the accrued bonus for Rs. 600, and received in addition Rs. 50 for rebate of premium. In June 1910 the bankrupt and his wife applied to the assurance company for the issue of a new paid-up policy, and a memorandum was endorsed on the old one stating that it should hold good for Rs. 12,000, and be non-participating and fully-paid.

In the same month the assurance company granted an advance of Rs. 1,600 to the wife, with the assent of the bankrupt, upon the security of the policy. No disclosure of the policy was made in the bankruptcy, and the trustee, having become aware of the transactions, applied for a declaration against the wife and the assurance company that the policy belonged to the estate, and that the company was liable to repay the sums of Rs. 600 and Rs. 50 paid to the wife, and that the present mortgage for Rs. 1,600 was void.

The case came before Phillimore, J., on 10 February 1913, and he decided in favour of the trustee. In delivering judgment to this effect, he said that "the assignment of the policy to the wife was obviously void under section 47 of the Bankruptcy Act, 1883. It was said on behalf of the company that the issue of a new policy in 1910 was a mere act of grace, and involved no contract upon which they could be sued. He did not think that that attitude could be sustained in view of the subsequent mortgage, which treated the policy as valid for all purposes. They must be taken to have had good consideration for what they did, and were as much bound by the new policy as by the old. The claim of the trustee to the policy, and to the two sums of Rs. 600 and Rs. 50, was therefore valid. With regard to the advance of Rs. 1,600, the company, under the doctrine of *In re Hart* ([1912] 3 K.B. 6, *J.I.A.* vol. xlvii, p. 92) would be entitled to protection as purchasers if they could prove that they made the advance without notice of the bankruptcy." The case therefore stood over for further evidence on this point, in order that the documents of the assurance company in India might be produced.

Deed of assignment retained by assignee on payment of policy moneys under assignment. Claim of assurance company to acknowledge right to production.

I am indebted to Mr. C. R. V. Coutts, F.I.A., of the National Mutual Life Assurance Society, and to Mr. H. Foot, F.I.A., of the Northern Assurance Company, for calling my attention to the case of *Dockray v. Refuge Assurance Company*, and to Mr. Foot also for a copy of the judgment. The case relates to the claim of an assurance company, on making payment of the policy moneys, to an acknowledgment of their right to the production of a deed of assignment relating to the policy retained by the assignee. The case, which does not appear to have been reported, was tried at the Manchester Assizes on 10 December 1912. The

material facts are as follows :—A policy was effected with the defendant company on 1 November 1897 by a Mrs. Ann Weston. In May 1905, Mrs. Weston assigned the policy to the plaintiff, and written notice thereof was given to the defendant company. When the policy matured the plaintiff produced proof of her title, but the assurance company refused to pay over the policy moneys until they were furnished with an acknowledgment of their right to the production of the deed of assignment. The plaintiff denied their right to demand this, asserting that since the Judicature Act, 1873, the assignee of an assurance policy could give a legal discharge to the company affected, and was bound to do no more. The defendants called witnesses who testified that it was the custom of assurance companies, in such cases, to demand an acknowledgment, and that such an acknowledgment had always been given. Coleridge, J., before whom the case came, said that he could not find that such a custom had been established. Therefore he had to fall back upon the common law, which said that if a person could give a legal discharge, the other person was not entitled to demand anything more. He accordingly gave judgment for the plaintiff, with costs.

In the course of his judgment, Coleridge, J., said : “ In this case a curious point has arisen in regard to the refusal of the defendants to pay over the moneys under a policy of insurance to an assignee without covenant or acknowledgment or agreement for production of the document of assignment. We are now dealing with the year 1912, and undoubtedly in old days, before the Judicature Act of 1873, matters were upon a very different footing as regards such transactions as are the subject of the present dispute. . . . The difficulty which arose in old days was because in those days, in regard to a *chose in action* and assignment, there could be no legal discharge given by an assignee, and the assignor might . . . at any time rise up and demand that he, being the person with whom originally the contract was made, was the person who could sue and recover upon such a contract. . . . It was necessary . . . and . . . reasonable that in such cases there should be a similar protection to what has been set up as existing now. In those days it would have been a protection—I do not know whether it did exist or not—at any rate it might have existed. But since the law was altered, a person who takes an absolute assignment by writing under the hand of the assignor,

“ and gives notice of such assignment to the other person who  
“ is party to the contract, is given all the rights of the assignor,  
“ and so far as the person who is the other party to the contract  
“ is concerned, it substitutes (without his choice) the assignee  
“ for the assignor. The assignee is to have all the rights as  
“ against the other contracting party, and the other contracting  
“ party could deal in safety with the assignee as if he were the  
“ original contracting party, where the due provisions of notice  
“ have been observed.

“ In this case there has been such an assignment, there has  
“ been such a notice, and the insurance company are placed  
“ in this position. The assignee is, so far as their obligations are  
“ concerned, placed in the position in law of the assignor. . . .  
“ It is said that, in spite of this, the insurance companies have  
“ established the practice or custom that the ordinary mode of  
“ dealing in this case should be altered in the case of policies  
“ of insurance, and that everybody knows it—because that is  
“ the meaning of ‘immemorial custom’; it is a custom which  
“ is known to everybody, and therefore the enforcement of  
“ which could take nobody by surprise. It is said that there  
“ has been a practice, too, in cases of assignment—it does not  
“ deal with any other matters—for the insurance company to  
“ obtain possession and keep in their custody the assignment  
“ as well as the cancelled policy, and that in the case of assign-  
“ ments by way of mortgage where the deed deals with other  
“ properties, and therefore is of a more or less complicated  
“ nature, it can demand production by the assignee of the  
“ mortgage as and when that shall be required.

“ First of all, I find as a matter of fact that such a custom  
“ has not been established. There have been called witnesses  
“ of large experience in various companies, but the evidence  
“ does not satisfy me that the course of business, which I daresay  
“ may exist between the assurance companies, arises, and is  
“ assumed by reason of an immemorial custom. . . . The  
“ evidence does not satisfy me that any such custom exists, and  
“ certainly one thing has not been proved, that if it has ever been  
“ resisted, it has been successfully proved. Of course it would  
“ be an argument in favour of immemorial custom that nobody  
“ would be so foolish as to resist it, but on the other hand, if  
“ there is no reason for resisting it, that argument loses the force  
“ which it would have if it were resisted and successfully proved  
“ on the ground that it was an immemorial custom. Therefore

"I find as a matter of fact that this practice has not been proved.

"Then what remains is that we have to fall back upon the common law. The common law, as I understand it, is that if a person can give a legal discharge the other party is not entitled to demand anything more. If he gives a legal discharge . . . he is entitled to the custody of the document.

"I think the plaintiff must succeed in this action for the amount which is claimed. . . . Therefore I give judgment . . . for the plaintiff with costs."

It was stated at the time that an appeal would probably be lodged, but I understand that as a matter of fact, no further action was taken in the matter.

I am indebted to Mr. S. J. Rowland, A.I.A., of the Equitable Life Assurance Society, for calling my attention to a case dealing with the withdrawal of a statutory deposit and the effect to be given to the depreciation which had taken place in the securities since they were deposited. The case in question arose out of the application of the Royal London Mutual Insurance Society, Limited, for the payment out of Court of £20,000, representing moneys that had been paid in by the company under the Employers' Liability Insurance Companies Act, 1907, on the ground that the Company had not carried on, and did not intend to carry on the business of employers' liability insurance. Counsel for the Board of Trade said that there had been a difference in the practice between the Courts as to whether only half of the fund should be transferred back, or whether so much should be transferred back as to leave £20,000 at the present prices, there having been a large fall in the value of the securities since they were deposited. The insurance company asked for the transfer of half the face value of the stock, the other half having been deposited in respect of life assurance business which the company was carrying on. Swinfen Eady, J., before whom the case came, said that *prima facie* it is half the fund that has to go back in such circumstances, although there may be special features in particular cases leading to a different conclusion. He therefore ordered the payment out of one half of the fund, the other half to go to the credit of the life assurance fund. The case is of interest as indicating the view taken by the Court of the question of the

Repayment of  
statutory deposit.  
Effect of  
depreciation.

depreciation of the securities deposited by assurance companies under the Assurance Companies Act, 1909.

Income tax in respect of double endowment assurance premiums. The appeal of the Crown against the decision in *Gould v. Curtis* ([1912] 1 K.B. 635, *J.I.A.*, vol. xlvi, p. 191), which is concerned with the rebate of income tax in respect of double endowment assurance premiums, was heard on 18 April 1913 (29 T.L.R. 469), when the Court of Appeal, consisting of Cozens-Hardy, M.R., Buckley and Kennedy, L.J.J., unanimously dismissed the appeal, and upheld the decision of the Court below, which was to the effect that a policyholder is entitled to a rebate of income tax in respect of the whole of such a premium, and not merely in respect of the temporary assurance portion, as contended by the Crown. The case has not yet been reported in the Law Reports, and in view of the importance of the decision and its bearing on ordinary endowment assurances, it has been thought advisable to postpone a more detailed note of the case until it is fully reported.

## REVIEWS.

### *The Mortality of Hungarian Assured Lives.*

(Published by the Central Office for Hungarian Mortality Investigation.)

THE four volumes comprised in this work are the result of the collaboration of eighteen companies transacting Life Assurance in Hungary. The observations cover a period of twenty-five years from 1 January 1876 to 31 December 1900 and are based on data relating to 374,835 male and 48,802 female lives.

The question as to whether the individual or the amount assured is the correct unit of observation is discussed in the preface. Some divergence of opinion having existed on the point, it was decided to compile the Tables in two ways, basing them upon individuals and upon the number of different occasions upon which each of the assured was medically examined. By this latter method each person is only included once in each age-group, but one person may be counted in several different age-groups. It is interesting to note that the two methods were found to give nearly identical results, from which it would appear that it is unnecessary to establish the identity of persons who are insured under several policies.

Aggregate, truncated and select tables are given for all business, and for old business, new business, whole-life and endowment

assurances separately. The age on the birthday nearest to the date of entry is taken as the age at entry and the duration is taken in complete years, periods of less than six months being disregarded and of more than six months being counted as a full year. The graduation throughout the tables is effected by means of Makeham's modification of the Gompertz formula.

The following extracts will give some idea of the results obtained :

#### HUNGARIAN TABLES.

##### *Graduated Rates of Mortality.*

##### *Male Lives—Whole Life Insurances. Aggregate Tables.*

Age	UNITS OF OBSERVATION		CORRESPONDING VALUES FROM BRITISH TABLES	
	Medical selections	Individuals	H <sup>M</sup>	O <sup>M</sup>
20	.00703	.00736	.00633	.00404
30	.00896	.00940	.00772	.00595
40	.01298	.01355	.01031	.00915
50	.02127	.02200	.01595	.01504
60	.03830	.03904	.02968	.02887
70	.07280	.07296	.06219	.06207
80	.14065	.13864	.14465	.13844
90	.26633	.25879	.27945	.30075

##### *Male Lives—Whole Life Insurances. Truncated Tables.*

Age	UNITS OF OBSERVATION— MEDICAL SELECTIONS	H <sup>M(5)</sup>	O <sup>M(5)</sup>
	First 5 years excluded		
20	.00961	.00833	.00652
30	.01109	.00920	.00747
40	.01441	.01132	.00978
50	.02180	.01712	.01545
60	.03815	.03064	.02921
70	.07377	.06284	.06219
80	.14879	.14577	.13850
90	.29544	.28244	...

The original data are given in full detail. There are Summary Tables showing the numbers of Deaths and Exposed to risk at the respective ages at entry and at observation and these are readily traceable into the Mortality Tables. The latter show the numbers observed of Deaths and Exposed to risk, unadjusted and graduated values of  $p_x$  and  $q_x$  and a graduated table of  $l_x$ . Following the

Mortality Tables are Select Tables giving unadjusted and graduated probabilities of death in each of the first ten years of insurance. Tables are given of the numbers of entries and exits at successive ages based respectively on different medical examinations and upon individuals and of the rates of mortality in successive years after entry. There are also detailed Tables of the numbers passing out of observation while living, and by death after all durations of insurance from one to sixty years. An analytical description of the Tables is given in Hungarian, German, English and French, and although the headings throughout are in Hungarian the difficulty of the foreign student is overcome by the reproduction at the end of each volume of the headings in the four languages just mentioned.

Herr Wilhelm von Ormody contributes an account of the methods of tabulating and enumerating the data and Herr Julius Altenburger explains the hypotheses upon which the work is based and the programme followed in its execution. It would have increased the interest of the work from an English point of view, had these accounts, which are given in Hungarian and German, been translated at length into English instead of being condensed into very brief extracts. There is, however, a mine of information available for those to whom the study of Hungarian Mortality is a matter of either scientific or professional interest.

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*Versicherungs-Lexikon.* Professor Dr. Alfred Manes.

(J. C. B. Mohr (Paul Siebeck). Tübingen.)

THE two original half-volumes of Dr. Manes' valuable work were reviewed in the *Journal* of the Institute so far back as July 1910. A supplemental volume has now been issued covering the period from 1908 to 1912 and containing 430 pages of text.

The plan of the Systematic Register is continued, the subjects dealt with in the original volumes are further elaborated and many others treated, while twelve biographies are added to the original number.

In the preface Dr. Manes points out that the past five years have been more fruitful in insurance development than many previous periods of a quarter of a century. New conditions have arisen in Germany as regards insurance business, the Government insurance schemes have been codified and extended, and social insurance has been adopted in other countries.

The new volume presents the same thoroughness of execution as its predecessors, and the wide field of insurance literature laid under contribution is apparent from a list, occupying nearly eleven pages, of the names of writers of varying degrees of eminence.

Among the minor branches of insurance additional articles are included on the following : Insurance against the Loss of Railway Season Tickets; against the Additional Responsibilities Imposed by Law upon German Employers in respect of Sickness and

Ineapacity of Workpeople; against Loss arising from Strikes and Popular Disturbances; against Death by Accident, and Loss of Baggage in the case of Emigrants; against Loss by Disease among Bees; against Insanity; against Loss by Floods and Inundations and Destruction of Crops; and against Loss to Silkworm Breeders from Unfavourable Harvests.

Apart from the importance of the work as a book of reference, its value as a guide to the sources of more detailed information is remarkable, and Dr. Manes is entitled to the thanks of the insurance profession on a notable achievement. It is hardly to be supposed that he will regard his work as ended, and if during the next five years the development of insurance is maintained, the preparation of further volumes is an obligation which he will no doubt readily fulfil.

*Nozioni di Scienza Attuariale.* Gennaro Minutilli.

(Ulrico Hoepli. Milan.)

THE Actuarial Student who reads Italian and appreciates a pocket volume will find this a very interesting little book which he can use from time to time to refresh his recollections of formulae. It maintains the high standard of the 1200 volumes comprised in the series of Manuali Hoepli, and although there are only 190 pages of reading matter the author has contrived to give a fairly comprehensive outline of the mathematical methods of Actuarial Science. The first three chapters forming the first part are devoted to the subject of probabilities simple and compound and of Bernouilli's theorem. Part II is divided into eight chapters dealing with Compound Interest, Mortality Tables, Commutation Tables, Annuities, Single and Annual Premiums, Assurances with varying Capital Sums or Premiums, Loading and Office Premiums and Reserves. The reasoning is clear and the printing accurate, although we observe a few errors likely to perplex the student. The derivation on pp. 11 and 12 of

$$-n \log \frac{99}{100} = -\log 3 \text{ from } 1 - \left(\frac{99}{100}\right)^n = \frac{1}{3}$$

the equation on p. 48

$$\frac{1}{i} \cdot \frac{1}{1 - \frac{1}{r^n}} = \frac{i}{1 - r^{-n}}$$

and the absence of brackets in the factors  $1 - S$ ,  $1 - 2S$ , etc., on pp. 149 and 150 are examples which need correction in any subsequent edition, while the explanation that the symbol  $l_x$  is derived from the English word "leaving" is an odd slip not unlikely to puzzle an Italian reader.

Throughout the book  $(1+i)$  is replaced by  $r$  and this leads to the use of  $r$  with negative indices instead of the more convenient symbol  $r$  of the Institute Notation.

The last 137 pages are devoted to an interesting collection of 87 tables which are alone sufficient to make the work a very handy book of reference. These comprise a table of the values of

$$\frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx$$

for values of  $x$  from .01 to 3.72, values of

$$(1+i)^n, (1+i)^{-n}, \frac{r^n - 1}{i} \text{ and } a_n$$

for rates of  $3, 3\frac{1}{4}, 3\frac{1}{2}, 3\frac{3}{4}, 4$  and  $5$  per-cent, from  $n=1$  to  $n=100$ ; Mortality Tables of Deparcieux (1746), the Carlisle Table (1787), Tables by Duvillard (1806) and Finlaison (1840), the 17 Offices (1843), the American Table (1868), Beauvisage (1869), H<sup>M</sup> Tables (1869), 23 German Companies (1883), O<sup>M</sup> Table (1900), Assurés Français and Rentiers Français (1895) and Italian Tables (1881 and 1901); a Table of Mortality per 1,000 from each of the foregoing, a Commutation table and a Table of Annuities and Premiums based on the Italian Table at  $3\frac{1}{2}$  per-cent. Following these are 63 tables containing values of  $l, d$  and  $p_{xz}$ , I, N and M,  $a_{xz}$ , A<sub>xz</sub> and P<sub>xz</sub>, for combinations of two lives,  $x$  and  $z$ , from  $x=z$  to  $x=z+20$ , at  $3\frac{1}{2}$  per-cent.

Pocket volumes such as these of the Manuali Hoepli in Italy and the German series of the Sammlung Göschen, both excellent in their presentation and condensation of mathematical and other subjects, seem to be somewhat lacking in this country. While they do not supply the wider range of information contained in the larger English text-books they provide the student with a portable guide to the main outlines of the subject of his study and enable him to utilize odd moments in revision. Both series contain short treatises on the Calculus and probably something of a similar nature would appeal to English students of the scientific side of insurance.

W. R. S.

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[Mr. W. Palin Elderton has kindly supplied the following notes on papers of actuarial interest read at the International Congress of Mathematicians, held at Cambridge in August 1912.—ED. J.I.A.]

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*Calculation of Moments of an Abrupt Frequency-Distribution.* By  
W. F. SHEPPARD.

WHEN the moments of a frequency-distribution have to be calculated it is usual to assume that the frequencies are concentrated about the central ordinate in much the same way as we assume in

the construction of mortality tables that the "nearest" age is the same as the "exact" age. The assumption is, however, not sufficiently accurate in many cases, and the resulting moments should be adjusted. Some few years ago Dr. Sheppard gave the correction required when the frequency-distribution has close contact at both extremities with the base, and one or two attempts have since been made to give adjustments in other circumstances. In the paper before us Dr. Sheppard gives formulæ in terms of the differences (or differential coefficients) of the successive frequencies. These differences can be calculated either directly or by means of an auxiliary function if one is available, and the resulting formulæ, though formidable in appearance, enable us to make accurate adjustments when the distribution rises abruptly and its starting points are known. In practice the latter condition is not very frequently fulfilled, and Dr. Sheppard's formulæ will not, therefore, carry us very far, but they are a step in the direction of a more complete solution of the problem.

*On the fitting of Makeham's Curve to Mortality Observations.* By  
J. F. STEFFENSEN.

In the paper before us Dr. Steffensen assumes that it has been decided to use Makeham's curve for graduating a mortality table, and shows how the method of least squares can be used to obtain the graduation in a rather less laborious way than those in which it has been applied in the past. The fundamental condition is that

$$\Sigma(y_x - \sigma_x)^2 \Gamma_x$$

is a minimum where  $\sigma_x$  is the observed value of  $y_x$  and  $\Gamma_x$  is proportional to the weight of  $\sigma_x$ ,  $y_x$  being, of course,  $a + \beta c^x$  in the case of Makeham's curve, and Dr. Steffensen proceeds by showing a form in which the condition can be put which can be solved approximately.  $\Gamma_x$  is taken as  $\frac{p_x E_x}{q}$  where  $p$  and  $q$  are found from a rough first graduation.

The method gives good results judging by the example in the paper, but the approximate solution of the equations seems to involve more work than the methods used by Mr. Hardy in the graduation of the British Offices tables, while we do not think there can be much gained in accuracy, as Mr. Hardy's graduation depends on the method of moments which gives practically the same results as the method of least squares in the cases in which the methods have been compared.

At the same time Dr. Steffensen's paper is interesting in showing how the method of least squares can be applied to a somewhat complicated problem and provides a solution which is apparently simpler than those previously given.

*Notes upon the Curves of Certain Functions involving Compound Interest and Mortality.* By R. R. BRODIE.

MR. BRODIE discusses the curves of the reserve values of single and annual premium whole-life assurances, endowment assurances, etc., and the changes in the curves resulting from increases in the rate of interest. The paper will be interesting to those who are helped by the graphical representation of algebraical or arithmetical work.

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# JOURNAL

OF THE

## INSTITUTE OF ACTUARIES.

*An investigation into the effects of Family and Personal History upon the rates of mortality experienced in various classes of Life Assurance risks, with special reference to Tuberculosis.*  
*By EDWARD A. RUSHER, F.I.A., Assistant Actuary of the Prudential Assurance Company, Limited, and CHARLES WILLIAM KENCHINGTON, F.I.A., of the Prudential Assurance Company, Limited.*

[Read before the Institute, 28 April 1913.]

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**Introduction.** An investigation into the effects of family and personal history upon assurance risks has long been desired by Actuaries, but hitherto in this country the material at their disposal has not been sufficient in volume to admit of analytical treatment. The only attempt the Institute as a body has made to deal with this subject is in the old Institute experience, which contains the records of 11,146 rated-up lives, in which all kinds of risks are combined without any attempt at classification.

**Object of  
Investigation.**

In the hope that this long-felt want might, to some extent, be met, an investigation was projected into the experience of a large number of lives assured in the Ordinary Branch of the Prudential Assurance Company whose family or personal history showed some marked peculiarity, by means of which they could be separated into more or less well-defined classes. The actual work of compiling the necessary data was commenced in 1903, and by the courteous permission and public-spirited action of the Directors we have now the pleasure of placing the main results of that investigation before the Institute. No claim is made in this Paper either to exhaustive analysis or to complete solution of the numerous and important problems that arise in practice, but it is thought that, at least, the results may form a useful aid to the Actuary in arriving at those conclusions which necessarily have to be formed—mainly from *a priori* reasoning—when risks come before him for adjudication.

**Actuarial and  
Medical  
Standpoint.**

That is to say, the primary object of the enquiry was to exhibit the experience in such a way as to enable the results to be applied practically in the assessment of risks. In regard to this it must be pointed out that the actuary has not altogether the same object in view as the doctor. The former is concerned mainly, if not entirely, with the financial result, so that what concerns him is the way in which a number of lives—fairly homogeneous in family and personal history—are subjected to special rates of mortality; at what age-group or groups that special rate first begins to show itself; and throughout what age-period it exists. The doctor, on the other hand, has in mind, to a large extent, the resulting benefits to humanity in the alleviation, and it may be even the elimination, of causes that tend to increase the death roll. He would thus seek for some light to be thrown on the causes themselves, as well as on the period of life at which they exert their most powerful influence. Too much of purely medical interest must not be expected, then, from the statistics given, though it is hoped that some light may be thrown upon some important problems—such as the effect on mortality of family and personal history, and the age-incidence of various causes of death.

**Source of  
Data.**

The information on the Office valuation cards was sufficient to indicate any defect, either in the family history or personal condition, where it existed, thus rendering it possible to pick out all the cases required; but, unfortunately, the

actual nature of the defect was not specified. To remedy this it was necessary to examine the actual proposal papers. The observations started with policies existing in 1893, and Valuation Cards of all kinds from that date onwards—whether existing, discontinued, or died—were carefully gone through to ensure the whole of the required data being brought into account.

A continuous record was maintained until it was thought that a sufficiently large body of data had accumulated, the observations eventually being closed on 31 December 1910. As those surviving at the commencement were brought under observation on their policy anniversaries in 1893, and the existing at the close were in similar manner taken to their policy anniversaries in 1910, the utmost period of observation in any one case was 17 policy-years.

The particulars having been filled in from the valuation cards, the experience cards were submitted together with the original proposal papers, to the Company's Medical Officer—the late Dr. E. M. Light—for classification according to the class of risk. The death of Dr. Light, after he had expended so much time and thought on the investigation and before he could see the fruit of his labour in its completion, was particularly untimely. Not only was the classification made according to a plan formulated by him, but every individual case passed through his hands. Had he lived to record the ripe experience thus gained, many obscure points would, doubtless, have been elucidated. The enormous volume of work he undertook in this connection may be appreciated from the fact that the records of the 140,898 policies included in the experience were all scrutinized and classified by him personally, in addition to which he was continuing the records down to the time of his death.

The classification adopted was based broadly on Bertillon's system, an adaptation of which by Melville-Dewey was described in Dr. Carruthers' Paper, read before this Institute in April 1905 (*J.I.A.*, vol. xxxix, p. 306), the extreme elaboration suggested by the latter, however, being avoided. At the outset the basis was determined by *a priori* reasoning, modifications or extensions being made as the work proceeded. The main groups were indicated by letters of the Alphabet, subsidiary sub-groups being indicated by Numerals. The form which this classification finally took is given in the following Table of

Classification. As it was necessarily prepared before it was possible to know whether the extent of the data would permit of sufficient facts being obtained in the respective sub-groups, it is not a matter for surprise that a large number of the groups could not be dealt with, owing to the paucity of data.

#### TABLE OF CLASSIFICATION.

##### A.—Relatives who died of consumption or childbirth.

0. Some distant relative, such as uncle or grandfather.
  - (a) *Lineals.*
  1. Father only.
  2. Mother only (consumption).
  3. Mother only (childbirth).
  4. Father and mother (consumption).
  - (β) *Lineals plus Collaterals.*
  5. Father and brother or sister (consumption).
  6. Mother and brother or sister (consumption).
  7. Mother (childbirth) and sister (childbirth).
  8. Mother (childbirth) and brother or sister (consumption).
  9. Father and mother, and brother or sister (consumption).
  - (γ) *Collaterals.*
  10. Sister (childbirth).
  11. One brother or sister (consumption).
  12. Two or more brothers or sisters (consumption).

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  13. Family History associated with some pulmonary disease, other than that of a tuberculous character.
  14. Unknown Family History.
  15. Obscure Family History.

##### B.—Personal History of Rheumatic Fever, and Lesions of the Circulatory System.

1. Rheumatic Fever (without acquiring Morbus Cordis).
2. Rheumatic Fever (with secondary Morbus Cordis).
3. Some form of Heart Disease without a history of Rheumatism or Rheumatic Fever.
4. Family History with three or more cases of Morbus Cordis.
5. Irregular or intermittent Pulse only.
6. Tachycardia.

7. Weak Heart.
  8. Fatty Heart.
  9. Hypertrophied Heart (without a valvular lesion).
  10. Dilated Heart.
  11. Bradycardia.
  12. Heart functionally deranged through abuse of Tobacco.
  13. Pericarditis.
  14. Arterio-Sclerosis.
  15. Aneurism.
  16. Angina pectoris.
- 

C.—Relatives who died of Cancer. (Two cases or more.)

(α) *Lineals.*

1. Father and mother.

(β) *Lineals plus Collaterals.*

2. Father and brother or sister.
3. Mother and brother or sister.
4. Father and mother, and brother or sister.

(γ) *Collaterals.*

5. Two or more brothers or sisters.
- 

D.—Obesity.

E.—Some personal blemish other than already tabulated.

F.—Formerly intemperate.

G.—Gouty Diathesis.

1. Family History of Gout.

H.—History of Glycosuria.

1. Family History of Diabetes.

I.—History of Asthma.

1. Family History of Asthma.

K.—Family History associated with diseases of the Nervous System—Mental or Spinal.

M.—Alcoholic Family History. (Two cases.)

N.—Apoplectic Family History. (Two cases.)

S.—Syphilitic Family History.

Z.—History of Zymotic disease.

1. Typhoid Fever.
2. Malarial Fever.
3. Scarlet Fever.
4. Influenza.
5. Dysentery.

In any case where the life belonged to more than one class the matter of treatment received careful consideration, and a small number of such cases were excluded altogether. Those that were retained consisted mainly of lives that combined a family history of tuberculosis with some personal blemish, indicated in the results as Group A+E. The rest were in Group E, the double taint referring to sub-groups of the same class. Throughout the observations, whole-life cases were kept distinct from endowment assurances, and, in addition, the sexes were distinguished. It is to be regretted that, even in so large an experience, the data both for whole-life assurances on male lives and for assurances on female lives, were often too small for the deduction of detailed conclusions.

*Classification of Causes of Death.* It was thought that some useful information might be obtained from an analysis of the causes of death.

For this purpose some standard of grouping was necessary, and, upon consideration, the list on page 439 was drawn up by Dr. Light, based mainly on the Registrar-General's classification. The numerical index was added to facilitate tabulation.

*Actuarial Treatment of Data.* The advantages of tabulating in "select" form are obvious, and in order to give effect to this, the data were tabulated in all cases according to duration of assurance, and in the final tables the analysis was retained as far as the first ten years following entry. In the principal groups additional tabulations were made in full aggregate form, and also in truncated form, eliminating the first five and the first ten years of assurance. In order to facilitate this the policy-year system was adopted. Fractional periods of exposure were eliminated by the adoption of the nearest duration method for withdrawals, and the existing were traced to their policy anniversaries in 1910. For the deaths the curtate duration was taken. No attempt was made to eliminate duplicates, as it was thought that the effect of their exclusion would not be sufficient to warrant the extra labour involved.

*Age at Entry.* The age at entry was taken throughout as the Office Age, that is to say, the age next birthday. It was felt that this was quite sufficient for the purpose in view. The work was considerably simplified thereby, and as quinquennial age-groups had ultimately to be employed, the use of a method involving further labour seemed an unnecessary refinement.

*Table of Causes of Death.*

Infective Diseases	1 Enteric Fever (Typhoid)—Dysentery—Diarrhoea (not otherwise defined)—Cerebro - Spinal Fever — Diphtheria — Erysipelas—Malaria, and other infective processes. 2 Pyæmia (not puerperal) — Septicæmia—Septic Intoxication. 3 Influenza.
Tuberculosis (all forms)	4 Phthisis—Pulmonary Tuberculosis—Tuberculosis of Spine and Joints—Tuberculosis of other organs.
Alcoholism	5 Alcoholism — Delirium Tremens — Cirrhosis of Liver.
Other General Diseases	6 Rheumatic Fever—Acute Rheumatism—Chronic Rheumatism—Gout—Osteo-arthritis. 7 Carcinoma—Sarcoma. 8 Anæmia—Leucocytæmia—Pernicious Anæmia 9 Diabetes Mellitus.
Diseases of the Nervous System	10 Paralysis—General Paralysis of the Insane—Locomotor Ataxia—Neuritis—Epilepsy—Meningitis, and other diseases of the Nervous System.
Diseases of the Heart	11 Valvular Disease — Pericarditis — Angina Pectoris—Syncope—Fatty Degeneration of the Heart, and other organic diseases of the Heart.
Diseases of Blood-Vessels	12 Cerebral Hæmorrhage—Apoplexy—Arterio-Sclerosis. 13 Embolism — Thrombosis — Phlebitis—and other diseases of the blood-vessels. 14 Aneurism.
Diseases of Respiratory System	15 Laryngitis — Bronchitis — Emphysema — Asthma—Congestion of Lungs, and other diseases of Respiratory System (not otherwise grouped). 16 Pneumonia—Pleurisy and Empyema.
Diseases of Digestive System	17 Quinsy—Gastric Ulcer—Gastritis—Enteritis and other diseases of the Digestive System. 18 Appendicitis and Perityphlitis. 19 Hernia. 20 Intestinal obstruction. 21 Peritonitis. 22 Diseases of the Liver—Gall-stones.
Diseases of Urinary System	23 Acute and chronic Nephritis (Bright's disease)—Renal Calculus, and other diseases of the Kidney. 24 Diseases of the Bladder, Prostate and Urethra.
Diseases of the Female Generative System	25 Fibroid Tumour of the Uterus, and other diseases (not malignant) of the Uterus and Ovaries.
Diseases of Pregnancy and Childbirth	26 Childbirth—Abortion—Misearriage—Puerperal Mania, and other accidents of pregnancy and childbirth.
Other Specified Diseases	27 Lymphadenoma (Hodgkin's disease)—Raynaud's disease—Grave's disease—Plumbism, and other specified diseases.
Ill-defined or not Specified Diseases	28 Debility—Old age—Dropsy—Tumour, and other ill-defined causes.
Suicide ... ...	29
Accident ... ...	30

*Summary  
of Data.*

To assist in forming some idea of the weight to be attached to the observations, Tables I to VII, giving a general summary of the data, are here set out.

TABLE I.

*Summary of Data.**Whole-Life Assurances.**Males.*

Group (1)	ENTRANTS			Deaths (5)	Average duration during period of observation (6)
	Survivors (2)	New Entrants (3)	Total (4)		
A <sub>0</sub> to A <sub>12</sub> ...	3,534	6,369	9,903	1,434	8·91
A <sub>13</sub> ...	65	751	816	79	7·73
A <sub>14</sub> and A <sub>15</sub> ...	533	1,917	2,450	343	8·11
A + other cause ...	326	621	947	201	9·58
B ...	938	736	1,674	509	9·88
C ...	16	147	163	15	5·3½
D ...	233	1,666	1,899	440	7·00
E ...	1,311	1,856	3,167	762	9·21
F ...	358	832	690	262	8·70
G ...	160	214	374	147	8·22
Total ...	7,474	14,609	22,083	4,192	8·71

TABLE II

*Summary of Data.**Endowment Assurances.**Males.*

Group (1)	ENTRANTS			Deaths (5)	Average duration during period of observation (6)
	Survivors (2)	New Entrants (3)	Total (4)		
A <sub>0</sub> to A <sub>12</sub> ...	7,393	48,372	55,765	2,424	7·36
A <sub>13</sub> ...	86	2,855	2,941	102	4·70
A <sub>14</sub> and A <sub>15</sub> ...	790	7,918	8,708	450	6·00
A + other cause ...	703	3,112	3,815	297	9·52
B ...	1,359	3,401	4,760	368	8·06
C ...	17	542	559	26	5·32
D ...	139	3,087	3,226	292	6·62
E ...	1,513	8,159	9,672	640	7·01
F ...	391	637	1,028	138	7·93
G ...	67	245	312	51	6·62
Total ...	12,458	78,328	90,786	4,788	7·20

TABLE III.

*Summary of Data.**Whole-Life Assurances.**Females.*

Group (1)	ENTRANTS			Deaths (5)	Average duration during period of observation (6)
	Survivors (2)	New Entrants (3)	Total (4)		
A <sub>0</sub> to A <sub>12</sub> ...	753	1,398	2,151	322	8·76
A <sub>13</sub> ...	21	207	228	23	7·61
A <sub>14</sub> and A <sub>15</sub> ...	140	511	651	95	8·00
A + other cause ...	50	111	161	45	9·01
B ...	177	147	324	131	9·31
C ...	7	44	51	9	6·69
D ...	141	863	1,004	235	7·45
E ...	332	427	759	221	9·12
Total ...	1,621	3,798	5,329	1,081	8·44

TABLE IV.

*Summary of Data.**Endowment Assurances.**Females.*

Group (1)	ENTRANTS			Deaths (5)	Average duration during period of observation (6)
	Survivors (2)	New Entrants (3)	Total (4)		
A <sub>0</sub> to A <sub>12</sub> ...	1,877	12,144	14,021	549	7·48
A <sub>13</sub> ...	22	736	758	31	4·77
A <sub>14</sub> and A <sub>15</sub> ...	153	1,893	2,046	74	5·83
A + other cause ...	127	799	926	67	8·72
B ...	287	733	1,020	66	7·66
C ...	8	157	165	2	6·05
D ...	81	911	992	103	7·35
E ...	452	2,320	2,772	156	7·24
Total ...	3,077	19,693	22,700	1,048	7·25

TABLE V.

*Summary of Data.**Males.*

Group (1)	WHOLE-LIFE ASSURANCES		ENDOWMENT ASSURANCES		TOTAL	
	Exposed (2)	Deaths (3)	Exposed (4)	Deaths (5)	Exposed (6)	Deaths (7)
A <sub>0</sub> to A <sub>12</sub> ...	88,219	1,434	410,551	2,424	498,770	3,858
A <sub>13</sub> ... ...	6,311	79	13,828	102	20,139	181
A <sub>14</sub> and A <sub>15</sub> ...	19,866	343	52,257	450	72,123	793
A + other cause... ...	9,073	201	36,307	297	45,350	498
B ... ...	16,536	509	38,379	368	54,915	877
C ... ...	871	15	2,975	26	3,846	41
D ... ...	13,287	440	21,350	292	34,637	732
E ... ...	29,161	762	67,799	640	96,960	1,402
F ... ...	6,001	262	8,148	138	14,149	400
G ... ...	3,074	147	2,065	51	5,139	198
Total ... ...	192,399	4,192	653,659	4,788	846,058	8,980

TABLE VI.

*Summary of Data.**Females.*

Group (1)	WHOLE LIFE ASSURANCES		ENDOWMENT ASSURANCES		TOTAL	
	Exposed (2)	Deaths (3)	Exposed (4)	Deaths (5)	Exposed (6)	Deaths (7)
A <sub>0</sub> to A <sub>12</sub> ...	18,838	322	104,554	549	123,692	871
A <sub>13</sub> ... ...	1,735	23	3,619	31	5,354	54
A <sub>14</sub> and A <sub>15</sub> ...	5,208	95	11,936	74	17,144	169
A + other cause... ...	1,451	45	8,076	67	9,527	112
B ... ...	3,018	131	7,811	66	10,829	197
C ... ...	341	9	998	2	1,339	11
D ... ...	7,479	235	7,290	103	14,769	338
E ... ...	6,924	221	20,076	156	27,000	377
Total ... ...	44,094	1,081	164,660	1,048	209,654	2,129

TABLE VII.

Summary of Data.

Both Sexes.

Group	WHOLE-LIFE ASSURANCES		ENDOWMENT ASSURANCES		TOTAL	
	Exposed	Deaths	Exposed	Deaths	Exposed	Deaths
(1)	(2)	(3)	(4)	(5)	(6)	(7)
A <sub>0</sub> to A <sub>12</sub>	107,057	1,756	515,405	2,973	622,462	4,729
A <sub>13</sub>	8,016	102	17,447	133	25,493	235
A <sub>14</sub> and A <sub>15</sub>	25,074	438	64,193	524	89,267	962
A + other cause	10,524	246	44,383	364	54,907	610
B	19,554	640	46,190	434	65,744	1,074
C	1,212	24	3,973	28	5,185	52
D	20,766	675	28,610	395	49,406	1,070
E	36,085	983	87,875	796	123,960	1,779
F	6,001	262	8,148	138	14,149	400
G	3,074	147	2,065	51	5,139	198
Total ...	237,393	5,273	818,319	5,836	1,055,712	11,109

It will be seen that Group A, consisting of cases having a family history of tuberculosis, is by far the largest. The great preponderance of endowment assurances is very noticeable, the total years of life under this class on male lives in Groups A<sub>0</sub> to A<sub>12</sub> (where there is some definite family history of tuberculosis) amounting to no less than 410,551, the corresponding deaths being 2,424. The magnitude of the data in this group rendered it possible to make a fairly detailed analysis of the sub-groups and to place confidence in the results.

In other groups detailed analysis was not feasible, and groups H, I, K, M, N, S, and Z did not afford sufficient data to warrant investigation.

To this list of massacred innocents must, unfortunately, be added Group C, in which there was a family history of cancer. In view of the importance, at the present time, of statistics throwing any light whatever upon this disease, it is much to be regretted that the number of cases were so few, amounting in all to 938 only. The total years of life amounted only to 5,185, so that, in addition to paucity of data, there is a lack of extended duration in the very class in which long periods of exposure might be expected to teach some useful lessons. It may, however, be noted that of the 52 deaths recorded, 11 were due to cancer and 6 to phthisis. Of 11 deaths from all causes amongst female lives, 5 were due to cancer. Comparatively large as the data are in total in the groups

History of Cancer.

A, B, D, E, F and G, they are insufficient to admit of the construction or graduation of full Select Tables.

Tables showing the exposed to risk and deaths in Aggregate Data. the main groups of the experience are furnished in an Appendix in the form of Full Aggregate and Truncated Aggregate Tables. It is hoped that the information there given may assist those who desire to further investigate the questions raised in the Paper.

The O<sup>(M)</sup> as a Standard of Comparison. It was thought that the best way of showing results was by a comparison of the actual deaths with those expected according to some standard Table. The standard adopted was the O<sup>(M)</sup>. There are certain obvious advantages in using this Table. Not only is it, at the present time,

the recognized standard of mortality amongst healthy lives assured under whole-life policies, but the rates of mortality during the first ten years of assurance were readily available for measuring the corresponding rates in the various groups.

It may here be stated that the expected deaths were calculated by multiplying the total exposed to risk in quinquennial groups of age by the appropriate value of  $q_x$  for the central age of the group. Investigation was made in a number of cases to determine the error introduced by this process, and it was found to be of no practical importance. In a few cases at the extremities of the Tables where the exposed to risk were increasing or decreasing rapidly the expected deaths were calculated for each individual age.

Light Mortality under Endowment Assurances. The O<sup>(M)</sup> Table, however, has the defect of relating only to whole-life assurances, whereas in the present experience a great preponderance of the cases were

under endowment assurances. Upon investigation it was found, as shown in Table VIII, that the actual deaths in the British Offices' Endowment Assurance Experience, Male Lives, New Assurances (O<sup>(EM)</sup>) during the first 10 years of assurance, were only 75 per-cent of the expected deaths calculated by the O<sup>(M)</sup> Table; after the expiration of 10 years the percentage increases with the age attained up to age 45, after which age it is practically a constant ratio of about 83 per-cent.

The opinion has been expressed that the light rate of mortality shown by the British Offices' experience under endowment assurances would not be a permanent feature of the experience of this class of assurance, as it was thought that the self-selection which had undoubtedly been operative in former times would

not be maintained, seeing that endowment assurances have to a large extent replaced whole-life assurances in modern practice. The same feature is, however, observed in such a marked degree in the present investigation that we are inclined to think that it is of a more permanent character than has been generally believed to be the case. Thus, whilst the O<sup>(M)</sup> has, for the sake of convenience, been retained throughout as the standard of comparison, a sufficiently accurate estimate of the effect of employing the British Offices' endowment assurance experience as a standard may be obtained by taking the expected deaths as three-fourths of those tabulated, or the percentages of actual deaths to those expected as increased by one-third.

TABLE VIII.

*Comparison of Actual Deaths in the British Offices' Endowment Assurance Experience (1863-1893) (Males), with those expected according to O<sup>(M)</sup> Table.*

(A) *First 10 years of assurance. (New Assurances only).*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
-37	2008·2	1506	75·0	1452·1	1088	74·9	3460·3	2594	75·0
38-52	523·5	393	75·1	421·3	302	71·7	944·8	695	73·6
Total	2531·7	1899	75·0	1873·4	1390	74·2	1405·1	828·9	74·7

(B) *Ultimate Table, excluding first 10 years of assurance. (Combined Old and New Assurances).*

Central Age Attained	Expected Deaths by O <sup>(M)</sup> Ultimate.	Actual Deaths	Percentage
(1)	(2)	(3)	(4)
35	281·0	201	71·5
40	498·0	391	78·5
45	611·2	532	83·0
50	632·5	526	83·2
55	516·1	405	79·1
60	255·7	229	89·6
65	53·5	44	82·2
Total	2878·0	2331	81·0

## FAMILY HISTORY OF TUBERCULOSIS.

Tuberculosis.  
Combined  
Experience.

Coming now to the actual groups, the first to be dealt with is that containing a family history of tuberculosis, *i.e.*, Group A. To obtain a first general impression, sub-groups  $A_0$  to  $A_{12}$ , which include all cases showing a definite family history of tuberculosis, were combined. Sub-groups  $A_{13}$ ,  $A_{14}$ , and  $A_{15}$  were dealt with separately, as the family history is not of quite the same definite character.

Tables IX and X show the combined experience on male lives for each of the first ten years of assurance; Table IX referring to whole-life, and Table X to endowment assurances. The data for female lives were too small to show any useful result when set out in such detail. Each central age at entry is shown, and summaries are given for central age-groups 20 to 35 (*i.e.*, ages at entry 18 to 37), 40 to 50 (*i.e.*, ages at entry 38 to 52), and 55 to 60 (*i.e.*, ages at entry 53 to 62). A final summary is also given for all ages at entry. These summary age-groups were chosen as it was found that they best enabled the general trend of the curves to be seen.

For each of the ten years of assurance the deaths expected according to the O<sup>(M)</sup> Table are shown in one column, and the corresponding actual deaths in the next succeeding column. In addition, for each of the summary age-groups the percentages of actual to expected deaths are set out in distinctive type.

TABLE IX  
*Family History of Tuberculosis. Combined Experience A<sub>0</sub> to A<sub>12</sub>.*  
*Comparison of Actual Deaths with those expected according to O'N<sub>1</sub>* Table  
*Whole-Life Assurances*

YEARS OF ASSURANCE													Central Age at Entry		Central Age at Entry							
0		1		2		3		4		5		6		7		8		9		0-9		
Expected Deaths		Actual Deaths		Expected Deaths		Actual Deaths		Expected Deaths		Actual Deaths		Expected Deaths		Actual Deaths		Expected Deaths		Actual Deaths		Expected Deaths		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	
20	.2	..	.3	.2	.4	.4	.4	..	.5	..	.5	..	.5	..	.6	1	.6	2	4.5	10	222.2	
25	1.4	1	2.1	2	2.4	4	2.5	3	2.7	4	2.9	5	3.0	1	3.2	4	3.3	3	26.1	30	114.9	
30	2.9	5	4.3	9	4.8	7	5.0	8	5.4	4	6.0	6	6.5	9	6.8	9	7.0	6	7.5	7	56.2	
35	4.1	10	5.8	11	6.8	14	7.3	12	7.7	9	8.4	7	9.2	9	10.0	15	10.5	7	10.9	12	80.7	
20-35	8.6	16	12.5	24	14.1	25	15.2	27	16.1	16	17.6	17	19.1	23	20.4	26	21.3	18	22.3	24	167.5	
Per-cent	186.0	192.0	173.6	177.6	99.4	96.6	120.4	127.5	84.5	84.5	107.6	129.0	129.0	129.0	129.0	129.0	129.0	129.0	129.0	129.0	20.35	
Per-cent	40	4.9	10	6.5	15	7.2	8	7.7	13	8.1	7	8.8	8	9.7	14	10.2	11	10.7	9	11.2	15	85.0
45	6.3	8	8.3	10	9.5	12	10.1	16	11.0	15	11.6	9	12.1	12	12.9	14	13.3	11	13.9	5	109.0	
50	6.1	9	7.3	7	8.1	7	8.4	8	9.0	11	9.4	8	10.0	16	10.4	14	10.4	9	10.4	13	89.5	
40-50	17.3	27	22.1	32	24.8	32	26.2	37	28.1	33	29.8	25	31.8	42	33.5	39	34.4	29	35.5	33	283.5	
Per-cent	156.1	144.8	108.9	141.2	117.4	83.9	132.1	116.4	84.3	84.3	93.0	93.0	93.0	93.0	93.0	93.0	93.0	93.0	93.0	93.0	114.3	
55	3.0	3	3.4	5	3.7	4	3.8	1	3.9	3	3.9	1	4.2	4	4.4	4	4.3	4	4.0	4	386.6	
60	.7	1	.9	2	1.0	..	1.0	..	1.2	3	1.3	1	1.6	1	1.5	2	1.6	3	11.7	13	111.1	
55-60	3.7	4	4.3	5	4.6	6	4.8	1	4.9	3	5.1	4	5.5	5	6.0	5	5.8	6	5.6	7	50.3	
Per-cent	108.1	116.3	130.4	20.8	61.2	28.4	90.9	83.3	103.4	103.4	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	125.0	91.5	
Total	29.6	47	38.9	61	43.8	58	46.2	65	49.1	52	52.5	46	56.4	70	59.9	70	61.5	53	63.4	64	501.3	
Per-cent	158.8	156.8	122.4	140.7	105.9	87.6	116.9	124.1	86.2	86.2	100.9	116.9	116.9	116.9	116.9	116.9	116.9	116.9	116.9	116.9	116.9	

Actual to

Expected Deaths,

Percentage of

Deaths.

Males,

Endowment Assurances. Combined Experience A<sub>0</sub> to A<sub>12</sub>. Comparison of Actual Deaths with those Expected according to O<sup>[M]</sup> Table.

TABLE X.

Family History of Tuberculosis. Combined Experience A<sub>0</sub> to A<sub>12</sub>.

Percentage of Actual Deaths.

Central Age at Entry	Years of Assurance												Central Age at Entry	Central Age at Entry										
	0	1	2	3	4	5	6	7	8	9	0-9	(23)	(24)	(25)										
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)
20	13·5	14	19·3	20	20·5	15	20·3	13	20·0	28	19·6	13	19·1	12	18·2	5	16·8	10	15·4	13	182·7	143	78·3	20
25	33·4	31	48·0	52	52·0	33	51·8	57	50·9	28	50·4	48	49·0	40	47·3	41	44·3	31	40·5	19	467·6	380	81·3	25
30	33·2	31	48·5	31	53·9	42	55·8	41	56·8	35	57·0	44	56·5	29	55·4	34	53·4	40	50·1	26	520·6	353	67·8	30
35	29·0	33	41·1	37	45·7	33	47·9	35	49·3	42	50·2	18	50·7	36	50·3	37	48·6	36	46·7	39	459·5	346	75·3	35
20-35	109·1	109	156·9	140	172·1	123	175·8	146	177·0	133	177·2	123	175·3	117	171·2	117	163·1	117	152·7	97	1630·4	1222	75·0	20-35
Per-cent	99·9	89·2	71·5	51·5	83·0	75·1	69·4	66·7	68·3	71·7	68·3	71·7	69·4	66·7	64·3	61·7	60·3	57·7	55·3	53·0	50·7	48·3	46·0	44·7
40	21·1	23	32·7	21	36·1	23	37·9	17	39·3	27	40·6	29	41·2	25	41·3	21	40·8	18	38·9	24	372·9	228	61·1	40
45	17·6	12	22·5	10	24·8	14	26·0	26	26·9	16	28·1	15	28·7	19	28·8	13	28·3	19	26·9	21	258·6	165	63·8	45
50	7·1	5	8·4	5	8·8	6	9·2	9	9·6	3	9·9	7	10·4	7	10·4	8	10·1	8	9·5	7	93·4	65	69·6	50
40-50	48·8	40	63·6	36	69·7	43	73·1	52	75·8	46	78·6	51	80·3	51	80·5	42	79·2	45	75·3	52	724·9	458	63·2	40-50
Per-	80·8	56·1	61·2	70·5	61·4	64·3	64·3	51·7	56·3	68·5	56·3	51·7	56·3	51·7	56·3	51·7	56·3	51·7	56·3	56·3	63·2	63·2	63·2	Per-
Total	157·9	149	220·5	176	241·8	166	248·9	198	252·8	179	255·8	174	255·6	168	251·7	159	242·3	162	228·0	149	2355·3	1680	71·3	Total
Per-	94·4	79·8	68·7	79·6	70·8	68·0	65·7	63·2	66·9	65·4	65·4	65·4	65·4	65·4	65·4	65·4	65·4	65·4	65·4	65·4	71·3	71·3	71·3	Per-
centage																								Percentage of Actual to Expected Deaths.

Tables XI and XII give the summary results for years of assurance 0 to 4 and 5 to 9 for male and female lives respectively.

TABLE XI.

*Family History of Tuberculosis. Combined Experience A<sub>0</sub> to A<sub>12</sub>*

*Comparison of Actual Deaths with those Expected according to O<sup>[M]</sup> Table.  
First 10 years of assurance.*

(A). *Whole-Life Assurances.*

*Males.*

Ages at Entry	YEARS OF ASSURANCE									
	0-4			5-9			0-9			(10)
	Expected Deaths	Actual Deaths	Per- centage	Expected Deaths	Actual Deaths	Per- centage	Expected Deaths	Actual Deaths	Per- centage	
	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	
-37	66·8	108	161·7	100·7	108	107·2	167·5	216	129·0	
38-52	118·5	156	131·6	165·0	168	101·8	283·5	324	114·3	
53-62	22·3	19	85·2	28·0	27	96·4	50·3	46	91·5	
Total.	207·6	283	136·3	293·7	303	103·2	501·3	586	116·9	

(B). *Endowment Assurances.*

*Males.*

-37	790·9	651	82·3	839·5	571	68·0	1630·4	1222	75·0
38-52	331·0	217	65·6	393·9	241	61·2	724·9	458	63·2
Total	1121·9	868	77·4	1233·4	812	65·8	2355·3	1680	71·3

TABLE XII.

*Family History of Tuberculosis. Combined Experience A<sub>0</sub> to A<sub>12</sub>.*  
*Comparison of Actual Deaths with those Expected according to O<sup>[M]</sup> Table.*  
*First 10 years of assurance.*

(A). *Whole-Life Assurances.**Females.*

Ages at Entry ( )	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths (2)	Actual Deaths (3)	Per- centage (4)	Expected Deaths (5)	Actual Deaths (6)	Per- centage (7)	Expected Deaths (8)	Actual Deaths (9)	Per- centage (10)
-37	10·0	18	180·0	15·2	21	138·2	25·2	39	154·8
38-52	33·5	41	122·4	49·7	49	98·6	83·2	90	108·2
53-62	9·4	8	85·1	11·4	10	87·7	20·8	18	86·5
Total	52·9	67	126·7	76·3	80	104·8	129·2	147	113·8

(B). *Endowment Assurances.**Females.*

-37	187·9	141	75·0	206·0	122	59·2	393·9	263	66·8
38-52	106·6	57	53·5	142·0	70	49·3	248·6	127	51·1
Total	294·5	198	67·2	348·0	192	55·2	642·5	390	60·7

The figures relating to the "ultimate" experience, excluding the first ten years of assurance, are given in Table XIII. It is to be observed that in this Table the ages are ages "attained" and not ages at entry.

TABLE XIII.

*Family History of Tuberculosis. Combined Experience A<sub>0</sub> to A<sub>12</sub>. Comparison of Actual Deaths with those Expected according to O<sup>[M]</sup> Table. Ultimate Experience, excluding first 10 years of assurance.*

(A). *Whole-life Assurances.* Males.

Central Age attained (1)	Expected Deaths (2)	Actual Deaths (3)	Percentage (4)
30	1·3 } 13·0	1 } 16	123·1
35	11·7 } 13·0	15 } 16	
40	39·3 }	38 }	
45	84·0 } 123·3	87 } 125	101·4
50	125·9 }	126 }	
55	161·3 }	137 }	91·6
60	173·1 }	199 }	
65	127·2 }	124 }	107·6
70	72·0 }	70 }	
75	34·7 }	30 }	93·7
80	14·4	17	118·1

(B). *Endowment Assurances.* Males.

30	24·9 }	15 }	65·7
35	122·8 }	82 }	
40	206·1 }	119 }	
45	243·5 }	155 }	60·9
50	230·7 }	148 }	
55	172·6 }	128 }	68·4
60	90·8 }	82 }	
65	14·1 }	11 }	83·7

It is hoped that the arrangement of the results in the form of percentages of a standard Table may enable the facts to be readily grasped; but it is

Caution necessary in using Percentages of Totals. necessary to point out that the percentages deduced from the totals for all ages at entry must be used with caution in making comparisons between different sections of the data.

This caution is required because the varying age constitution of the groups is quite lost sight of in these percentages of totals.

The point is well illustrated by reference to the percentages

given in Table XI for years of assurance 0 to 4. Under whole-life assurances the percentage in total is 136·3, while that for endowment assurances is 77·4. Taken by themselves, these figures would appear to show a difference of 58·9 per-cent in favour of the endowment assurances. If, however, the whole-life experience had not extended beyond age 52 at entry the percentage for that Table would have been increased to 142·5, thus showing the larger difference of 65·1 per-cent between the classes of assurance. Even this statement does not fully express the difference between the rates in the two particular sections, for the average age at entry in the whole-life experience was higher than that under endowment assurances, and a redistribution of the whole-life experience according to age at entry, in the same proportion as the exposed to risk under the endowment assurances, would have led to an even greater divergence in the percentages—as we proceed to show.

The exposed to risk which formed the basis of Table XI for years of assurance 0 to 4 are set out in Table XIV.

TABLE XIV.

*Exposed to Risk for Years of Assurance 0 to 4.*

*Male Lives.* *Group A<sub>0</sub> to A<sub>12</sub>.*

Ages at Entry	WHOLE-LIFE ASSURANCES		ENDOWMENT ASSURANCES	
	Exposed to Risk	Percentage on Total Exposures	Exposed to Risk	Percentage on Total Exposure
(1)	(2)	(3)	(4)	(5)
-37	12,024	47·9	152,216	79·0
38-52	13,094	52·1	40,518	21·0
Total	25,118	100·0	192,734	100·0

Applying the percentages shown in column 5 of Table XIV to redistribute the expected and actual deaths in columns 2 and 3 of Table XI (A), the percentage of actual to expected deaths for ages at entry below 53 is 152·5. This, then, would have been the percentage in the whole-life class if the age-distribution had been identical with that under endowment assurances, and the difference between the classes, instead of 58·9 as it at present stands, would have been no less than 75·1 per-cent.

Owing to the variations in age distribution this warning against the use of percentages of totals is also required in a modified degree in making comparisons between the sub-groups for the same class of assurance.

The percentages of totals are useful for comparing the figures within the group to which the totals refer, but cannot be relied upon for other purposes, and comparisons between the different classes of risk should be made, as far as possible, by means of the percentages for corresponding age-groups.

Difference in Mortality between Whole-Life and Endowment Assurances. The first point that attracts attention is the great difference between the mortality in the two classes of assurance. From Table XI it will be seen that in years of assurance 0 to 4 the mortality in the whole-life class is approximately double that in the endowment assurance class, and even for years 5 to 9 there is a difference of about 40 in the percentages, whilst much greater differences are shown in Table XII.

As will be seen later on, this feature is not peculiar to the Combined Table, but appears in every sub-group, and is throughout more marked amongst female than male lives. Nor is it confined only to the first ten years of assurance, as it is just as clearly a feature of the ultimate experience given in Table XIII. Again, it will be noted that the whole-life male experience follows the  $O^{[M]}$  mortality fairly closely after the first five years of assurance, whilst the endowment assurances for ages at entry under 38 follow that of the  $O^{[EM]}$ . Above that age the endowment assurances show more favourable results, and in the "Ultimate" experience the mortality is lighter than that of the  $O^{[EM]}$ . That endowment assurances should show such light rates after the first five years is somewhat unexpected, and we suggest, as an explanation, that the selection exercised by the assured is a most potent factor and practically of equal importance to the medical selection. No doubt, had it been possible to distinguish between the various terms of years for which the endowment assurances were effected, the mortality would have shown results corresponding to such terms. One thing seems obvious: that in the assessment of risks there should be a differentiation in practice as between whole-life and endowment assurances.

Male and Female Mortality compared.

Another feature of these Tables is the lighter rate of mortality experienced by female lives than by male lives. This feature is brought out clearly by

comparing Table XI with Table XII, and is persistent for each group of entry-ages and for each section of the Tables for the first ten years of assurance, with the single exception of ages at entry below 38 under whole-life assurances. The exception is noteworthy, as the excess mortality is considerable, and coincides with the results of previous experiences under whole-life assurances on female lives. It may possibly point to the fact that under endowment assurances the spinsters are in greater proportion than the married women.

**High Mortality Rate in Early Years of Assurance.** Returning to the high rate of mortality during the first five years of assurance, it will be seen that whilst the endowment assurance rates are but little in excess of  $O^{(EM)}$ , the whole-life are considerably in excess of  $O^{(M)}$ . From Tables IX and X it will be seen that this excess in both whole-life and endowment assurances is at its maximum in year of assurance 0, and that it decreases fairly rapidly with the duration. In regard to the year of assurance 0, it is to be remarked that the  $O^{(EM)}$  experience also shows a relatively heavy rate of mortality at this period as compared with  $O^{(M)}$ . This raises a doubt as to whether, from some unexplained cause, the  $O^{(M)}$  may not give too small a value for  $q_{[x]+6}$ , and, consequently, whether it is a suitable standard at this point. After making all allowances for this possibility, however, it would still appear that the mortality experienced in the early years of assurance was heavy. The excess is greatest for the younger ages at entry. Combining these facts, we may say that the effect of a definite family history of tuberculosis shows itself mainly in the younger ages at entry and the early years of assurance.

**Tuberculosis. Sectional Experiences.** Passing from the consideration of Group A as a whole, we consider next the three main sections of which it is composed. These are ( $\alpha$ ) Lineals, ( $\beta$ ), Lineals plus Collaterals and ( $\gamma$ ) Collaterals. The same general arrangement of the Tables has been adopted as for the combined results, except that owing to paucity of data, the detailed facts for each of the first ten years of assurance are shown only for endowment assurances on male lives.

It may be well to point out here that the terms "lineal" and "collateral" are used throughout this Paper in the special and restricted sense indicated in the Table of Classification.

Males.

(a) Family History of Tuberculosis in Lineals.  
Comparison of Actual Deaths with those Expected according to Old Table.

Endowment Assurances.

YEARS OF ASSURANCE

Central Age at Entry	C-9									Central Age at Entry
	0	1	2	3	4	5	6	7	8	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
6·9	7	9·7	12	10·1	8	9·9	12	9·6	5	9·2
15·0	14	21·0	24	22·6	19	21·7	11	21·3	26	20·5
30	12·7	15	18·2	10	19·9	18	20·4	15	19·5	9
35	8·9	6	12·2	11	13·3	7	13·7	6	13·9	4
20-35	43·5	42	61·1	57	66·1	52	66·4	42	65·1	49
Per-centage	96·6	93·3	78·7	78·3	63·6	75·3	62·0	49·5	78·9	76·6
40	6·0	7	8·0	6	8·6	9	8·7	4	8·8	8
45	3·8	2	4·7	1	5·1	5	5·2	4	5·2	1
50	1·1	1	1·3	1	1·4	...	1·6	...	1·5	...
40-50	10·4	10	14·0	8	15·1	14	15·7	9	15·9	13
Per-centage	91·7	57·1	92·7	51·9	57·3	81·8	37·7	58·1	47·0	80·3
Total	54·4	52	75·1	65	81·2	66	81·7	51	81·0	62
Per-centage	95·6	86·6	81·3	73·3	62·4	76·5	57·1	51·2	72·3	77·4

Endowment Assurances.

TABLE XV.

(a) Family History of Tuberculosis in Lineals.

Comparison of Actual Deaths with those Expected according to Old Table.

Central Age at Entry	C-9									Central Age at Entry
	0	1	2	3	4	5	6	7	8	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
6·9	7	9·7	12	10·1	8	9·9	12	9·6	5	9·2
15·0	14	21·0	24	22·6	19	21·7	11	21·3	26	20·5
30	12·7	15	18·2	10	19·9	18	20·4	15	19·5	9
35	8·9	6	12·2	11	13·3	7	13·7	6	13·9	4
20-35	43·5	42	61·1	57	66·1	52	66·4	42	65·1	49
Per-centage	96·6	93·3	78·7	78·3	63·6	75·3	62·0	49·5	78·9	76·6
40	6·0	7	8·0	6	8·6	9	8·7	4	8·8	8
45	3·8	2	4·7	1	5·1	5	5·2	4	5·2	1
50	1·1	1	1·3	1	1·4	...	1·6	...	1·5	...
40-50	10·4	10	14·0	8	15·1	14	15·7	9	15·9	13
Per-centage	91·7	57·1	92·7	51·9	57·3	81·8	37·7	58·1	47·0	80·3
Total	54·4	52	75·1	65	81·2	66	81·7	51	81·0	62
Per-centage	95·6	86·6	81·3	73·3	62·4	76·5	57·1	51·2	72·3	77·4

Central Age at Entry	C-9									Central Age at Entry
	0	1	2	3	4	5	6	7	8	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
6·9	7	9·7	12	10·1	8	9·9	12	9·6	5	9·2
15·0	14	21·0	24	22·6	19	21·7	11	21·3	26	20·5
30	12·7	15	18·2	10	19·9	18	20·4	15	19·5	9
35	8·9	6	12·2	11	13·3	7	13·7	6	13·9	4
20-35	43·5	42	61·1	57	66·1	52	66·4	42	65·1	49
Per-centage	96·6	93·3	78·7	78·3	63·6	75·3	62·0	49·5	78·9	76·6
40	6·0	7	8·0	6	8·6	9	8·7	4	8·8	8
45	3·8	2	4·7	1	5·1	5	5·2	4	5·2	1
50	1·1	1	1·3	1	1·4	...	1·6	...	1·5	...
40-50	10·4	10	14·0	8	15·1	14	15·7	9	15·9	13
Per-centage	91·7	57·1	92·7	51·9	57·3	81·8	37·7	58·1	47·0	80·3
Total	54·4	52	75·1	65	81·2	66	81·7	51	81·0	62
Per-centage	95·6	86·6	81·3	73·3	62·4	76·5	57·1	51·2	72·3	77·4

Central Age at Entry	C-9									Central Age at Entry
	0	1	2	3	4	5	6	7	8	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
6·9	7	9·7	12	10·1	8	9·9	12	9·6	5	9·2
15·0	14	21·0	24	22·6	19	21·7	11	21·3	26	20·5
30	12·7	15	18·2	10	19·9	18	20·4	15	19·5	9
35	8·9	6	12·2	11	13·3	7	13·7	6	13·9	4
20-35	43·5	42	61·1	57	66·1	52	66·4	42	65·1	49
Per-centage	96·6	93·3	78·7	78·3	63·6	75·3	62·0	49·5	78·9	76·6
40	6·0	7	8·0	6	8·6	9	8·7	4	8·8	8
45	3·8	2	4·7	1	5·1	5	5·2	4	5·2	1
50	1·1	1	1·3	1	1·4	...	1·6	...	1·5	...
40-50	10·4	10	14·0	8	15·1	14	15·7	9	15·9	13
Per-centage	91·7	57·1	92·7	51·9	57·3	81·8	37·7	58·1	47·0	80·3
Total	54·4	52	75·1	65	81·2	66	81·7	51	81·0	62
Per-centage	95·6	86·6	81·3	73·3	62·4	76·5	57·1	51·2	72·3	77·4

Central Age at Entry	C-9									Central Age at Entry
	0	1	2	3	4	5	6	7	8	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
6·9	7	9·7	12	10·1	8	9·9	12	9·6	5	9·2
15·0	14	21·0	24	22·6	19	21·7	11	21·3	26	20·5
30	12·7	15	18·2	10	19·9	18	20·4	15	19·5	9
35	8·9	6	12·2	11	13·3	7	13·7	6	13·9	4
20-35	43·5	42	61·1	57	66·1	52	66·4	42	65·1	49
Per-centage	96·6	93·3	78·7	78·3	63·6	75·3	62·0	49·5	78·9	76·6
40	6·0	7	8·0	6	8·6	9	8·7	4	8·8	8
45	3·8	2	4·7	1	5·1	5	5·2	4	5·2	1
50	1·1	1	1·3	1	1·4	...	1·6	...	1·5	...
40-50	10·4	10	14·0	8	15·1	14	15·7	9	15·9	13
Per-centage	91·7	57·1	92·7	51·9	57·3	81·8	37·7	58·1	47·0	80·3
Total	54·4	52	75·1	65	81·2	66	81·7	51	81·0	62
Per-centage	95·6	86·6	81·3	73·3	62·4	76·5	57·1	51·2	72·3	77·4

Central Age at Entry	C-9									Central Age at Entry
	0	1	2	3	4	5	6	7	8	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
6·9	7	9·7	12	10·1	8	9·9	12	9·6	5	9·2
15·0	14	21·0	24	22·6	19	21·7	11	21·3	26	20·5
30	12·7	15	18·2	10	19·9	18	20·4	15	19·5	9
35	8·9	6	12·2	11	13·3	7	13·7	6	13·9	4
20-35	43·5	42	61·1	57	66·1	52	66·4	42	65·1	49
Per-centage	96·6	93·3	78·7	78·3	63·6	75·3	62·0	49·5	78·9	76·6
40	6·0	7	8·0	6	8·6	9	8·7	4	8·8	8
45	3·8	2	4·7	1	5·1	5	5·2	4	5·2	1
50	1·1	1	1·3	1	1·4	...	1·6	...	1·5	...
40-50	10·4	10	14·0	8	15·1	14	15·7	9	15·9	13
Per-centage	91·7	57·1	92·7	51·9	57·3	81·8	37·7	58·1	47·0	80·3
Total	54·4	52	75·1	65	81·2	66	81·7	51	81·0	62
Per-centage	95·6	86·6	81·3	73·3	62·4	76·5	57·1	51·2	72·3	77·4

Central Age at Entry	C-9									Central Age at Entry
	0	1	2	3	4	5	6	7	8	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
6·9	7	9·7	12	10·1	8	9·9	12	9·6	5	9·2
15·0	14	21·0	24	22·6	19	21·7	11	21·3	26	20·5
30	12·7	15	18·2	10	19·9	18	20·4	15	19·5	9
35	8·9	6	12·2	11	13·3	7	13·7	6	13·9	4
20-35	43·5	42	61·1	57	66·1	52	66·4	42	65·1	49
Per-centage	96·6	93·3	78·7	78·3	63·6	75·3	62·0	49·5	78·9	76·6
40	6·0	7	8·0	6	8·6	9	8·7	4	8·8	8
45	3·8	2	4·7	1	5·1	5	5·2	4	5·2	1
50										

(β) Family History of Tuberculosis in Lineals plus Collaterals.

*Comparison of Actual Deaths with those Expected according to O'M Table.*

TABLE XVII.

(γ) Family History of Tuberculosis in Collaterals.

## Endowment Assurances.

Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.

Males.

YEARS OF ASSURANCE										Central Age at Entry		Central Age at Entry		
										Actual Deaths		Actual Deaths		
										Expected Deaths		Expected Deaths		
0	1	2	3	4	5	6	7	8	9	0-9	10-19	20-35	Per-cent	
Central Age at Entry										Deaths of Males	Deaths of Females	Deaths of Females	Percentage	
0	103·6	79·5	68·7	56·5	49·1	40·5	31·8	21·8	11·8	74·9	79·1	79·1	40	
10-35	52·1	51	45·5	39·0	33·0	27·9	20·5	14·4	7·9	74·9	79·1	79·1	40	
35	15·8	23	22·6	18	25·2	21	26·4	24	17	84·4	80·1	75·3	45	
30	16·2	12	23·7	14	26·4	21	27·5	18	17	86·3	61	60	50	
25	14·7	14	21·4	22	23·2	24	22·8	13	19	7·9	5	5·3	45	
20	5·4	5	7·8	6	8·3	4	8·3	7	8·0	7	7·5	5·7	25	
15	4·6	2	5·4	4	5·6	3	5·8	6	6·0	14	15·4	13·7	35	
10	1·0	10	11·0	10	11·4	7	15·4	8	16·2	18	17·6	13	20	
5	—	—	—	—	—	—	—	—	—	—	—	—	—	
Total	81·9	81	114·2	84	125·4	78	129·9	110	132·3	112	134·7	91	121·5	77
Per-cent	98·9	—	73·6	—	62·2	—	84·7	—	67·6	—	73·8	—	74·8	74·8
										70·7	70·2	63·4	Percentage of Actual Deaths to Expected Deaths	

In the five succeeding Tables the policy-years have been grouped in the manner already described; Tables XVIII and XIX show the results for whole-life assurances, and Tables XX and XXI for endowment assurances, respectively, on male lives, whilst Table XXII deals with endowment assurances on female lives.

TABLE XVIII.

*Family History of Tuberculosis. Sectional Tables.**Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.**First 10 years of Assurance.**Whole-Life Assurances**Males.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

(a) *Lineals.*

-37	26·6	47	176·7	37·4	46	123·0	64·0	93	145·3
38-52	27·4	37	135·0	31·2	39	125·0	58·6	76	129·7
53-62	3·7	3	81·1	4·3	3	69·8	8·0	6	75·0
Total	57·7	87	150·8	72·9	88	120·7	130·6	175	134·0

(β) *Lineals plus Collaterals.*

-37	9·5	13	136·8	15·4	18	116·9	24·9	31	124·5
38-52	21·8	37	169·7	33·1	30	90·6	54·9	67	122·0
53-62	5·0	3	60·0	6·7	9	134·3	11·7	12	102·6
Total	36·3	53	146·0	55·2	57	103·3	91·5	110	120·2

(γ) *Collaterals.*

-37	29·4	46	156·5	47·1	44	93·4	76·5	90	117·6
38-52	68·6	82	119·5	100·0	98	98·0	168·6	180	106·8
53-62	13·2	13	98·5	15·8	12	75·9	29·0	25	86·2
Total	111·2	141	126·8	162·9	154	94·5	274·1	295	107·6

TABLE XIX.

*Family History of Tuberculosis. Sectional Tables.**Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.**Ultimate Experience, excluding first 10 years of Assurance.**Whole-life Assurances.**Males.*

Central Age attained (1)	Expected Deaths (2)	Actual Deaths (3)	Percentage (4)
<i>(a) Lineals.</i>			
40	16·2	14	102·4
45	29·7	33	
50	36·2	37	92·8
55	34·9	29	
60	30·4	37	129·8
65	18·9	27	
70	9·4	9	96·6
75	5·1	5	
<i>(β) Lineals plus Collaterals.</i>			
40	5·2	7	98·8
45	12·0	10	
50	17·7	25	98·6
55	24·9	17	
60	30·7	40	110·3
65	22·8	19	
70	11·8	17	120·3
75	4·0	2	
<i>(γ) Collaterals.</i>			
40	17·4	17	103·2
45	41·7	44	
50	71·3	64	90·0
55	101·0	91	
60	111·1	122	101·7
65	84·6	77	
70	50·5	44	88·3
75	25·4	23	

Tuberculosis in  
Lineals and  
Collaterals  
compared.

It will be seen from Table XVIII that in the whole-life experience, up to age 52, the percentages of actual deaths to those expected amongst lineals are in every instance greater than the corresponding percentages amongst collaterals. In the "ultimate" experience given in

Table XIX, the differences, though still in the same direction, are not quite so marked.

Taken by itself this would seem to indicate that a history of tuberculosis in a parent should be viewed more seriously than a similar history in brothers or sisters. But upon turning to the corresponding figures for endowment assurances in Tables XX and XXI the results are reversed, with the exception of attained ages below 48 in the Ultimate Table.

TABLE XX.

*Family History of Tuberculosis. Sectional Tables.*

*Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.*  
*First 10 years of Assurance.*

*Endowment Assurances.**Males.*

Ages at Entry	YEARS OF ASSURANCE									
	0-4			5-9			0-9			Percentage
	Expected Deaths	Actual Deaths	Per-cent	Expected Deaths	Actual Deaths	Per-cent	Expected Deaths	Actual Deaths	Per-	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	

(a) *Lineals.*

-37 38-52	303·1 71·1	245 49	80·8 68·9	297·8 75·9	203 46	68·2 60·6	600·9 147·0	448 95	74·6 64·6
Total	374·2	294	78·6	373·7	249	66·6	747·9	543	72·6

(β) *Lineals plus Collaterals.*

-37 38-52	99·2 57·5	78 26	78·6 45·2	121·8 75·7	62 40	50·9 52·8	221·0 133·2	140 66	63·3 49·5
Total	156·7	104	66·4	197·5	102	51·6	354·2	206	58·2

(γ) *Collaterals.*

-37 38-52	382·1 201·6	323 142	84·5 70·4	412·6 241·8	306 155	74·2 64·1	794·7 443·4	629 297	79·1 67·0
Total	583·7	465	79·7	654·4	461	70·4	1238·1	926	74·8

TABLE XXI.

*Family History of Tuberculosis. Sectional Tables.**Comparison of Actual Deaths with those Expected according to O<sup>[M]</sup> Table.**Ultimate Experience, excluding first 10 years of Assurance.**Endowment Assurances. Males.*

Central Age attained (1)	Expected Deaths (2)	Actual Deaths (3)	Percentage (4)
(a) <i>Lineals.</i>			
30	11·1}	8}	72·0
35	50·0}	36}	
40	71·7}	38}	61·9
45	65·7}	47}	
50	49·1}	39}	70·2
55	29·2}	16}	
60	13·0}	9}	59·2
65	2·2}	...	
(β) <i>Lineals plus Collaterals.</i>			
30	2·4}	2}	63·2
35	16·6}	10}	
40	31·0}	16}	72·7
45	40·5}	36}	
50	43·1}	26}	60·6
55	32·8}	20}	
60	19·3}	20}	107·0
65	2·2}	3}	
(γ) <i>Collaterals.</i>			
30	10·4}	5}	62·6
35	53·5}	35}	
40	100·8}	65}	57·9
45	135·7}	72}	
50	137·7}	83}	70·6
55	110·2}	92}	
60	57·9}	52}	88·9
65	9·6}	8}	

TABLE XXII.

*Family History of Tuberculosis. Sectional Tables.**Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.**First 10 years of Assurance.**Endowment Assurances.**Females.*

Ages at Entry (1)	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths (2)	Actual Deaths (3)	Percentage (4)	Expected Deaths (5)	Actual Deaths (6)	Percentage (7)	Expected Deaths (8)	Actual Deaths (9)	Percentage (10)
-37 38-52	76·6 26·0	53 12	69·2 46·2	78·4 31·5	42 13	53·6 41·3	155·0 57·5	95 25	61·3 43·5
Total	102·6	65	63·4	109·9	55	50·0	212·5	120	56·5

(a) *Lineals.*

-37 38-52	23·6 19·2	20 8	84·7 41·7	29·2 26·4	18 16	61·6 60·6	52·8 45·6	38 24	72·0 52·6
Total	42·8	28	65·4	55·6	34	61·2	98·4	62	63·0

(β) *Lineals plus Collaterals.*

-37 38-52	23·6 19·2	20 8	84·7 41·7	29·2 26·4	18 16	61·6 60·6	52·8 45·6	38 24	72·0 52·6
Total	42·8	28	65·4	55·6	34	61·2	98·4	62	63·0

(γ) *Collaterals.*

-37 38-52	85·0 60·9	68 36	80·0 59·1	95·2 83·5	58 41	60·9 49·1	180·2 144·4	126 77	69·9 53·3
Total	145·9	104	71·3	178·7	99	55·4	324·6	203	62·5

It should also be noted that the figures in Table XXII, deduced from endowment assurances on female lives, support in every particular the results shown in Table XX as between lineal and collateral tuberculous family history for male lives under endowment assurances. The contrary results shown as between whole-life and endowment assurances on male lives prevent any dogmatic assertion on the basis of the figures alone as to the relative weight to be attached to tuberculous history in lineals and collaterals respectively. In view, however, of the very much larger numbers exposed to risk in the endowment assurance as compared with the whole-life experience and the support accorded to the former by the experience on female lives under endowment assurances, we have no hesitation in expressing it as our opinion that at least as much attention—if not more—must be given to a history of tuberculosis amongst brothers and sisters as amongst parents.

*Lineals plus Collaterals.* In view of the fact that for a life to be included in Section  $\beta$ , two at least of the assured's family must have died from tuberculosis, a heavier rate of mortality might reasonably have been expected than under either of the other two sections.

It will be seen, however, that in the whole-life experience on male lives the percentages shown in Table XVIII for this section lie, on the whole, practically midway between the corresponding figures in the other two sections. When we come to endowment assurances on male lives (Table XX), not only do we find light rates of mortality, but we have the unexpected result that for each group of ages at entry and for all durations the rates are considerably below those in either of the other sections.

It is true that the percentages are derived from a much smaller number of exposures than in either of the other two sections, and this may possibly account for the apparent anomaly; but, no doubt, other causes were also in operation. It is probable, for instance, that in the medical selection of these lives special care was taken to ensure that in all respects, other than the defects in the family history—such as personal condition, occupation, &c.—the applicants were unimpeachably first-class lives. Or, it may be, that the assured, knowing the family weakness, have taken great personal care not to incur risks likely to injure their health. It is just possible, also, that there may be some truth in the modified form of the "immunisation" theory put forward by some medical writers, namely, that a family

taint of tuberculosis may have worked itself out in other members of the family, and have left the surviving members actually less liable to the ravages of this disease than the average man. In regard to this, moreover, it should be borne in mind that the family history dealt with in this experience is the history as it stands at the date of application for assurance—a period at which, in a large number of cases, it is not by any means complete. In this particular section it is more complete than in the others.

Another likely cause may have been the terms upon which lives were accepted under this section. They were only accepted subject to a surcharge in the nature of a contingent debt. This debt formed a heavy initial deduction from the sum assured rapidly diminishing with the duration of the policy. In the circumstances, it is arguable that only lives who considered themselves to have a reasonable expectation of surviving the contingent debt period would take up the policies.

The figures given in Table XXII for female lives under endowment assurances do not support the results shown in Tables XX and XXI in regard to the light rate of Section  $\beta$ . With a single exception the percentages of actual to expected deaths are higher under Section  $\beta$  than under either Section  $\alpha$  or  $\gamma$ ; there is, however, practically no difference between the percentages under Sections  $\beta$  and  $\gamma$  and this would tend to show that as with male so also with female risks, tuberculosis in collaterals should be given at least equal weight to that in lineals.

Analysis in Sub-groups. We now pass to a brief consideration of the individual sub-groups. The only section of the data that permitted of analysis in such detail was that relating to endowment assurances on male lives, and Sub-group A<sub>0</sub> was omitted owing to paucity of data. Even then it would have been useless to set out the results for each year of assurance. The results for grouped periods are set out in Tables XXIII, XXIV, and XXV in a similar manner to those given for the Combined and Sectional Tables.

TABLE XXIII.

*Family History of Tuberculosis. Sub-groups A<sub>1</sub> to A<sub>4</sub>.**Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table  
First 10 years of Assurance.**Endowment Assurances. (a) Lineals. Males*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<i>A<sub>1</sub>—Father only.</i>									
-37	143·9	124	86·2	137·9	101	73·2	281·8	225	79·8
38-52	31·4	25	79·6	32·0	24	75·0	63·4	49	77·3
Total	175·3	149	85·0	169·9	125	73·6	345·2	274	79·4
<i>A<sub>2</sub>—Mother only (consumption).</i>									
-37	98·6	87	88·2	96·2	63	65·5	194·8	150	77·0
38-52	22·9	16	69·9	23·8	11	46·2	46·7	27	57·8
Total	121·5	103	84·8	120·0	74	61·7	241·5	177	73·3
<i>A<sub>3</sub>—Mother only (childbirth).</i>									
-37	33·2	25	75·3	30·9	24	77·7	64·1	49	76·4
38-52	8·7	8	92·0	9·9	6	60·6	18·6	14	75·3
Total	41·9	33	78·8	40·8	30	73·5	82·7	63	76·2
<i>A<sub>4</sub>—Father and Mother (consumption).</i>									
-37	27·0	9	33·3	32·3	15	46·4	59·3	24	40·5
38-52	7·7	2	26·0	10·3	3	29·1	18·0	5	27·8
Total	34·7	11	31·7	42·6	18	42·3	77·3	29	37·5

TABLE XXIV.

*Family History of Tuberculosis. Sub-groups A<sub>5</sub> to A<sub>9</sub>.**Comparison of Actual Deaths with those Expected according to O<sup>[M]</sup> Table.**First 10 years of Assurance.**Endowment Assurances.**Males.*(β) *Lineals plus Collaterals.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Per- centage	Expected Deaths	Actual Deaths	Per- centage	Expected Deaths	Actual Deaths	Per- centage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
-37 38-52	38·1 21·3	29 15	76·1 70·4	46·5 26·6	31 19	66·7 71·4	84·6 47·9	60 34	70·9 71·0
Total	59·4	44	74·1	73·1	50	68·4	132·5	94	70·9

*A<sub>5</sub>—Father plus Brother or Sister (consumption).*

-37 38-52	37·3 20·2	29 8	77·7 39·6	46·2 27·9	18 7	39·0 25·1	83·5 48·1	47 15	56·3 31·2
Total	57·5	37	64·3	74·1	25	33·7	131·6	62	47·1

*A<sub>6</sub>—Mother plus Brother or Sister (consumption).*

-37 38-52	37·3 20·2	29 8	77·7 39·6	46·2 27·9	18 7	39·0 25·1	83·5 48·1	47 15	56·3 31·2
Total	57·5	37	64·3	74·1	25	33·7	131·6	62	47·1

*A<sub>7</sub>—Mother (childbirth) plus Sister (childbirth).*

Total	...	...	...	...	...	...	9·0	6	66·7
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*A<sub>8</sub>—Mother (childbirth) plus Brother or Sister (consumption).*

-37 38-52	15·7 9·6	15 2	95·5 20·8	19·1 12·7	9 5	47·1 39·4	34·8 22·3	24 7	69·0 31·4
Total	25·3	17	67·2	31·8	14	44·0	57·1	31	54·3

*A<sub>9</sub>—Father and Mother plus Brother or Sister (consumption).*

-37 38-52	5·0 4·0	3 ...	60·0 ...	7·4 6·2	3 5	40·5 80·6	12·4 10·2	6 5	48·4 49·0
Total	9·0	3	33·3	13·6	8	58·8	22·6	11	48·7

TABLE XXV.

*Family History of Tuberculosis. Sub-groups A<sub>10</sub> to A<sub>12</sub>.**Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.**First 10 years of Assurance.**Endowment Assurances.**(γ) Collaterals.**Males.*

Ages at Entry (1)	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths (2)	Actual Deaths (3)	Per-centage (4)	Expected Deaths (5)	Actual Deaths (6)	Per-centage (7)	Expected Deaths (8)	Actual Deaths (9)	Per-centage (10)
-37 38-52	7·5 4·4	16 8	213·3 181·8	7·2 5·8	13 6	180·6 103·4	14·7 10·2	29 14	197·3 137·3
Total	11·9	24	201·7	13·0	19	146·2	24·9	43	172·7

A<sub>10</sub>—*Sister (childbirth).*

-37 38-52	7·5 4·4	16 8	213·3 181·8	7·2 5·8	13 6	180·6 103·4	14·7 10·2	29 14	197·3 137·3
Total	11·9	24	201·7	13·0	19	146·2	24·9	43	172·7

A<sub>11</sub>—*One Brother or Sister (consumption).*

-37 38-52	282·5 116·4	247 89	87·4 76·5	288·4 131·2	211 97	73·2 73·9	570·9 247·6	458 186	80·2 75·1
Total	398·9	336	84·2	419·6	308	73·4	818·5	644	78·7

A<sub>12</sub>—*Two or more Brothers or Sisters (consumption).*

-37 38-52	92·5 80·6	60 45	64·9 55·8	116·9 105·3	82 52	70·1 49·4	209·4 185·9	142 97	67·8 52·2
Total	173·1	105	60·7	222·2	134	60·3	395·3	239	60·5

To enable these sub-groups to be the more easily compared, Table XXVI is added, giving the percentages only. Sub-group A<sub>7</sub> is omitted as the facts are too small for inclusion.

TABLE XXVI.

*Family History of Tuberculosis. Sub-groups A<sub>1</sub> to A<sub>12</sub>.*

*Comparison for each Sub-group of the Percentages of Actual Deaths to those Expected by O<sup>(M)</sup> Table.*

*First 10 years of Assurance.*

*Endowment Assurances.*

*Males.*

Sub-Groups	AGES AT ENTRY UNDER 37		AGES AT ENTRY 38-52	
	Years of Assurance		Years of Assurance	
	0-4 (1)	5-9 (2)	0-4 (4)	5-9 (5)
A <sub>1</sub>	86·2	73·2	79·6	75·0
A <sub>2</sub>	88·2	65·5	69·9	46·2
A <sub>3</sub>	75·3	77·7	92·0	60·6
A <sub>4</sub>	33·3	46·4	26·0	29·1
A <sub>5</sub>	76·1	66·7	70·4	71·4
A <sub>6</sub>	77·7	39·0	39·6	25·1
A <sub>8</sub>	95·5	47·1	20·8	39·4
A <sub>9</sub>	60·0	40·5	...	80·6
A <sub>10</sub>	213·3	180·6	181·8	103·4
A <sub>11</sub>	87·4	73·2	76·5	73·9
A <sub>12</sub>	64·9	70·1	55·8	49·4

Low Mortality  
where Family  
History of two  
Tuberculous  
Deaths.

The first fact that strikes the attention is that in the sub-groups A<sub>4</sub>, A<sub>9</sub>, and A<sub>12</sub>, in all of which there is a family history of at least two deaths from tuberculosis, the mortality is exceptionally light. The data in A<sub>1</sub> and A<sub>9</sub> are small, but, on the other hand,

the percentages in all three sub-groups are fairly consistent with one another.

The most striking difference is shown in the comparison of sub-groups A<sub>11</sub> and A<sub>12</sub>, in both of which the data are substantial. It will be seen that the mortality is throughout and in a marked degree in favour of Sub-group A<sub>12</sub>. That is to say,

the mortality shown is considerably better when two or more brothers or sisters have died from tuberculosis than when only one such relative has died from that disease. Probably causes similar to those suggested in connection with the mortality of Section ( $\beta$ ) (Lineals plus Collaterals) are also in operation here.

Another striking feature is the heavy rate experienced in sub-group  $A_{10}$ , where the family history is that a sister died in childbirth. This heavy rate, exceptionally high at the younger ages at entry, is observable throughout the Table. It will be noticed that in sub-group  $A_3$ , where the mother died in childbirth, the mortality is also comparatively heavy. The smallness of the data warns us against drawing too hasty conclusions; perhaps this may be a case of the inheritance of a poor constitution with consequent inability to resist infection or to recover from serious illness. However this may be, it seems clear that the death of either the mother or sister in childbirth must be given special attention in estimating assurance risks.

Comparing  $A_1$  with  $A_2$  it will be noticed that the rates are higher in the former than in the latter. This, as far as it goes, would tend to show that tuberculosis in a father is of more importance than in a mother. To some extent this tendency seems to be supported by the figures in  $A_5$ , when compared with those in  $A_6$  and  $A_8$ , in which sub-groups the mother died from tuberculosis or in childbirth.

Sub-group  $A_{11}$ —brother or sister died from tuberculosis—is entitled to considerable weight in view of the fact that it records the experience of a large body of data. Comparing the percentages with those for Sub-groups  $A_1$  and  $A_2$  it will be seen that there is very little difference in the first five years of assurance, but in years of assurance 5 to 9 the mortality is substantially higher than it is in sub-group  $A_2$ . Thus where there is only one tuberculous death in the family it does not appear that there is any material difference whether that death is of a lineal or of a collateral relation.

Throughout the whole of the comparative Tables on male lives it will be noticed that the excess mortality is greatest in the youngest group of ages at entry, and that it is practically confined to the first 10 years of assurance. This feature is unmistakeable, and it is also shown in a marked degree in the Tables for female lives.

*High Mortality  
where Family  
History of death  
in Childbirth.*

*Definite  
Tuberculous  
Family History—  
General  
Summary.*

When each experience is compared with the normal mortality for the corresponding class of assurance it will be seen that there is no trace of excess mortality in the endowment assurances above age 37 at entry, and in the whole-life assurances after about age 50 at entry.

Furthermore, the *degree* of tuberculous family history is of much less importance than the age at entry, and may, in fact, be said to have no practical value for assurance purposes over age 35 at entry.

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#### FAMILY HISTORY OF NON-TUBERCULOUS PULMONARY DISEASE AND OF PRESUMPTIVE TUBERCULOUS TAINT.

Hitherto no account has been taken of the sub-groups numbered A<sub>13</sub>, A<sub>14</sub>, and A<sub>15</sub>. The figures for A<sub>13</sub>, and for the combined groups A<sub>14</sub> and A<sub>15</sub>, together with a further group designated A+E, are set out in Tables XXVII, XXVIII, and XXIX. This latter group was composed of such cases as combined a tuberculous family history with some personal defect in the life; these defects were, mainly, indifferent health or poor physique, and include weights markedly below the standard. Groups A<sub>14</sub> and A<sub>15</sub> have been combined, as the family history in both is of an indefinite character.

It will be seen that the mortality throughout these sub-groups is heavy, particularly so in the male whole-life group which combines a tuberculous family history with some personal defect. In none of these groups were the data sufficient to warrant tabulation in "ultimate" form.

TABLE XXVII.

*Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.  
First 10 years of Assurance.*

*Whole-Life Assurances.**Males.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9 *			0-9		
	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
<i>A<sub>13</sub>—Family History of Non-Tuberculous Pulmonary Disease.</i>									
-37	7·6	7	92·1	8·2	10	122·0	15·8	17	107·6
38-52	13·1	23	175·6	15·0	11	73·3	28·1	34	121·0
53-62	1·4	1	71·4	1·4	2	142·9	2·8	3	107·1
Total	22·1	31	140·3	24·6	23	93·5	46·7	54	115·6
<i>A<sub>14</sub> and A<sub>15</sub>—Unknown or Obscure Family History.</i>									
-37	16·4	26	158·5	20·1	27	134·3	36·5	53	145·2
38-52	36·0	68	188·9	44·2	43	97·3	80·2	111	138·4
53-62	7·6	6	78·9	8·2	13	158·5	15·8	19	120·3
Total	60·0	100	166·7	72·5	83	114·5	132·5	183	138·1
<i>A + E—Tuberculous Family History plus Personal Defect</i>									
-37	7·2	15	208·3	11·0	18	163·6	18·2	33	181·3
38-52	11·6	12	103·4	17·8	34	191·0	29·4	46	156·5
53-62	3·7	10	270·3	4·8	12	250·0	8·5	22	258·8
Total	22·5	37	164·4	33·6	64	190·5	56·1	101	180·0

TABLE XXVIII.

*Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.**First 10 years of Assurance.**Endowment Assurances.**Males.*

Ages at Entry (1)	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths (2)	Actual Deaths (3)	Percentage (4)	Expected Deaths (5)	Actual Deaths (6)	Percentage (7)	Expected Deaths (8)	Actual Deaths (9)	Percentage (10)
-37 38-52	31.5 10.7	35 16	111.1 149.5	26.2 10.8	20 13	76.3 120.4	57.7 21.5	55 29	95.3 134.9
Total	42.2	51	120.9	37.0	33	89.2	79.2	84	106.1

A<sub>13</sub>—*Family History of Non-Tuberculous Pulmonary Disease.*

-37 38-52	31.5 10.7	35 16	111.1 149.5	26.2 10.8	20 13	76.3 120.4	57.7 21.5	55 29	95.3 134.9
Total	42.2	51	120.9	37.0	33	89.2	79.2	84	106.1

A<sub>14</sub> and A<sub>15</sub>—*Unknown or Obscure Family History.*

-37 38-52	98.4 50.3	115 57	116.9 113.3	96.9 56.3	109 57	112.5 101.2	195.3 106.6	224 114	114.7 106.9
Total	148.7	172	115.7	153.2	166	108.4	301.9	338	112.0

A+E—*Tuberculous Family History plus Personal Defect.*

-37 38-52	56.8 26.9	52 29	91.5 107.8	74.0 36.0	68 37	91.9 102.8	130.8 62.9	120 66	91.7 104.9
Total	83.7	81	96.8	110.0	105	95.5	193.7	186	96.0

TABLE XXIX.

*Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.**First 10 years of Assurance.**Endowment Assurances.**Females.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)

A<sub>13</sub>—*Family History of Non-Tuberculous Pulmonary Disease.*

-37	7·5	12	160·0	6·5	5	76·9	14·0	17	121·4
38-52	3·5	6	171·4	3·4	5	147·1	6·9	11	159·4
Total	11·0	18	163·6	9·9	10	101·0	20·9	28	134·0

A<sub>14</sub> and A<sub>15</sub>—*Unknown or Obscure Family History.*

-37	22·6	13	57·5	21·1	20	94·8	43·7	33	75·5
38-52	13·0	9	69·2	15·0	18	120·0	28·0	27	96·4
Total	35·6	22	61·8	36·1	38	105·3	71·7	60	83·7

A+E—*Tuberculous Family History plus Personal Defect*

-37	12·8	14	109·4	15·0	17	113·3	27·8	31	111·5
38-52	9·2	7	76·1	12·9	10	77·5	22·1	17	76·9
Total	22·0	21	95·5	27·9	27	96·8	49·9	48	95·2

*Family History of Non-Tuberculous Pulmonary Disease.*

With regard to group A<sub>13</sub>, which relates to family history associated with some pulmonary disease other than tuberculosis, and includes bronchitis, it may be said, on the whole, that the mortality both for male and female lives is worse than in the combined group A<sub>0</sub> to A<sub>12</sub>, particularly at the older ages at entry. It is, therefore, apparent that at least as much weight must be given to the death of a relative from non-tuberculous pulmonary disease as from tuberculosis. It is possible that this may point rather to a family peculiarity, displaying itself in a general weakness in the lungs than to an actual hereditary transmission of the disease. As to the possibility of such family peculiarities, Sir Clifford Allbutt gives an instance in *The Practitioner* for January, 1913. "Such 'a peculiarity,' he says, 'may be found in the particular 'bronchiole of Hirschfeld, which in certain families is apt to 'sag, and thus to make a dip in which the bacillus may defy 'the cleansings of Nature. Such a tube may be more defective 'on one side of the chest than on the other; and Turban showed, 'accordingly, that a tendency to pulmonary tubercle is not 'only an inherited proclivity, but in this family or that pre-'dominates on one side or the other.'"

*Obscure and Unknown Family History.* It is generally considered that where the family history is obscure or unknown, there is a strong presumption

that it is of a tuberculous nature. For this reason, such cases have been included in the general Group A, being classed as Sub-groups A<sub>14</sub> and A<sub>15</sub>. It will be noticed here, that for male lives the mortality is higher, on the whole, than in the case of the non-tuberculous pulmonary lives. Giving weight to the amount of data for male lives under endowment assurances, it would seem that the mortality is at least as unfavourable as that of lives combining tuberculous history with personal blemish.

In view of these facts it would appear that there are strong grounds for the presumption referred to, and more caution would seem to be needed in accepting such risks than where there is a known family history of pulmonary disease, whether tuberculous or not.

*Tuberculous Family History coupled with Personal Blemish.* Finally, with regard to tuberculous family history associated with some personal blemish, there seems no doubt that the figures given may be relied upon as confirming the opinion, generally held, that from such a combination high rates of mortality are to be looked for.

Weight in  
Relation to  
Tuberculous  
History.

No doubt some valuable lessons might have been obtained by an investigation into Group A with regard to weight, but unfortunately, the data were not in a sufficiently advanced form to be included in this Paper.

#### PERSONAL BLEMISH OF A TUBERCULOUS OR PULMONARY NATURE.

In dealing with Group E (page 505) several of the Sub-groups have been combined into sections. Of these sections those numbered II and IV refer to personal history of diseases of a tuberculous or pulmonary nature, and Tables relating to those sections are inserted here (Tables XXX and XXXI). It will be observed that only the endowment assurance experience is given, as the whole-life data were too small to admit of any deductions being drawn.

TABLE XXX.

#### GROUP E.—SECTION II.

##### *Personal Blemishes of a Tuberculous Nature.*

*Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.*

##### (A) First 10 years of Assurance.

*Endowment Assurances.*

*Males.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Per-cent	Expected Deaths	Actual Deaths	Per-cent	Expected Deaths	Actual Deaths	Per-cent
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
-37	21·0	26	123·8	19·4	23	118·6	40·4	49	121·3
38-52	6·4	14	218·8	5·8	9	155·2	12·2	23	188·5
Total	27·4	40	146·0	25·2	32	127·0	52·6	72	136·9

##### (B) Ultimate Experience, excluding first 10 years of Assurance.

Central Age Attained	Expected Deaths		Actual Deaths	Percentage
	(1)	(2)		
40	5·3	10·1	8	
45	4·8	7	15	
50	3·6		1	
55	3·0		2	
60	1·8		5	
65	.4		2	

148·5

113·6

TABLE XXXI.

## GROUP E.—SECTION IV.

*Personal Blemishes of a Tuberculous or Pulmonary Nature.**Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.*(A) *First 10 years of Assurance.**Endowment Assurances.**Males.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
-37	17·9	30	167·6	21·7	30	138·2	39·6	60	151·5
38-52	9·6	22	229·2	13·0	25	192·3	22·6	47	208·0
Total	27·5	52	189·1	34·7	55	158·5	62·2	107	172·0

(B) *Ultimate Experience, excluding first 10 years of Assurance.*

Central Age Attained	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)
40	8·0	17·7	197·7
45	9·7	17	17
50	8·1	15·5	8
55	7·4	14	6
60	5·5	7	4
65	1·5	5	1

High Mortality  
where Personal  
History of  
Tuberculosis or  
Pulmonary  
Disease.

The exposures are very few, but, as might have been expected, the Tables confirm the commonly expressed opinion that, where there is a personal history of tuberculosis or pulmonary disease, the rates of

mortality are much higher than where there is a family history of these diseases. This is the case even when the latter is associated with indifferent health or poor physique. In both these Tables it will be observed that the mortality is particularly high in the first five years of assurance for entry ages 38 to 52, and also in the Ultimate Table for central ages attained 40 and 45. These facts indicate that the excess mortality in these sections attains its maximum intensity between ages 40 and 50, after which there seems to be a marked improvement.

*Group A.  
Causes of Death.*

A common method of enquiry into problems connected with tuberculosis has been by analysing the causes of death amongst persons having a tuberculous family history. This method, of course, is not entirely satisfactory from an actuarial point of view; but, as it was possible that such an investigation might throw further light on some of these problems, the causes of death were examined and tabulated for endowment assurances on male lives in Sub-groups A<sub>1</sub> to A<sub>12</sub>. The few cases in Sub-group A<sub>9</sub> were omitted, and all duplicates were excluded. As the object was to ascertain whether a tuberculous family history was associated with a preponderance of deaths from the same disease, the deaths from tuberculosis were compared with those arising from all causes.

The resulting totals, obtained in sections, are shown in Table XXXII, together with the figures obtained from a similar investigation of those deaths in 1910 amongst endowment assurances on male lives in which there was no tuberculous taint at the date of proposal.

TABLE XXXII.

*Comparison of Deaths from Tuberculosis with Deaths from All Causes, in Groups of Ages at Death.*

*Endowment Assurances.*

*Males.*

Data	AGES AT DEATH									
	20-24		25-34		35-44		45-54		55-64	
	Deaths from Tuberculosis	Deaths from All Causes								
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Tuberculous Families :										
(a) Lineals ... ...	22	42	125	280	84	244	27	118	2	21
(β) Lineals plus Collaterals ...	4	8	44	72	36	104	29	103	1	34
(γ) Collaterals ... ...	22	34	146	279	168	500	66	352	18	127
Tuberculous Families ...	48	84	315	631	288	848	122	573	21	182
Non-Tuberculous Families (Deaths in 1910)	31	95	113	363	143	630	82	667	24	368

For purposes of comparison these totals require to be brought to a common denominator. Accordingly Table XXXIII sets out the comparison in the form of deaths from tuberculosis per 1,000 deaths from all causes. Included in this Table are the corresponding rates deduced from the Returns of the Registrar-General for England and Wales for the first, last, and central calendar years of the investigation.

TABLE XXXIII.

*Comparison of Deaths from Tuberculosis with 1,000 Deaths from All Causes, in groups of ages at Death.*

*Endowment Assurances.*

*Males.*

Data (1)	AGES AT DEATH				
	20-24 (2)	25-34 (3)	35-44 (4)	45-54 (5)	55-64 (6)
Tuberculous Families : (a) Lineals ... ... (β) Lineals plus Collaterals (γ) Collaterals ... ...	523	446	344	229	95
Tuberculous Families ...	571	499	340	213	115
Non-Tuberculous Families ... (Deaths in 1910)	326	311	227	123	65
Returns of Registrar-General of England and Wales : Deaths in 1893 ... ... ,, 1901 ... ... ,, 1910 ... ...	407	363	269	161	80
	399	375	291	183	81
	428	389	274	167	77

Heavy  
Proportion of  
Deaths from  
Tuberculosis.

It will be seen at once that the mortality from tuberculosis amongst those having a family history of that disease is very much greater than amongst a similar body of assured lives having no such tainted history. It is even considerably greater than that shown

amongst the general population. There can be no doubt, in fact, that a family history of tuberculosis predisposes to death from that cause. As between the three sections—lineals, lineals plus collaterals, and collaterals—there does not seem to be any very decided difference after age 35. Below that age, there is apparent a tendency to a greater proportion of tuberculous deaths where the relative affected was a collateral. It is possible that the higher ratio of deaths below age 35 is due to the closer intercourse with the affected relative.

A very noticeable and somewhat disturbing feature of Table XXXIII, is the fact that the proportion of deaths due to tuberculosis shown in the Registrar-General's Returns, was practically the same in 1901 as in 1893, and in 1910 had even increased at ages below 35.

An investigation of a similar character was made by Dr. E. J. Marsh of the causes of death amongst persons assured in the Mutual Life Insurance Company of New York. It refers to the fifteen years 1879 to 1893, and apparently to lives of both sexes. The results are here reproduced in Tables XXXIV and XXXV, in a form slightly modified from the original. It will be seen that the age-groups lie midway between those given in this Paper, and that the resulting ratios also lie practically midway between those given in Table XXXIII. Such close correspondence confirms in a remarkable way the conclusions already drawn on this point.

TABLE XXXIV.

*Experience of Mutual of New York (1879–1893).*

*Deaths from Consumption and from All Causes.*

Data	AGES AT DEATH										
	20–29		30–39		40–49		50–59		60–69		
	Con-sump-tion	All Causes									
Consumptive Families	...	31	60	77	194	70	284	77	489	47	569
Non-consumptive Families	...	38	106	98	372	100	566	49	724	36	613

Reduced to the form of Deaths from Consumption to 1,000 Deaths from All Causes the Table is as follows :

TABLE XXXV.

*Experience of Mutual of New York (1879-1893).  
Deaths from Consumption per 1,000 Deaths from all causes.*

Data	AGES AT DEATH				
	20-29	30-39	40-49	50-59	60-69
Consumptive Families ...	516	396	246	157	82
Non-consumptive Families ...	358	263	176	67	58

*Deaths from Tuberculosis according to Duration of Assurance.* The deaths in Group A were also examined according to the duration of assurance, and the resulting ratios per 1,000 deaths from all causes, are given in Table XXXVI.

TABLE XXXVI.

*Family History of Tuberculosis. Combined Experience A<sub>0</sub> to A<sub>12</sub>. Deaths from Tuberculosis compared with 1,000 deaths from All Causes, in years of Assurance.*

Central Ages at Entry	YEARS OF ASSURANCE.					Males.
	0	1	2-4	5-9	0-9	
20-25	467	583	552	522	535	
30-35	281	412	390	351	363	
40-50	225	250	213	216	218	
All Ages	322	449	396	360	377	

It will be seen that the ratio of deaths from tuberculosis to those from all causes reaches a maximum in year of assurance "1," and then diminishes.

The Table may be said to show that the comparatively high rate of mortality exhibited for the younger ages at entry in the early years of assurance is due to an excess of deaths from tuberculosis.

*Correlation of  
Ages at Death.*

An investigation was made to ascertain whether there was any correlation between the ages at death of those who died from tuberculosis and the age at death of the tuberculous relative.

The male lives assured under endowment assurances were selected for this enquiry and the groups investigated were those in which only one relative had died from tuberculosis. The results are shown in Tables XXXVII and XXXVIII, and are interesting chiefly from their negative character. In neither of the Tables is there any indication of correlation between the ages at death of the relative and of the assured.

It would have been interesting, had it been possible, also to have investigated the effect of the length of the period elapsed between the death of the relative and the date of entry into assurance, but the proposals did not give the necessary information.

TABLE XXXVII.  
*Deaths from Tuberculosis.* Sub-groups A<sub>1</sub> and A<sub>2</sub>.  
*Age at Death of Assured, where Death resulted from Tuberculosis, compared with Age at Death of Father or Mother from Tuberculosis.*

*Endowment Assurances.*

Central Age at Death of Assured	CENTRAL AGE AT DEATH OF FATHER OR MOTHER						Total	Mean Age at Death of Father or Mother
	20	25	30	35	40	45		
20	...	...	1	1	2	3	...	42.1
25	...	...	3	13	12	6	...	41.0
30	1	1	5	16	7	9	1	41.2
35	1	3	5	9	11	3	2	41.0
40	...	...	4	9	5	4	2	41.8
45	...	...	4	5	5	4	...	39.5
50	...	...	1	...	1	3	1	44.2
55	...	1	...	...	4	1	6	42.5
60	...	...	...	...	...	...	1	55.0
Total	1	5	23	53	43	36	13	41.3
Mean Age at Death of Assured	35.0	38.0	35.2	32.5	32.8	37.1	33.2	34.1

TABLE XXXVIII.

*Deaths from Tuberculosis. Sub-group A<sub>11</sub>.*  
*Age at Death of Assured, where Death resulted from Tuberculosis, compared with age at Death of Brother or Sister from Tuberculosis.*

*Endowment Assurances.**Males.*

Central Age at Death of Assured	CENTRAL AGE AT DEATH OF BROTHER OR SISTER.						Total	Mean Age at Death of Brother or Sister
	10	15	20	25	30	35		
20	..	..	2	4	..	1	..	6
25	4	2	20	8	3	1	..	23.3
30	4	8	18	14	11	2	..	20.9
35	1	6	21	13	14	5	3	22.6
40	1	6	12	5	8	4	2	25.1
45	..	2	6	18	4	5	..	46
50	..	3	2	8	2	1	1	26.8
55	1	1	..	1	2	5	..	37
60	..	..	..	2	..	..	..	27.6
Total	11	28	81	73	44	24	14	1
Mean age at Death of Assured.	31.8	37.0	32.9	37.5	36.5	40.6	42.1	27.9

Historical  
Summary of  
Attitude of  
Medical  
Profession.

### TUBERCULOSIS AND THE MEDICAL PROFESSION.

Before finally leaving this subject, it may be thought not out of place to say a few words with regard to the medical point of view. Since Koch's discovery of the tubercle bacillus in 1882 and the advent of tuberculin treatment, the whole attitude of the medical profession has completely changed. Prior to that date, however, very widely differing views had been held all down the ages, from the very dawn of medical science. They are well summed up by Dr. L. K. Harrison in a Paper published in Vol. XIII of the *Journal of the Insurance Institute*: " Hippocrates, 400 b.c., believed in the curability of phthisis in all its stages, and in the benefits of change of residence. Aristotle also noted that, among the Greeks of his day, it was generally considered to be contagious. The theories as to its causation have undergone many changes. The celebrated Sylvius, writing in 1695, believed it to be contagious. Morton believed in contagion and in hereditary influences. Morgagni regarded it as extremely infectious, and refrained from doing autopsies on consumptives. . . . At the end of the eighteenth century the belief in contagion was universal, but in varying degree. . . . During the early part of the nineteenth century, the belief in contagion was lessened by Laennec, who drew particular attention to the tubercular diathesis, predisposition, and heredity. . . . Virchow believed in an inherited and acquired pre-disposition, which required no mysterious contagion to excite the disease. Writing in 1893, Dr. F. Taylor says, 'Nothing seems better established than the truth of the hereditary transmission of phthisis.' . . . It was Koch's discovery of the bacillus that formed the death-blow to the theory of heredity."

*Differences of Opinion at the present time.* At the present time there may be said to be two schools or camps of opinion. Both are agreed that tuberculosis is of an infectious nature. They differ mainly on the subject of heredity. Into the merits of these conflicting opinions we do not propose to enter. We propose merely to set out, briefly, their differing points of view, and the consequent effect on the treatment of proposals for life assurance.

*Medical Opposition to "Heredity" Theory.* The case of those who are opposed entirely to any theory of heredity, and believe solely in infection, is summed up in Dr. Harrison's Paper,

to which we have already referred : "Upon enquiry into "the facts upon which the theory of heredity in phthisis "was based, I found that the one argument was the "repeated occurrence of the disease in several members of one "family ; and it at once suggested itself to me, that as phthisis "used to be so common, it would follow, as a matter of course, "that a history of consumption 'in the family' would almost "of necessity be frequently found." "To prove hereditary "transmission we must show that consumption is distinctly more "common in the families of the tubercular than the normal "incidence in the population generally, and this fact has not "yet been demonstrated. And, even if we did find that the "children of consumptives were more frequently affected than "those of non-consumptives, this would not prove hereditary "transmission, as the greater frequency would be due to greater "opportunities of infection. If hereditary influence were a "real and potent cause, we should expect to find that after "removal of the child from his tubercular parent, and pro- "tecting him as far as possible from infection, he would still "become tubercular, but this is far from being the case." "Many so-called cases of heredity have included parents "who did not become phthisical till long after their children "had been born, and could not possibly be examples of "heredity."

He goes so far, however, as to admit that "There is no "doubt that the children of consumptive parents are often "delicate and of poor physique, and have a low resisting-power "and peculiar receptivity to all sorts of disease."

He qualifies this, however, by the further statement that "Tuberculosis does lead to an appreciable amount "of immunity in the offspring, of course not absolute, but "sufficient for us to notice an increased resistance to infection, "and an increased tendency to recover."

Effect of  
"Non-Heredity"  
Theory on  
Life Assurance. The effect of the acceptance of the theory of this school upon the attitude of Life Assurance Companies cannot be put better than in the words of Dr. Otto May, in a recent Paper, "*Some Notes on Life Assurance*" : "There is, however, ample and conclusive "evidence that the disease can be contracted by close "contact with persons suffering from phthisis, and that "such infection is facilitated by insanitary surroundings. "Yet most Assurance Companies at the present time pay no

"attention to these questions ; they seek no information as to "the nature of the contact between the healthy and the infected "members of the family ; the surcharge is the same whether "the tuberculous member lived in the same house with the "proposer, or in another continent, at and since the time the "infection declared itself. A similar anomaly shows itself in "the Companies' dealings with tuberculosis in married life. A "widower is often accepted at first-class rates three months "after the death from phthisis of his wife with whom he had "been living in the closest relationship up to the day of her "death. True, that in this case the hypothetical 'heredity' "factor is absent, but could one conceive more favourable "conditions for direct infection ? And it is quite impossible "to measure in months the latent period before such infection "may become manifest to physical examination.

"In all probability future work will reduce the 'heredity' "factor to a much lower rank, if it does not abolish it altogether, "and will focus attention on the influence of environment, "including contact with infection, nature of work, and mode of "livelihood."

On the other hand, many eminent authorities consider that heredity is an essential element to be reckoned with. It must be remembered, however,

Definition of Heredity. that the term "heredity" in this connection is usually employed in a special sense to denote, not the actual transmission of the disease itself, but the transmission of a more or less enfeebled constitution, or, rather, of a condition of the system which offers favourable soil for the propagation of the bacillus.

Thus Dr. Leslie Ogilvie in the *Transactions of the Actuarial Society of Edinburgh*, Vol. IV, says : "I do not deny that "perhaps in the great majority of instances it is not the disease "itself which is inherited, but either an inability of the cellular "structures of the body to resist the inroads of special organisms, "or a liability to degenerate in special ways. . . . The crucial "question is, 'Do we, or do we not, inherit enfeebled or robust "constitutions from our ancestors ?' I emphatically answer "'Yes' ; and I therefore think that the family history is a "consideration of great weight in medical examination for life "Assurance." In regard to this, the words of Dr. Ferdinand Hueppe may be called to mind : "Disease is the result of a "number of factors, of which external conditions form one,

"the condition of the body another, and the presence of bacteria,  
"a third."

Sir R. Douglas Powell in a Paper on "*The Prevention of Consumption*", read before the Sanitary Congress at Glasgow in 1904, remarks: "Looked at more broadly, however, the facts shape themselves rather to the conclusion that the poison is a distributed one and only in exceptional cases communicated directly from person to person. . . . Now this almost universal potential presence of the disease suggests, amongst many other lessons, an almost universally distributed virus rather than what we understand by person-to-person infection." He quotes in support of this the absence of the influence of husband upon wife, or *vice versa*, where one of them is consumptive.

Dr. Horder, writing in *The Practitioner* for <sup>The "Heredity" Theory.</sup> January 1913—which contains a mine of information

on the subject of tuberculosis—cautiously states the case for heredity: "The existence of tuberculous families is one of the cardinal facts of clinical medicine. It may be that a part of this familial incidence to tuberculosis is explained by similarities in the life and environment of the various individual members of the family, allowing of the same sources of infection, but this cannot explain the undoubtedly prevalence of tuberculosis in parents, brothers and sisters, where the family is scattered, a circumstance commonly seen. There is a tissue susceptibility, a 'favourable soil', that constitutes a potent element in the pathology of the disease." He adds, however, that the essence of this pre-disposition is at present unknown to us."

Sir Clifford Allbutt, in his masterly summing up of the discussion in *The Practitioner*, says: "The hereditary aspect of tuberculosis takes no great space: congenital tubercle is treated as little more than a curiosity; and if one or two writers are disposed to ignore the influence of heredity, others confine it to an inheritance of defective resistance. . . . I would oppose the too frequent assumption that the victims of tuberculosis are, after all, weaklings whose weeding out, if painful to their friends, is, on the whole, good for the stamina of the race. It is true that by inheritance, or by privation, a person may be so debilitated as to offer a lessened resistance to any injurious influence from without; but, on the other hand, in respect of tubercle, such a lack may mark a family

" from generation to generation, as a peculiarity rather than a  
" frailty. . . . Such a constitution is as a lock, which may be  
" shut to every key but one. Bar the tubercle, and such an  
" one may continue strong and beautiful."

Dr. Lister, writing in the same Journal on "*Pulmonary Tuberculosis in relation to Life Insurance*", says: "To-day, " I recognise that in spite of the great importance of the hygienist " and immunist in modern treatment, such matters as hereditary " and acquired vulnerability, and bad environment must " still, in our insurance practice, be granted at least as much " consideration as I ascribed to them in my lecture published in " *The Practitioner* of 1903. They are the essential factors in the " causation of infection by a bacillus which is ubiquitous, and " afford us the measure of relative immunity in any individual " coming before us for life insurance examination."

The controversy between Prof. Karl Pearson and  
Professor Pearson  
on Marital  
Infection. Dr. Newsholme on the subject is well known, and

it is not intended to discuss it here, but it led to a series of investigations by the Francis Galton Laboratory, the results of which are appearing in a series of "*Research Memoirs*." In one of the first to appear, entitled, "*A Second Study of the Statistics of Pulmonary Tuberculosis: Marital Infection, by the late E. G. Pope*", 1908, edited by Prof. Karl Pearson, the latter refers to the subject of marital infection, where obviously infection should have its most marked effect. "A belief in infection between man and wife", he says, "has been accepted by many on account of their clinical experience, but until such experience has been definitely recorded and analysed, it can in no way be considered as a scientific demonstration. It may, therefore, be said that, up to the present, no logical proof of marital infection in tuberculosis has been given." He summarizes his conclusions in the following words: "Let us assume that the whole degree of resemblance as to the tuberculous condition is due to infection. Then we are confronted with the remarkable result, that while the degree of resemblance between husband and wife is not more than ·17 or, possibly, ·25 at a maximum, that between parent and child is about ·4 to ·6, and that between brothers about ·4 to ·5. . . . To rationalize such results we are bound to consider that the inheritance of the constitution is the vital matter, and that infective action plays a subordinate roll."

**Professor Pearson  
on Tuberculosis  
and Heredity.**

In a further brochure, published in 1911, he comes to the conclusion "that clinically obvious and apparent tuberculosis appears to be inherited " at precisely the same rate as stature or cubit, or " insanity, impressed me from the outset as remarkable evidence " for the constitutional-immunity factor being of more " importance than the infection factor. . . . I failed to " find in current medical works on the subject anything in- " dicative of a logical statistical investigation of the problem."

Closely connected with the medical aspect is the **Tuberculin Test  
for Proposers.** question of the use of tuberculin as a test in examinations for life assurance. It would have been scarcely necessary to refer to this matter, but for the fact that within the past few months this test has been seriously put forward in two prominent assurance journals—one in this country and the other in Canada. In the one, the tuberculin test for proposers is suggested as the one sure and certain panacea for all problems as to tuberculosis which confront us in the acceptance of risks, and it is suggested that Offices not adopting it should be treated as too old-fashioned to be deserving of consideration. In the other, the demand—put forward by a medical man—is equally clamant: "In fact I would go further, and I think that the time " is not far distant, when each and every Insurance Company " shall insist on a tuberculin test being used in lung examinations, " even where no physical defect can be found. The test is " simple, and I do not see why it is not insisted on now. . . . " There are other tests to answer the same purpose, but none " so simple. Now if the results are beyond doubt, and the " procedure is simplicity itself, why is it not generally adopted " when such an issue means so much for all concerned? A " benefit for all and hardship for none."

Here there is not the least warning that using tuberculin for such purposes is playing with an exceedingly dangerous edged-tool. The fact is that any one of the dozen or so different kinds in use at the present time may have the effect of setting latent or sealed foci into action, thus producing active tuberculosis where it might have remained inactive for years, if not for ever. It is easier to imagine than to describe the estimation in which the public would hold a Company adopting a test, producing, even in only a few instances, such a result. As Dr. Priestley remarks, in referring generally to the use of tuberculin, the schemes are young, and caution is needed before

making *ex-cathedra* statements. Sir Clifford Allbutt leaves us no reasonable doubt on the matter : "The reader will look eagerly "for some lead as to early diagnosis by tuberculin, but not "altogether with reward. If he collects opinions from many "writers in this series, he will find rather a discordant note ; "some of the authors are emphatically in favour of its use, others "are more cautious or more timid. . . . On one point there is "a curious difference of opinion; namely, that while most "physicians seem agreed that the use of tuberculin in diagnosis "is followed by an exacerbation, or, at least, an awakening, of "the focus, Dr. Wilkinson denies this, and asserts that, on the "contrary, it has a sedative effect upon it. Those who think "there is a local awakening generally comfort us by the "assurance that this re-awakening does no harm. It may do no "harm in a focus already not inactive ; it may even modify "it in some favourable way ; but what about some other focus, "dormant and unsuspected, on the way to obsolescence ? The "general practitioner will desire some more solid assurances "than are provided in these Papers, before he will trust himself, "for diagnostic purposes, to arouse reactions which might be "injurious. There is too much at stake ; and even for the "expert, the number of 'normals' (or persons practically normal) "who react is embarrassing."

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#### PERSONAL HISTORY OF RHEUMATIC FEVER, AND LESIONS OF THE CIRCULATORY SYSTEM.

It will be seen from the Summary of Data (Table VII) that in total Group B, dealing with Personal History of Rheumatic Fever and Lesions of the Circulatory System, included 65,744 years of life, and that 1,074 deaths were recorded.

An investigation was first made of the experience of the group as a whole under whole-life and endowment assurances respectively on male lives, and under endowment assurances on female lives. This was carried out on precisely the same lines as in Group A, and the following Tables show the results for the first 10 years of assurance.

TABLE XXXIX.

*Group B. Rheumatic Fever and Circulatory Lesions.**Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.**First 10 years of Assurance.**Whole-Life Assurances.**Males.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
-37	7·2	8	111·1	14·6	30	205·5	21·8	38	174·3
38-52	16·6	37	222·9	31·8	57	179·2	48·4	94	194·2
53-62	11·1	18	162·2	26·4	31	117·4	37·5	49	130·7
Total	34·9	63	180·5	72·8	118	162·1	107·7	181	168·1

TABLE XL.

*Group B. Rheumatic Fever and Circulatory Lesions.**Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.**First 10 years of Assurance.**Endowment Assurances.**Males.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
-37	56·8	60	105·6	71·8	62	86·4	128·6	122	94·9
38-52	33·2	46	138·6	49·1	59	120·2	82·3	105	127·6
Total	90·0	106	117·8	120·9	121	100·1	210·9	227	107·6

TABLE XLI.

*Group B. Rheumatic Fever and Circulatory Lesions.**Comparison of Actual Deaths with those Expected according to O<sup>[M]</sup> Table.**First 10 years of Assurance.**Endowment Assurances.**Females.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
-37 38-52	11.3 7.8	12 5	106.2 64.1	14.8 11.7	17 10	114.9 85.5	26.1 19.5	29 15	111.1 76.9
Total	19.1	17	89.0	26.5	27	101.9	45.6	44	96.5

Except that the Tables indicate a high rate of mortality, particularly in the early years of assurance, and to a much greater extent under whole-life

**High Mortality in Group B.** than under endowment assurances, there is no very clear indication of the tendency of the results, and it was felt that further investigation was desirable. Examination of the figures in Tables XXXIX and XL with the corresponding Ultimate experiences suggested that the mortality of the early years of assurance was no better than that experienced after the expiration of 10 years. To test this, the data were recast into aggregate form showing in parallel columns the expected and actual deaths for quinquennial groups of ages attained in the full aggregate Table, and the truncated aggregate Tables excluding the first 5 and excluding the first 10 years of assurance respectively. The expected deaths were throughout calculated by means of the O<sup>[M]</sup> Ultimate Table, thus eliminating the effect of selection.

This experiment showed that, on the whole, and particularly at the younger attained ages, both for whole-life and for endowment assurances, a heavier rate of mortality was exhibited during the first 5 and first 10 years of assurance than in the Ultimate Tables excluding the first 10 years of assurance.

Personal History  
of Rheumatic  
Fever.

The data, though considerable, were not sufficient to enable a detailed examination to be made of the rates of mortality in the sub-groups indicated in the Classification Schedule, but it will be observed that the Sub-groups fall naturally into two sections, namely, (1) those which had a personal history of Rheumatic Fever which apparently had not affected the heart (Sub-group B<sub>1</sub>), and (2) the remainder of the Group, namely, those in which there was some definite, though slight, impairment of the circulatory system. As each of these sections consisted of a fair number of cases, it was considered desirable to investigate them separately to see whether there were any marked differences between them.

The results of this enquiry are set out in the following Tables, which all refer to male lives. Tables XLII and XLIII give the figures for whole-life assurances, and Tables XLIV and XLV for endowment assurances. It should be pointed out that the same standard of comparison has been adopted throughout in the calculation of the expected deaths under these Tables, namely, the O<sup>(M)</sup> Ultimate Table.

TABLE XLII.

*Sub-Group B<sub>1</sub>. Personal History of Rheumatic Fever.  
Comparison of Actual Deaths with those Expected according to OIM Ultimate Table.  
Whole-Life Assurances.  
Aggregate Tables.  
Males.*

Central Age Attained. (1)	Full Aggregate			Excluding First 5 Years of Assurance			Excluding First 10 Years of Assurance		
	Expected Deaths (2)	Actual Deaths (3)	Percentage (4)	Expected Deaths (5)	Actual Deaths (6)	Percentage (7)	Expected Deaths (8)	Actual Deaths (9)	Percentage (10)
30	2.6	8.8	...	11	125.0	'9	1.2	1.3	76.9
35	6.2	11	...	20	161.8	3.2	1.2	1.1	103.7
40	11.6	30.9	30	50	147.0	6.3	3.8	4	144.9
45	19.3	50	42	96	147.8	15.2	9.7	10	123.2
50	28.0	65.3	54	96	112.5	23.1	16.7	24	123.1
55	37.3	65.3	47	83	112.2	31.7	24.7	36	123.1
60	39.6	73.8	36	83	117.2	35.5	28.1	38	123.1
65	34.2	73.8	33	52	119.0	33.1	26.3	29	123.1
70	27.5	43.7	19	52	117.2	27.3	24.2	30	123.1
75	16.2	43.7	19	51	119.0	16.2	15.6	19	123.1

TABLE XI.III.

*Group B (excluding Sub-group B<sub>1</sub>). Personal History of Diseases of the Circulatory System.  
Comparison of Actual Deaths with those Expected according to O'M Ultimate Table.  
Whole-Life Assurances.*

*Males.  
Aggregate Tables.*

Central Age Attained (1)	FULL AGGREGATE			EXCLUDING FIRST 5 YEARS OF ASSURANCE			EXCLUDING FIRST 10 YEARS OF ASSURANCE		
	Expected Deaths (2)	Actual Deaths (3)	Percentage (4)	Expected Deaths (5)	Actual Deaths (6)	Percentage (7)	Expected Deaths (8)	Actual Deaths (9)	Percentage (10)
30	1.6	4	7	129.6	2.2	3	136.4	7	1
35	3.8	3	7	2.2	3	7	2.3	1	142.9
40	6.9	7	28	159.1	4.9	5	192.3	2.3	2
45	10.7	21	28	8.1	13.0	25	5.1	14	16
50	14.9	33	26	11.0	26.1	54	7.4	15	116.2
55	20.4	35.3	64	181.3	15.1	28	9.9	17.3	33
60	24.9	28	57	118.3	21.2	24	13.4	16	190.8
65	23.3	29	57	22.4	43.6	53	16.8	30.2	35
70	18.0	27	39	17.7	29	39	16.2	22	115.9
75	9.2	12	39	143.4	9.2	26.9	7.8	23.0	32

MINIMI

*Sub-Group B<sub>1</sub>. Personal History of Rheumatic Fever.  
Comparison of Actual Deaths with those Expected according to One Ultimate Table.*

Windom Insurance

TABLE XI

*Group B (excluding Sub-group B<sub>1</sub>). Personal History of Diseases of the Circulatory System.  
Comparison of Actual Deaths with those Expected according to Old Ultimate Table.  
Aggregate Tables.  
Endowment Assurances.*

**Rheumatic Fever and Heart Disease.**

Comparing Table XLII with Table XLIII, and Table XLIV with Table XLV, it will be seen that the mortality amongst those who had definite heart lesions is higher on the whole than amongst those who had a history of rheumatic fever without resulting heart affection. The higher mortality is particularly marked below age 60 in the Ultimate Table under whole-life assurances. For central ages 50 and 55 under whole-life assurances, and for ages 40 and 45 under endowment assurances, in the Full Aggregate Tables, the difference in favour of Sub-group B<sub>1</sub> is very marked. These are the ages at which the difference is of most consequence, and it will be observed that they relate to the groups which contain the largest number of actual deaths.

In each of these Tables the excess mortality attains a maximum between ages 45 and 60 and subsequently diminishes, though rather slowly. In the whole-life assurances it is still a prominent feature of the curves at age 75.

The Tables show that a personal history of rheumatic fever, though the heart exhibits no sign of impairment, must be viewed with considerable caution. Those cases in which the heart is actually affected, though the impairment is slight and not sufficient to warrant rejection, must be treated even more stringently, particularly in middle life.

**Causes of Death in Group B.** An investigation was made into the causes of death in Group B, amongst whole-life assurances on male lives, and the results are shown in Table XLVI.

TABLE XLVI.

*Group B. Comparison of Deaths from Rheumatic Fever and Heart Disease with Deaths from All Causes.*

*Whole-Life Assurances.*

*Males.*

Data	Causes of Death	AGES AT DEATH					
		25-34	35-44	45-54	55-64	65-74	75-
Sub-group B <sub>1</sub>	Rheumatic Fever ...	...	3	7	2	1	...
	Heart Disease	1	6	26	25	20	5
	All Causes ...	6	35	83	96	67	22
Group B excluding Sub-Group B <sub>1</sub>	Rheumatic Fever ...	...	...	3	1	...	...
	Heart Disease	2	7	20	19	12	3
	All Causes ...	4	17	57	59	48	15

In Table XLVII the figures are given in the form of ratios to 1,000 deaths from all causes, and corresponding figures, extracted from the Returns of the Registrar-General for England and Wales for 1901 and 1910, are furnished for comparison.

TABLE XLVII.

*Group B. Comparison of Deaths from Rheumatic Fever and Heart Disease with 1,000 Deaths from All Causes.*

*Males.*

Data	Causes of Death	AGES AT DEATH					
		25-34	35-44	45-54	55-64	65-74	75-
Sub-Group B <sub>1</sub> Whole Life Assurances	Rheumatic Fever ... Heart Disease	... 167	86 171	85 313	21 260	15 299	... 227
Group B excluding Sub-Group B <sub>1</sub> Whole Life Assurances	Rheumatic Fever ... Heart Disease	... 500	... 412	53 351	17 322	... 250	... 200
Returns of Registrar- General for England and Wales, 1901	Rheumatic Fever ... Heart Disease	15 74	12 99	12 130	12 176	13 180	8 112
Returns of Registrar- General for England and Wales 1910	Rheumatic Fever ... Heart Disease	10 73	8 104	7 141	8 183	9 181	6 113

It will be seen that the assured have died in much greater proportion from the causes for which they were classed as sub-normal than have persons in the general population, and that the maximum intensity is attained at an earlier age, namely, in the age group 45-54 instead of in the age group 55-64 as shown by the Registrar-General's figures.

It is necessary to bear in mind that, as in the case of tuberculosis, the medical point of view has radically changed in recent years. This may have a considerable effect in modifying the future practice of Life

Offices. The change is due mainly to the researches of Dr. James Mackenzie. His observations showed him that "the forces of the heart may be considered as being composed of two kinds—a 'rest' force, sufficient to maintain an efficient circulation when the body is at rest, and a 'reserve' force, which is called into play when the body or any part of it is called into activity." That is to say, that the condition of the heart muscle, and its action, are the two all-important considerations. Arguing from this basis, he concludes that "the presence of any sign revealed by physical examination, however abnormal it may seem, is of no serious significance so long as it is the only sign present, or so long as there is no limitation of the reserve force of the heart." In a Paper recently read before the Life Assurance Medical Officers' Association, he applies his principles to life assurance examinations in the following words: "If we search medical literature for any description of these manifestations of diseased heart muscle sufficiently precise to enable the reader to recognize and measure their value, we shall search in vain. As a consequence, it is rarely used in a practical manner. If you desired evidence for this statement, I would refer you to the reports, which you, as medical examiners, are asked to fill up. There are questions relating to the size of the heart, its rate and rhythm, to the character of the sounds, but not a single question directed to the most important class of evidence, to the evidence which gives you the really essential information on which to base a prognosis of the patient's future. Nay, more, some Insurance Companies have special heart forms, designed to be filled up when any abnormal sign is perceived. I have seen such forms, and, though there was a series of questions to be answered, there was not one designed to supply the essential information."

#### OBESITY.

In the group referring to obesity (Group D), Hutchinson's Tables were taken as a rough standard of the relation of weight to height. Considerable deviations were, however, allowed on either side; but it is impossible to give any statistical statement as to the amounts of such deviations, as each case was considered on its merits with special reference to occupation, general habits, family history, and any other matters bearing on the subject. The cases actually included in the

group were those that were considerably over the standard weight and outside the limits of such deviations, and accordingly consisted of all those that were rated up as sub-standard lives. The results are shown in Tables XLVIII, XLIX, and L.

The mortality for whole-life assurances is heavy throughout; also it is noticeable that the rates in the ultimate experience are very high. Under endowment assurances the rates both for male and female lives are much lighter for the first ten years of assurance. Here again the assured have discriminated in their choice of policy, the pessimists choosing whole-life contracts while the optimists have cheerfully paid the higher premiums under endowment assurances.

TABLE XLVIII.

*Group D.—Obesity.**Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.*(A) *First 10 years of Assurance.**Whole-Life Assurances.**Males.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Per- centage	Expected Deaths	Actual Deaths	Per- centage	Expected Deaths	Actual Deaths	Per- centage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
-37	3·8	6	157·9	4·3	7	162·8	8·1	13	160·5
38-52	37·8	63	166·7	43·5	85	195·4	81·3	148	182·0
53-62	37·0	58	156·8	40·9	63	154·0	77·9	121	155·3
Total	78·6	127	161·6	88·7	155	174·7	167·3	282	168·8

(B) *Ultimate, excluding first 10 years of Assurance.*

Central Age Attained	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)
40	·6	2·4	250·0
45	1·8	2	
50	4·4	13	243·1
55	10·0	22	
60	17·2	27	158·8
65	23·1	37	
70	19·9	33	159·7
75	8·9	13	

TABLE XLIX,

*Group D. Obesity.**Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.*(A) *First 10 years of Assurance.**Endowment Assurances.**Males.*

Ages at Entry (1)	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths (2)	Actual Deaths (3)	Percentage (4)	Expected Deaths (5)	Actual Deaths (6)	Percentage (7)	Expected Deaths (8)	Actual Deaths (9)	Percentage (10)
-37	29·1	29	99·7	28·2	36	127·7	57·3	65	113·4
38-52	55·2	53	96·0	58·2	87	149·5	113·4	140	123·5
Total	84·3	82	97·3	86·4	123	142·4	170·7	205	120·1

(B) *Ultimate, excluding first 10 years of Assurance.*

Central Age Attained (1)	Expected Deaths (2)	Actual Deaths (3)	Percentage (4)
40	3·2	6 } 11	
45	5·6	5 } 11	125·0
50	8·6	16 } 42	
55	10·3	26 } 42	222·2
60	9·3	12 } 18	
65	1·8	6 } 18	162·2

TABLE L.

*Group D. Obesity.*

*Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.  
First 10 years of Assurance.*

*Endowment Assurances.**Females.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
-37	5·9	4	67·8	6·6	7	106·1	12·5	11	88·0
38-52	21·6	23	106·5	26·1	39	149·4	47·7	62	130·0
Total	27·5	27	98·2	32·7	46	140·6	60·2	73	121·3

In this Group also an analysis was made of the causes of death. This is given in Table LI, which shows that they consist mainly of affections of the circulatory system. The deaths of a tuberculous nature were very few. These results are in general agreement with other published experiences of "heavy-weight" lives.

TABLE LI.

*Group D. Obesity.**Whole-Life Assurances.**Causes of Death.**Males.*

Cause of Death (Reference to Schedule page 7)	AGES AT DEATH							
	35-44		45-54		55-64		65-74	
	No. of Deaths	Per 1000 Deaths from All Causes	No. of Deaths	Per 1000 Deaths from All Causes	No. of Deaths	Per 1000 Deaths from All Causes	No. of Deaths	Per 1000 Deaths from All Causes
6-9	2	87	14	113	25	135	15	163
11	5	217	31	250	42	227	29	315
12-14	3	130	14	113	20	108	15	163
15-16	2	87	9	73	29	156	5	54
23-24	1	44	15	121	30	162	11	120
Other Causes	10	435	41	330	39	212	17	185
All Causes	23	1,000	124	1,000	185	1,000	92	1,000

## PERSONAL BLEMISHES NOT INCLUDED IN OTHER GROUPS.

In point of magnitude of material the Group relating to general personal blemishes (Group E) comes next to that dealing with tuberculous family history. But even so, it was insufficient to justify investigation into each of the 52 sub-groups into which it had been divided. The blemishes were classified as they came to the notice of Dr. Light in the examination of the proposals, and this accounts for the somewhat disconnected appearance of the classification.

Schedule showing the Sub-groups of which Group E was composed.

1. Presence of Cicatrices of old Tuberculous Glands.
2. Not Robust—Poor Physique, Delicate, Anæmic-looking.
3. Dyspepsia.
4. Infantile Paralysis.
5. Hæmoptysis.
6. Nervous disposition.
7. Fair or indifferent health.
8. Old Hip-joint or other Tuberculous Bone Disease.
9. Pigeon Breast or other Chest malformation.
10. Varicose Veins.
11. Pleurisy, Pneumonia or Empyema.
12. Hernia.
13. Lupus.
14. Fistula.
15. Spinal Curvature—(a) Angular, (b) Lateral.
16. Bronchitis.
17. Lungs unsatisfactory (Emphysema, &c.).
18. Epilepsy.
19. Appendicitis.
20. Stricture.
21. Renal Calculus.
22. Ear Disease.
23. Goitre.
24. Gravel.
25. Syphilis.
26. Deafness.
27. Operation for some innocent growth.
28. Gall-stones.

29. Dumb.
30. Looks older.
31. Lead Poisoning.
32. Albumen.
33. Ulcer of the stomach.
34. Nephritis.
36. Tuberculous Lesion or Incipient Phthisis in the past.
37. Coloured Person.
38. Traumatic Lesion, Fractured Skull, Concussion, &c.
39. Practically Blind.
40. Congenital Malformation, other than Chest.
41. Hæmophilia.
42. Chronic Laryngitis, Hoarseness.
43. Intestinal obstruction.
44. Gastric Ulcer (Operation).
45. Thrombosis.
46. Some innocent growth.
47. Exposure to Infection from Tuberculosis.
48. Skin Affection.
49. Instrumental Labour or Labours.
50. Hydrocele.
51. Operation for Peritonitis.
52. Ex-ophthalmia.

An examination of this schedule shows the very varied character of the risks included in this group, rendering a combined experience of the whole of very little use. But it will be seen that many of the Sub-groups may be roughly classed together, and accordingly it was decided to combine them into the following five sections.

Group E.—Schedule of Sections into which various Sub-groups were combined.

#### GROUP E.—SECTION I.

2. Not Robust, Poor Physique, Delicate, Anæmic-looking.
7. Fair or indifferent health.
30. Looks older.

#### GROUP E.—SECTION II.

1. Tuberculous Glands.
5. Hæmoptysis.

8. Tuberculous Bone Disease.
13. Lupus.
36. Incipient Phthisis in the past.
47. Exposure to the infection of Tuberculosis.

GROUP E.—SECTION III.

24. Gravel.
32. Albuminuria.
34. Nephritis.

GROUP E.—SECTION IV.

9. Chest Malformation.
11. Pleurisy, Pneumonia or Empyema.
15. Spinal Curvature.
16. Bronchitis.
17. Lungs unsatisfactory.
42. Chronic Laryngitis.

GROUP E.—SECTION V.

The remainder of the E cases made up of personal defects and diseases, such as—

3. Dyspepsia.
4. Infantile Paralysis.
6. Nervous disposition.
10. Varicose Veins.
12. Hernia.

\* \* \* \* \*

52. Exophthalmia.

Insufficient Records of Albuminuria. Even after this amalgamation it was found that Section III had to be discarded. This is much to be regretted, as it includes the important class of cases in which there is a personal history of albuminuria. For the other four Sections it was possible to investigate only the endowment assurance data on male lives.

With regard to these four sections generally, remembering that the Tables refer entirely to endowment assurances, it will be seen that the rates of mortality are high.

Poor Physique. Beyond this there are, however, certain distinctive features in each section. Sections II and IV, dealing with blemishes of a tuberculous or pulmonary nature have already been discussed in connection with the general subject

of tuberculosis. In Section I the mortality, except in the early years of assurance, in which the rates are comparatively heavy, is not very different from the normal for endowment assurances. This would seem to indicate that persons of poor physique are not much inferior to normal risks provided they show no definite sign of impairment, and that in such cases only a temporary surcharge is required.

TABLE LII.

## Group E. Section I.

*Comparison of Actual Deaths with those Expected according to O<sup>[M]</sup> Table.*

## (A) First 10 years of Assurance.

*Endowment Assurances.**Males.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths (1)	Actual Deaths (2)	Percentage (4)	Expected Deaths (5)	Actual Deaths (6)	Percentage (7)	Expected Deaths (8)	Actual Deaths (9)	Percentage (10)
37	57.0	51	89.5	58.2	61	104.8	115.2	112	97.2
38-52	17.9	19	106.1	21.1	16	75.8	39.0	35	89.7
Total	74.9	70	93.5	79.3	77	97.1	154.2	147	95.3

(B) *Ultimate Experience, excluding first 10 years of Assurance.*

Central Age Attained (1)	Expected Deaths (2)	Actual Deaths (3)	Percentage (4)
40	15.1	6	63.8
45	14.7	19	
50	12.7	13	
55	9.4	15	67.9
60	5.3	9	
65	2.2	7	93.3

Section V consists of risks of such varied character that little reliance can be placed on the combined results. The blemishes in some of the Sub-groups included in this Section are quite serious, and medical opinion as to their prognosis must remain the only guide in dealing with such cases. The results are given in Table LIII.

TABLE LIII.

## Group E. Section V.

*Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.*(A) *First 10 years of Assurance.**Endowment Assurances.**Males.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
-37	30.7	24	78.2	31.3	36	115.0	62.0	60	96.8
38-52	18.1	22	121.5	20.6	23	111.7	38.7	45	116.3
Total	48.8	46	94.3	51.9	59	113.7	100.7	105	104.3

(B) *Ultimate Experience, excluding first 10 years of Assurance.*

Central Age Attained	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)
40	8.2	8	92.0
45	9.2	16	
50	8.6	8	
55	7.9	4	
60	6.9	10	60.6
65	2.7	6	
		5	
		2	
		7	72.9

## FORMERLY INTEMPERATE.

In Group F were included those male lives who appeared, from an examination of the proposals and other reports, to have been formerly intemperate. This Group is naturally a small one, consisting altogether of 1,718 lives, and the whole-life and endowment assurances were combined in one Table.

TABLE LIV.

*Group F.—Formerly Intemperate.**Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.*(A) *First 10 years of Assurance.**Whole-Life and Endowment Assurances.**Males*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Per- centage	Expected Deaths	Actual Deaths	Per- centage	Expected Deaths	Actual Deaths	Per- centage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
-37	10·5	26	247·6	15·6	23	147·4	26·1	49	187·7
38-52	20·7	47	227·1	40·2	83	206·5	60·9	130	213·5
53-62	9·4	13	138·3	18·6	33	177·4	28·0	46	164·3
Total	40·6	86	211·8	74·4	139	186·8	115·0	225	195·6

(B) *Ultimate Experience, excluding first 10 years of Assurance.*

Central Age attained	Expected Deaths		Actual Deaths	Percentage
	(1)	(2)		
40	4·5	4·5 } 12·0	1 } 13	108·3
45	7·5	7·5 } 12·0	12 } 13	
50	11·4	11·4 } 28·6	19 } 33	115·4
55	17·2	17·2 } 28·6	14 } 33	
60	24·4	24·4 } 52·1	39 } 75	143·9
65	27·7	27·7 } 52·1	36 } 75	
70	19·1	19·1 } 28·3	34 } 44	155·5
75	9·2	9·2 } 28·3	10 } 44	

During the first 5 years of assurance, it will be seen that the mortality at the principal insuring ages is exceptionally bad, being the worst of any of the Groups investigated. During the second 5 years the mortality, though still excessive, shows a slight improvement, but in the Ultimate Table there is shown a consistently progressive increase with the age attained.

The opinion may safely be ventured that these figures show that where there is the least suspicion of former intemperance, the lives are undesirable as assurance risks.

## GOUTY DIATHESIS AND FAMILY HISTORY OF GOUT.

Group G deals with gouty diathesis and family history of gout. This Group consists entirely of male lives, and as the data were so few, the whole-life and endowment assurances were combined. The results are shown in Table LV.

TABLE LV.

*Group G.—Gouty Diathesis and Family History of Gout.*  
*Comparison of Actual Deaths with those Expected according to O<sup>(M)</sup> Table.*

(A) *First 10 years of Assurance.**Whole-Life and Endowment Assurances.**Males.*

Ages at Entry	YEARS OF ASSURANCE								
	0-4			5-9			0-9		
	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
-37	1.9	3	157.9	2.0	7	350.0	3.9	10	256.4
38-52	11.0	22	200.0	14.9	30	201.3	25.9	52	200.8
53-62	7.0	18	257.1	11.7	31	265.0	18.7	49	262.0
Total	19.9	43	216.1	28.6	68	237.8	48.5	111	228.9

(B) *Ultimate Experience, excluding first 10 years of Assurance.*

Central Age Attained	Expected Deaths	Actual Deaths	Percentage
(1)	(2)	(3)	(4)
40	.6	2.0	100.0
45	1.4	2	
50	3.1	5	166.7
55	6.5	16	
60	11.1	16	121.0
65	13.7	30	
70	13.6	14	123.4
75	9.9	29	
		13	

The mortality throughout is very heavy, but is lower in the first 5 years of assurance than in years of assurance 5 to 9. From the Ultimate Table it appears that the maximum of excess mortality is attained between ages 50 and 60.

On investigating the deaths in this Group we found a large number where the cause of death was cirrhosis of liver. This at least raises a suspicion that the assured persons in this Group were not all total abstainers.

#### CONCLUSION.

Value of the Results. In concluding this Paper, we venture to express the hope that the results may be considered not merely of interest, but also of practical value. It appears to us that its main value lies in the investigation of the incidence of the extra risk in the various groups and classes dealt with. This incidence has been shown to vary not only in regard to age, but also in regard to duration of assurance. It has been shown to depend in large measure also on the self-selection of the assured as expressed by the class of policy chosen. We have deliberately abstained from making specific suggestions as to the methods of dealing with the extra risks, either by way of extra premiums, rating-up of age, or decreasing contingent debts, nor have we touched the equally important question of modification of reserves. These subjects form a separate branch of enquiry, and to have included them would have unduly extended the length of the Paper.

We have had the great advantage of being able to draw upon a very substantial body of data, and to treat it on a consistent plan under the supervision of a single medical officer fully conversant with the problems involved. These are almost ideal conditions. We venture to think that one of the results of the investigation should be to indicate the lines upon which further research can profitably be made, whether by individual offices or by collective action.

Many actuaries have made researches in the same subjects, but we have refrained from specific reference to their works, because it has been our aim to restrict the Paper within as reasonable bounds as possible. We hope, however, that the discussion which is to follow will throw further light on the many problems involved.

Finally, we would record our great indebtedness to Mr. G. A. Vokins, A.I.A., whose suggestions and enthusiastic help in dealing with the large amount of clerical work involved have been of the utmost value in the preparation of this Paper.

## APPENDIX.

## AGGREGATE TABLES.

Age next Birthday $x$	FULL AGGREGATE		EXCLUDING FIRST 5 YEARS OF ASSURANCE		EXCLUDING FIRST 10 YEARS OF ASSURANCE	
	$E_x$	$\theta_x$	$E_x^{(5)}$	$\theta_x^{(5)}$	$E_x^{(10)}$	$\theta_x^{(10)}$
20	14	...	...	...	...	...
21	38	...	...	...	...	...
22	87	2	2	...	...	...
23	141	1	2	...	...	...
24	223	1	4	...	...	...
25	301	4	17	...	...	...
26	416	3	46	1	3	1
27	534	3	83	1	3	1
28	660	8	137	3	6	...
29	830	1	193	...	9	...
30	970	8	275	3	23	...
31	1,171	13	379	4	46	1
32	1,348	14	515	5	90	...
33	1,541	12	621	6	137	2
34	1,738	13	772	7	191	2
35	1,936	20	920	8	265	3
36	2,129	16	1,109	8	340	3
37	2,318	22	1,265	12	453	5
38	2,508	29	1,450	12	557	4
39	2,646	24	1,624	15	667	4
40	2,796	36	1,784	22	789	13
41	2,890	29	1,933	17	928	6
42	2,971	29	2,037	22	1,046	11
43	3,095	29	2,129	22	1,161	16
44	3,189	36	2,216	26	1,302	17
45	3,283	38	2,299	25	1,424	12
46	3,349	38	2,373	29	1,521	21
47	3,401	42	2,384	31	1,563	21
48	3,452	41	2,448	29	1,605	20
49	3,430	51	2,484	39	1,645	22
50	3,348	49	2,453	37	1,639	24
51	3,218	63	2,444	48	1,628	31
52	3,067	40	2,423	37	1,627	29
53	2,909	36	2,383	28	1,624	19
54	2,729	38	2,321	31	1,591	19

*Family History of Tuberculosis. Combined Experience A<sub>0</sub> to A<sub>12</sub>.*  
*—continued.*

*Whole-Life Assurances.**Males.*

Age next Birthday <i>x</i>	FULL AGGREGATE		EXCLUDING FIRST 5 YEARS OF ASSURANCE		EXCLUDING FIRST 10 YEARS OF ASSURANCE	
	E <sub>x</sub>	θ <sub>x</sub>	E <sub>x</sub> <sup>(5)</sup>	θ <sub>x</sub> <sup>(5)</sup>	E <sub>x</sub> <sup>(10)</sup>	θ <sub>x</sub> <sup>(10)</sup>
55	2,538	62	2,221	56	1,560	42
56	2,328	39	2,062	34	1,505	25
57	2,136	44	1,929	42	1,479	32
58	1,929	49	1,762	45	1,417	37
59	1,699	52	1,573	49	1,319	38
60	1,485	56	1,402	54	1,209	51
61	1,285	47	1,231	47	1,071	41
62	1,095	38	1,066	38	939	32
63	932	27	917	27	818	23
64	790	26	785	26	705	23
65	650	23	646	23	595	23
66	546	35	544	35	504	32
67	437	23	437	23	412	23
68	351	12	351	12	338	11
69	289	13	289	13	283	12
70	230	20	230	20	227	20
71	181	18	181	18	178	17
72	144	10	144	10	141	10
73	119	11	119	11	116	9
74	92	5	92	5	91	5
75	73	7	73	7	73	7
76	57	6	57	6	57	6
77	41	3	41	3	41	3
78	35	5	35	5	35	5
79	26	1	26	1	26	1
80	21	5	21	5	21	5
81	13	3	13	3	13	3
82	10	3	10	3	10	3
83	5	...	5	...	5	...
84	3	...	3	...	3	...
85	2	1	2	1	2	1
86	1	1	1	1	1	1

*Family History of Tuberculosis. Combined Experience A<sub>0</sub> to A<sub>12</sub>.**Endowment Assurances.**Males.*

Age next Birthday <i>x</i>	FULL AGGREGATE		EXCLUDING FIRST 5 YEARS OF ASSURANCE		EXCLUDING FIRST 10 YEARS OF ASSURANCE	
	E <sub>x</sub>	θ <sub>x</sub>	E <sub>x</sub> <sup>(5)</sup>	θ <sub>x</sub> <sup>(5)</sup>	E <sub>x</sub> <sup>(10)</sup>	θ <sub>x</sub> <sup>(10)</sup>
16	3	...	...	...	...	...
17	5	...	...	...	...	...
18	34	...	...	...	...	...
19	161	2	...	...	...	...
20	806	1	...	...	...	...
21	2,608	11	3	...	...	...
22	4,828	19	9	...	...	...
23	6,632	20	68	...	...	...
24	8,261	33	208	1	...	...
25	9,959	41	679	3	...	...
26	11,454	54	1,786	5	3	...
27	12,899	56	3,122	12	11	...
28	14,172	55	4,282	21	62	...
29	15,141	67	5,321	28	180	1
30	16,053	61	6,363	31	480	3
31	16,744	71	7,206	33	980	2
32	17,289	86	8,026	52	1,582	9
33	17,552	80	8,637	47	2,038	16
34	17,655	84	9,159	46	2,475	12
35	17,729	92	9,739	46	2,972	15
36	17,497	91	10,081	55	3,344	18
37	17,273	86	10,280	57	3,688	21
38	16,829	100	10,273	57	3,957	23
39	16,284	88	10,203	57	4,197	24
40	15,537	86	9,853	54	4,171	25
41	14,734	87	9,565	62	4,261	25
42	13,965	70	9,267	55	4,320	22
43	13,210	89	8,970	68	4,336	31
44	12,377	96	8,571	74	4,268	43

*Family History of Tuberculosis.*      *Combined Experience A<sub>0</sub> to A<sub>12</sub>.*

—continued.

*Endowment Assurances.*

*Males.*

Age next Birthday <i>x</i>	FULL AGGREGATE		EXCLUDING FIRST 5 YEARS OF ASSURANCE		EXCLUDING FIRST 10 YEARS OF ASSURANCE	
	E <sub>x</sub>	θ <sub>x</sub>	E <sub>x(5)</sub>	θ <sub>x(5)</sub>	E <sub>x(10)</sub>	θ <sub>x(10)</sub>
45	11,384	75	8,007	57	4,002	32
46	10,539	59	7,548	46	3,857	26
47	9,606	58	7,043	50	3,746	23
48	8,708	67	6,513	48	3,586	28
49	7,750	65	5,992	53	3,385	35
50	6,627	76	5,240	62	2,936	41
51	5,638	51	4,660	42	2,646	23
52	4,762	43	4,084	36	2,368	21
53	4,027	61	3,566	59	2,163	35
54	3,348	50	3,067	46	1,950	34
55	2,612	42	2,491	42	1,614	27
56	2,072	19	2,009	19	1,380	14
57	1,664	25	1,627	25	1,196	18
58	1,342	35	1,322	34	1,040	30
59	1,035	25	1,020	25	850	20
60	659	15	653	15	579	14
61	426	10	423	10	394	8
62	283	10	280	10	262	10
63	188	7	187	7	178	7
64	128	1	128	1	122	1
65	30	1	30	1	27	1
66	8	1	8	1	6	1
67	5	1	5	1	3	1
68	3	...	3	...	2	...
69	2	...	2	...	2	...
70	1	...	1	...	1	...
71	1	...	1	...	1	...
72	1	...	1	...	1	...
73	1	...	1	...	1	...
74	1	1	1	1	1	1

*Personal History of Rheumatic Fever and Lesions of the Circulatory System.**Whole-Life Assurances.**Group B.**Males.*

Age next Birthday <i>x</i>	FULL AGGREGATE		EXCLUDING FIRST 5 YEARS OF ASSURANCE		EXCLUDING FIRST 10 YEARS OF ASSURANCE	
	$E_x$	$\theta_x$	$E_x^{(5)}$	$\theta_x^{(5)}$	$E_x^{(10)}$	$\theta_x^{(10)}$
20	1	...	...	...	...	...
21	4	...	...	...	...	...
22	10	...	...	...	...	...
23	13	...	1	...	...	...
24	19	...	1	...	...	...
25	31	...	3	...	...	...
26	40	1	7	...	...	...
27	52	1	12	1	...	...
28	64	1	16	...	...	...
29	90	...	26	...	...	...
30	109	1	36	1	3	...
31	138	...	49	...	9	...
32	157	2	69	...	14	...
33	180	2	85	2	24	1
34	206	2	99	1	32	...
35	236	1	125	1	44	...
36	259	6	155	5	57	1
37	302	3	180	2	71	...
38	314	2	200	2	81	...
39	346	5	241	3	102	1
40	371	3	268	2	116	...
41	413	6	299	6	150	4
42	437	11	330	6	172	1
43	456	8	356	7	204	5
44	473	8	376	6	232	2
45	504	13	390	11	251	6
46	522	10	399	9	261	6
47	535	12	412	6	280	5
48	552	17	440	12	298	9
49	557	13	442	9	309	5
50	559	14	447	13	308	7
51	551	18	442	17	323	13
52	559	13	437	9	324	5
53	571	13	455	12	335	8
54	571	16	453	14	339	10
55	566	22	450	17	337	11
56	543	18	448	18	335	16
57	525	16	443	14	318	9
58	507	17	433	14	313	12
59	482	15	418	14	295	12

*Personal History of Rheumatic Fever and Lesions of the Circulatory System.*

*Whole-Life Assurances.*

*Group B.—continued.*

*Males.*

Age next Birthday $x$	FULL AGGREGATE		EXCLUDING FIRST 5 YEARS OF ASSURANCE		EXCLUDING FIRST 10 YEARS OF ASSURANCE	
	$E_x$	$\theta_x$	$E_x^{(5)}$	$\theta_x^{(5)}$	$E_x^{(10)}$	$\theta_x^{(10)}$
60	447	15	391	14	283	9
61	405	14	361	12	270	9
62	377	14	348	14	267	12
63	334	17	319	16	244	14
64	298	7	285	7	219	5
65	278	14	270	13	208	10
66	242	14	236	13	182	11
67	220	13	215	13	175	8
68	198	15	193	14	166	12
69	172	11	171	11	148	10
70	143	13	142	13	126	11
71	124	16	124	16	109	14
72	100	5	100	5	90	5
73	83	5	83	5	74	5
74	70	9	70	9	64	8
75	56	9	56	9	51	8
76	37	4	37	4	35	4
77	31	4	31	4	31	4
78	26	4	26	4	26	4
79	22	8	22	8	22	8
80	12	1	12	1	12	1
81	9	1	9	1	9	1
82	5	...	5	...	5	...
83	5	1	5	1	5	1
84	4	1	4	1	4	1
85	4	1	4	1	4	1
86	3	1	3	1	3	1
87	2	...	2	...	2	...
88	2	1	2	1	2	1
89	1	...	1	...	1	...
90	1	1	1	1	1	1

*Personal History of Rheumatic Fever and Lesions of the Circulatory System.*

*Endowment Assurances.*

*Group B.*

*Males.*

Age next Birthday <i>x</i>	FULL AGGREGATE		EXCLUDING FIRST 5 YEARS OF ASSURANCE		EXCLUDING FIRST 10 YEARS OF ASSURANCE	
	$E_x$	$\theta_x$	$E_x^{(5)}$	$\theta_x^{(5)}$	$E_x^{(10)}$	$\theta_x^{(10)}$
16	1	...	...	...	...	...
17	4	...	...	...	...	...
18	9	...	...	...	...	...
19	27	...	...	...	...	...
20	73	...	...	...	...	...
21	173	...	1	...	...	...
22	329	2	6	...	...	...
23	434	1	12	...	...	...
24	586	1	33	...	...	...
25	704	6	71	1	...	...
26	793	1	157	1	3	...
27	917	7	241	2	7	...
28	1,004	8	315	3	12	...
29	1,135	5	426	3	27	...
30	1,185	3	502	1	53	...
31	1,260	1	585	...	98	...
32	1,345	6	675	3	150	1
33	1,394	11	731	7	184	2
34	1,450	8	833	6	257	4
35	1,499	9	904	5	313	2
36	1,532	12	964	5	366	2
37	1,542	9	1,023	5	436	2
38	1,551	15	1,055	8	478	4
39	1,536	15	1,068	8	528	3
40	1,482	17	1,046	15	541	7
41	1,439	11	1,030	7	566	6
42	1,409	10	1,016	8	590	3
43	1,382	19	1,015	7	592	1
44	1,316	16	999	15	571	7
45	1,233	11	946	6	546	4
46	1,131	10	879	6	511	5
47	1,076	10	828	8	487	3
48	1,015	14	814	11	503	6
49	950	11	767	10	480	7

*Personal History of Rheumatic Fever and Lesions of the Circulatory System.*

*Endowment Assurances.*

*Group B.—continued.*

*Males.*

Age next Birthday $x$	FULL AGGREGATE		EXCLUDING FIRST 5 YEARS OF ASSURANCE		EXCLUDING FIRST 10 YEARS OF ASSURANCE	
	$E_x$	$\theta_x$	$E_x^{(5)}$	$\theta_x^{(5)}$	$E_x^{(10)}$	$\theta_x^{(10)}$
50	837	18	638	15	395	6
51	745	13	584	11	362	8
52	660	10	540	9	325	5
53	585	9	477	7	308	6
54	509	9	435	9	286	6
55	419	11	374	11	228	5
56	367	6	334	5	214	3
57	325	4	300	4	206	1
58	286	15	266	14	187	9
59	232	6	223	6	162	5
60	163	6	156	6	119	5
61	113	5	111	5	85	4
62	87	3	86	3	66	..
63	62	2	62	2	52	2
64	41	1	41	1	36	..
65	13	1	13	1	11	..
66	5	...	5	...	5	..
67	2	...	2	...	2	..
68	2	...	2	...	2	..
69	2	...	2	...	2	..
70	2	...	2	...	2	..
71	1	...	1	...	1	..
72	1	...	1	...	1	..
73	1	...	1	...	1	..
74	1	...	1	...	1	..
75	1	...	1	...	1	..
76	1	...	1	...	1	..

*Obesity.**Whole-Life Assurances.**Group D.**Males.*

Age next Birthday $x$	FULL AGGREGATE		EXCLUDING FIRST 5 YEARS OF ASSURANCE		EXCLUDING FIRST 10 YEARS OF ASSURANCE	
	$E_x$	$\theta_x$	$E_x^{(5)}$	$\theta_x^{(5)}$	$E_x^{(10)}$	$\theta_x^{(10)}$
21	1	1	...	...	...	...
22	1	...	...	...	...	...
23	2	...	...	...	...	...
24	4	...	...	...	...	...
25	4	...	...	...	...	...
26	5	...	...	...	...	...
27	11	...	1	...	...	...
28	18	...	1	...	...	...
29	21	...	2	...	...	...
30	31	...	3	...	...	...
31	44	...	5	...	...	...
32	53	1	8	...	...	...
33	68	1	11	...	...	...
34	78	1	14	...	...	...
35	92	1	23	...	1	...
36	111	...	33	...	2	...
37	131	1	36	1	5	...
38	142	1	40	...	5	...
39	150	1	47	1	9	1
40	178	5	59	2	15	1
41	191	1	59	...	17	...
42	227	6	77	3	19	2
43	239	3	76	1	20	...
44	271	4	89	1	25	...
45	313	9	103	6	29	2
46	369	3	119	2	34	...
47	422	5	136	2	40	...
48	466	15	152	6	45	2
49	498	8	182	4	48	2
50	536	20	211	15	57	3
51	573	10	235	8	68	4
52	611	23	258	11	69	2
53	617	16	268	10	83	3
54	635	14	279	10	94	5

*Obesity.**Whole-Life Assurances. Group D.—continued. Males.*

Age next Birthday $x$	FULL AGGREGATE		EXCLUDING FIRST 5 YEARS OF ASSURANCE		EXCLUDING FIRST 10 YEARS OF ASSURANCE	
	$E_x$	$\theta_x$	$E_x^{(5)}$	$\theta_x^{(5)}$	$E_x^{(10)}$	$\theta_x^{(10)}$
55	653	16	304	9	98	2
56	640	19	322	9	98	2
57	624	19	334	15	109	10
58	590	25	337	13	116	4
59	546	26	330	17	116	7
60	493	15	311	10	112	5
61	449	18	300	15	125	9
62	402	16	289	11	123	2
63	343	21	262	16	114	7
64	296	11	250	11	115	5
65	254	15	230	14	114	4
66	199	15	192	15	108	7
67	160	18	156	18	101	14
68	124	8	123	8	82	5
69	98	8	98	8	74	6
70	79	8	79	8	64	8
71	61	7	61	7	57	7
72	47	8	47	8	45	7
73	32	4	32	4	32	4
74	23	1	23	1	23	1
75	20	4	20	4	20	4
76	14	3	14	3	14	3
77	8	1	8	1	8	1
78	6	2	6	2	6	2
79	4	1	4	1	4	1
80	3	...	3	...	3	...
81	3	...	3	...	3	...
82	1	...	1	...	1	...
83	1	...	1	...	1	...
84	1	1	1	1	1	1

*Obesity.**Endowment Assurances.**Group D.**Males.*

Age next Birthday $x$	FULL AGGREGATE		EXCLUDING FIRST 5 YEARS OF ASSURANCE		EXCLUDING FIRST 10 YEARS OF ASSURANCE	
	$E_x$	$\theta_x$	$E_x^{(5)}$	$\theta_x^{(5)}$	$E_x^{(10)}$	$\theta_x^{(10)}$
18	1	...	...	...	...	...
19	5	...	...	...	...	...
20	14	...	...	...	...	...
21	23	...	1	...	...	...
22	38	...	1	...	...	...
23	56	...	2	...	...	...
24	80	1	3	...	...	...
25	109	1	8	...	...	...
26	157	2	11	...	1	...
27	176	...	22	...	1	...
28	226	...	32	...	2	...
29	301	1	49	...	3	...
30	381	...	65	...	5	...
31	437	3	93	1	5	...
32	483	2	112	...	14	...
33	542	4	137	2	13	...
34	600	5	178	2	19	...
35	671	4	217	...	22	...
36	724	2	247	1	35	...
37	763	3	270	3	38	...
38	816	6	296	2	52	1
39	849	11	321	4	56	1
40	913	4	355	2	70	...
41	936	13	380	11	76	2
42	943	11	378	5	73	2
43	937	8	402	3	81	...
44	933	7	414	6	99	2
45	920	14	424	8	95	1
46	883	8	425	3	94	1
47	865	15	439	11	92	1
48	827	10	430	6	94	1
49	789	11	429	9	119	3

*Obesity.*

*Endowment Assurances.*      *Group D.—continued.*      *Males.*

Age next Birthday $x$	FULL AGGREGATE		EXCLUDING FIRST 5 YEARS OF ASSURANCE		EXCLUDING FIRST 10 YEARS OF ASSURANCE	
	$E_x$	$\theta_x$	$E_x^{(5)}$	$\theta_x^{(5)}$	$E_x^{(10)}$	$\theta_x^{(10)}$
50	782	13	406	11	118	4
51	718	16	388	12	114	2
52	627	16	360	12	114	6
53	556	12	364	9	121	4
54	496	15	344	12	111	5
55	396	16	311	15	98	10
56	335	11	267	10	85	4
57	286	8	237	7	82	3
58	231	6	199	6	90	1
59	193	6	169	5	83	1
60	132	10	116	10	71	7
61	89	4	86	4	51	2
62	46	3	46	3	26	1
63	30	3	30	3	17	1
64	20	3	20	3	15	2
65	11	3	11	3	7	2
66	3	1	3	1	3	1
67	1	...	1	...	1	...

## ABSTRACT OF THE DISCUSSION.

MR. H. E. W. LUTT in opening the discussion, said that the first point to which he proposed to refer was the classification of the causes of extra risk. He assumed that all the lives were rated-up lives. It would be noticed that there were over 100 divisions of extra risk specified, and that they were eventually dealt with in 19 classes. At the Vienna Congress Mr. Hardy suggested the classification of rated-up cases according to the incidence of the extra risk (*i.e.*, early, throughout, or late in life). In certain respects the method of classification adopted by the authors was similar to this. For instance, all the consumptive risks would go into the first class, where the extra risk was assumed to occur in early life. He thought, however, that if any valuable information was to be derived from the collection of such statistics, it would be necessary to limit the number of the divisions in every possible way, and to proceed upon lines of enquiry in certain definite directions, where sufficient statistics could be obtained to give fairly reliable results. The comparison which the authors had made in their Paper was based on the  $O^{(M)}$  mortality. It was well known, however, that the  $O^{(M)}$  mortality was not that which all offices experienced, and if the authors could have obtained a standard table from their own experience of normal lives he ventured to think that the comparisons which they had instituted would have been very much more valuable. Moreover, as regards endowment assurances there was admittedly an entirely different class of mortality, and it was with endowment assurances that the Paper chiefly dealt. All the comparisons given in the Paper, both for whole-life and for endowment assurances, were based on the  $O^{(M)}$  Table, and that had to be borne in mind.

A point that occurred to him was whether in dealing with under-average lives a discrimination had been exercised in suggesting endowment assurance rather than whole-life assurance for certain descriptions of extra risk. That might have an influence upon the mortality which had been experienced in special classes. The authors had endeavoured to institute a comparison of the causes of death among under-average lives on the lines of the causes for which they had been rated up, and had included in one table the rates deduced from the Registrar-General's Returns. Those rates, however, were not of great use for this purpose, and he was glad to see that in Tables XXXII–XXXIII, the authors had given their own experience in connection with the causes of death in non-tuberculous families. That, to his mind, constituted a much better basis of comparison. With regard to the question of the investigation of consumptive family history, he thought the lines which had been adopted by Messrs Rusher and Kenchington gave a very definite basis of division and pointed to a very definite conclusion. That particular line of enquiry could at all events be maintained and extended if necessary. He wished to draw attention in passing to the large class of A + E, which the authors mentioned specifically, but to which they only referred again in one paragraph,

on page 474, and in Tables XXVII to XXIX, where they gave the ratio of actual to expected deaths in the case of lives with tuberculous family history plus personal defect. Then, on page 451, they referred to the necessity for caution in using the statistics, because of the varying age-constitution of the groups, and suggested a method of remedying that by taking the exposed to risk as the same in both classes and deriving the expected and actual deaths therefrom for the purposes of comparison. But even then absolute finality was not reached, because in the three divisions that were made, *i.e.*, ages at entry up to 37, 38-52 and 53-62, there was also a great deal of scope for variation owing to the different constitution of the groups in relation to individual ages at entry.

Perhaps the most surprising result as regards consumptive family history was the comparison of the mortality where the family history taint was exhibited in lineals, lineals and collaterals, and collaterals alone. It seemed almost impossible, *a priori*, that the mortality of persons having consumptive indications in both ancestors and collaterals should be lighter than in the case of those who had only a lineal consumptive family history. If reference were made to Table XX, Sections  $\alpha$  and  $\beta$ , the number of actual deaths indicated in the latter seemed quite too small to rely upon for this inference. For duration of assurance 0-4, ages at entry up to 37, a percentage was obtained in the lineals of 80·8, in the lineals plus collaterals of 78·6, and in the collaterals of 81·5. Those differences were so very small that the only conclusion which could be arrived at was that the risks were about equal. In the duration of insurance 5-9, the figures were 68·2, 50·9 and 74·2. The latter figure was dependent upon quite a large number of deaths, and rather surprised him. The conclusion that it was necessary to pay as much attention to the consumptive history of collaterals as to that of lineals, was quite opposed to that which Mr. Manly came to in 1892, but Dr. Marsh, in an investigation of the Mutual of New York Experience, 1879-1893, to which the authors referred, rather corroborated that view. There the percentages of deaths from consumption at ages 20-29, 30-39 and 40-49, to those from all causes where one parent had died from that disease were 39·8, 17·1 and 8·6; and where one brother or sister had died 34·1, 20·6 and 10·2. It was, however, difficult to believe that a person whose brother or sister had died of consumption could be as dangerous a life from the insurance point of view as a person whose father or mother had died from the same cause, and in Table XXXIII, on page 478, where comparison was made of deaths from tuberculosis with 1,000 deaths from all causes, the proportions were about equal for the lineals and the collaterals, but the lineals plus collaterals showed decidedly larger percentages. The only exception was at age 20-24, where the proportion was 500 for section  $\beta$ . It would be found, however, that the figure of 500 was derived from 4 deaths out of 8, so that an addition of one to the number of deaths from consumption would increase the figure 500 to 625.

It seemed to him that perhaps an explanation of the results

in Tables XX and XXI might be that in a case where only one brother or sister had died from consumption it had not been the practice of the office to pay so much attention to the personal condition of the life as where the father or mother had died, and that the risks accepted were not on the average so good. He did not think the infection theory in regard to brothers and sisters was of so much importance as the possibility that they showed a similar weakness or predisposition to disease. So far as the medical view of consumption was concerned, he thought he was right in saying that originally heredity was deemed the most important feature, and that when Koch announced the discovery of the tubercle bacillus in 1882, heredity came to be rather disregarded, and infection was assumed to be the chief cause of the spread of consumption. He believed that a section of medical opinion now held that infection was not so much through the lungs as through the alimentary canal, and that the spread of consumption could be prevented by greater regard being paid to food, such as milk and meat, which had now for several years been done.

The authors compared the decrease of the general death rate with the decrease of that from consumption, and pointed out that the two rates had not followed each other very well. As a matter of fact, sanitation had had more effect on the reduction of the death rate than sanatoria for consumptive sufferers. The decrease was greater before 1882, when the bacillus was discovered, than it had been since. It seemed to him that the housing of the working classes, a measure which had attracted considerable attention in Parliament, was likely to give very much better results than the segregation of sufferers in sanatoria, and allowing them to return to their insanitary homes before they were cured—because most of them could not be cured. The authors had also referred to the tuberculin treatment. He believed he was right in saying that tuberculosis, in some shape or form, was probably present in a large proportion of the community in the way of some lesion which had been healed, but which responded to tuberculin, and any suggestion that offices should use a tuberculin test for their proposers would lead to very serious results, not only in the way of danger to themselves, but of danger to the proposers. He thought with Dr. Lister that, from the insurance point of view, they had still to ask two questions in regard to tuberculosis, namely, had the proposer had it, and was he likely to get it, having regard to his family history and conditions of life? He believed that offices would continue to treat these cases on much the same lines as formerly.

He desired, in conclusion, to refer to the question of obesity, which he thought was the best example of the great difference in practice that existed in different offices. It was almost impossible to estimate the value of the tables unless some idea could be obtained of the standard which had been adopted in assessing the risks. It might be that Dr. Light, who had to deal with the cases, took a lenient view of heavy weight or the reverse, and if these lives were consequently all very much over weight the mortality was only

what would be expected. He desired to point out that in many of the diseases with which the Paper dealt the rates of mortality had been altering during the last seventeen years, and that the tables in the Paper gave the average rather than the final results. In fact, it was hardly possible to look for more than tendencies. In that connection the question arose : Was the line of enquiry one that could be pursued with advantage ? Personally he thought it could, with very great advantage to the actuary, but owing to the huge amount of work that would have to be done it must be restricted. He suggested that the lines of such an investigation should be settled by a small Sub-Committee appointed for the purpose, with a view to their being adopted generally by offices which would like to take up that form of investigation, so that results might be obtained at a future period which would not require all the trouble and labour that the collection of the information in the Paper had evidently given to the authors.

Mr. A. D. BESANT said the subject under discussion was one of the most interesting that could possibly be brought before the Institute. It was of the utmost practical value ; for anything that threw light upon the problem of what additional charges should be made for invalid lives, or for lives which for one cause or another might be considered below the normal, was of the highest importance to all actuaries. The members, therefore, were fortunate in having placed before them statistics based upon so large an experience of assured lives. The most interesting and the most important section of the Paper was that dealing wth consumptive family history and its effect upon longevity. The authors had classified lives whose relatives had died of consumption or child-birth under no fewer than twelve or fifteen sub-headings as regards family history, but there was no mention whatever of the personal condition of the applicants included in such headings. In his own experience he had always been accustomed to regard the personal condition as of the utmost importance, in fact, as of even more importance than the family history, and he could not think it was sound policy to classify lives with regard to the family history alone. As, he thought, Mr. Lutt had already mentioned, one of the commonest types met with in consumptive family history cases was that of personal delicacy in the applicant, especially at the younger ages at entry, and he could not help feeling that if each of the twelve or fifteen divisions made by the authors on account of the family history had been sub-divided into sections containing lives whose personal history was either sub-normal or normal, or above normal, the members would have been better able to judge whether the high rate of mortality exhibited in the whole-life class was due to the defective family history or to personal delicacy, or to both combined. As the figures stood, there was no doubt that an undue mortality was experienced amongst those assured for the whole of life, especially noticeable at the younger ages at entry and during the early years of assurance, and gradually becoming less powerful until it became practically negligible for entrants above 35. He

gathered that the authors attributed that heavy early mortality to family history alone. It seemed to him that the element of personal delicacy was also involved, and that if the facts could have been made more homogeneous by the sub-division of the classification it might have been possible to determine more accurately whether the personal condition did not outweigh the family defect to such an extent that the latter might almost be ignored in assessing the charge to be made, even at the early ages at entry. To put the matter in more concrete form, taking Table XI, where during the first ten years of experience the death rate among the younger male entrants was 29 per cent. in excess of that expected under the O<sup>(M)</sup> Table, that would indicate that, as a type, such cases needed some addition to the premium ; but if the matter were analysed a little more deeply it might be found that a good many cases included under the observations were personally exceptionally sound, and that this counterbalanced their unfavourable family history, while others were delicate, a fact which accentuated the danger of their family history.

He would like to pass on to consider the very interesting problem as to the relative mortality experienced under whole-life and endowment assurance policies with a tubercular family history. The difference was remarkably striking, and it was especially noticeable that it extended throughout all the tables. The authors attributed the difference to the self-selection exercised by the more healthy members in choosing endowment assurances, and they suggested that that was a most potent factor, equal in importance to the medical selection. He thought that another explanation must be put forward as, at any rate, a reasonable hypothesis. It would be noticed from the figures given in Tables I to VII that endowment assurances formed five-sixths of the whole ; taking only the new entrants from 1893 to 1910, the proportion was much higher than five-sixths. Therefore, instead of regarding the whole-life mortality as the normal and the endowment assurance mortality as specially select, the matter might be regarded conversely, and the large endowment section be looked upon as exhibiting the normal mortality, and the small whole-life section as exhibiting an adverse selection against the office. In support of that view he would say that the tendency of the man who was somewhat delicate, but who had been just able to pass the doctor's tests, would naturally be to secure as much assurance cover as his money would provide, so that he would naturally gravitate towards the whole-life section, and in that way a good many lives of inferior vitality would become included in that section. That might account to a larger extent than was at first realized for the higher mortality of the whole-life section. However, whatever the explanation, the fact remained that with regard to over five-sixths of the combined experience of lives with a consumptive family history, the actual mortality, as shown in Tables XI, XII and XIII, was less than two-thirds of the O<sup>(M)</sup> and was even slightly less than the O<sup>(EM)</sup>. That appeared to be by far the most important result that the investigation had

disclosed. Without going so far as to say that it showed that the family history could be ignored altogether, it seemed to point quite definitely to the fact that its adverse influence had been greatly exaggerated, and that much less attention might be attached to it, provided that the applicant was personally healthy.

In his own office, where they had long devoted special attention to the assurance of invalid lives, provided that the candidate was of good average weight and physique, they almost invariably accepted at the tabular premium cases where the family history contained one death from consumption, and in many cases did so when the family history was even more adverse, and they saw no reason to make any change in that respect. He remembered a remark being made to him some time ago by a distinguished physician on that point. He said, "you may classify your lives under A or B or C or D as much as you like; but you will never get a sound view unless you treat an applicant as an individual, and not merely as a type." That, he thought, was the keynote of the whole problem. However detailed and elaborate the system of classification might be, there must be many considerations that fell outside any form of classification whatever, and there was always the danger that if actuaries depended too much on classification they might be led to regard the problem as a series of parts rather than to take a wide general view of the whole. The great value of the Paper lay in the light thrown upon the fact that it was clear that adverse family history alone did not necessarily cause undue mortality amongst at least five-sixths of the lives coming under observation, and all actuaries who regarded the personal factor as all-important in assessing the scale of premium to be charged would find a valuable support for their views in the results tabulated in the authors' important investigation.

Mr. H. W. MANLY said it might be expected that he should make a few observations upon the Paper, especially as the Paper he read just twenty-one years ago on the same subject was the first production of its kind. He defined it as "An Attempt to measure the Extra Risk arising from a Consumptive Family History where the Life proposed for Assurance is physically sound and healthy", and the Paper under discussion was entitled "An investigation into the effects of Family and Personal History upon the Rates of Mortality experienced in various classes of Life Assurance Risks, with special reference to Tuberculosis." It appeared to him that the statistics in each case were of the same class, because he described his lives as being in such a healthy condition, otherwise than with regard to the history of tuberculosis, that they were accepted at ordinary rates; while the authors stated, "It is probable, for instance, that in the medical selection of these lives special care was taken to ensure that in all respects, other than the defects in the family history—such as personal condition, occupation, &c.—the applicants were unimpeachably first-class lives" and they were all apparently charged with a decreasing debt, so that, to a certain extent they might all be classed as self-selected lives. With regard

to Mr. Besant's suggestion, that it would be desirable to find out the experience amongst those who had a family history of consumption but with a delicate constitution, the difficulty was that there were no facts to go upon. He did not know any office that would accept such lives, so the statistics could not come from the offices. Then the idea that the heavier mortality amongst whole-life assurances arose from the desire to obtain higher cover at once did not seem to hold good. Apparently they obtained very small cover at once if they took out a whole-life assurance, and it was not until some period after, when the debt was partly wiped off, that they obtained the larger cover of the whole-life assurance. He did not know what the proportion would be, but he imagined that in the whole-life assurance the insurance in the first year would be less than in the endowment assurance, so that he did not think there would be a selection in that way. It was a mystery to him that the experience amongst the endowment assurances was so light, while amongst the whole-life assurances it was comparatively heavy.

The authors had had the advantage of a much later experience and of far greater material than he (Mr. Manly) had, but he did not think their conclusions were so very different from those he came to 21 years ago. For instance, they stated at page 470 : " Furthermore, the *degree* of tuberculous family history is of much less importance than the age at entry, and may, in fact, be said to have no practical value for assurance purposes over age 35 at entry." In the Paper he read 21 years ago he stated with regard to the same subject : " There is still another theory, that the time of the greatest risk of the hereditary taint being developed is in early manhood ; and that when that time is past and the applicant is of sound constitution, and in every respect—but for the family history—a first-class life, then the risk, as is actually shown by these figures, is small. Personally, I have a great respect for the last theory ; and I am inclined to think that all persons under 26 years of age who had a parent die of consumption or child-birth should be charged the premium for age 30, but that after the age of 25 an addition of from 5 to 3 years is quite sufficient to cover the risk." There was a further remark made with regard to collaterals, on which he thought the authors' conclusions were very much the same as his own. For instance, he said in 1892 : " So far as these observations have any weight, cases of consumption in the family, other than in a parent, may be disregarded where the applicant is perfectly sound. As the total observations under the head of ' Collaterals ' embrace 6,714 years of life and 128 deaths, I think it will be considered that some weight does attach to them." And, finally, he said : " As we may safely assume that ' Collaterals ', especially in child-birth cases, do not count, I have combined Table III—mother of child-birth, and Table VII—mother, child-birth ; sister, child-birth ; thus giving a total of 2,853 years of life and 69 deaths, the observations extending over an average of 12 years." That, he thought, fairly summarized the results of the present Paper.

Then the authors referred to some remarks which had been

made by several medical men on the hereditary theory of tuberculosis, and he thought they might have referred to what Dr. Lyon said in a supplementary paper to his own, in which he mentioned that particular point. He did not think that any very great advance had been made on that since, and it must be borne in mind that Dr. Lyon's remarks were made before Sir William Broadbent made his celebrated announcement that consumption was infectious and not hereditary, and that it was a question of environment more than a question of family history. Dr. Lyon said : " Consumption is now clearly made out to be caused by the entrance into the body, and growth there, of a specific germ—a microscopic rod called the tubercle bacillus. The disease germ is frequently found in the expectoration of consumptive patients, and there is little doubt that the disease is disseminated chiefly by means of dust formed by the drying of the expectoration of phthisical patients. The degree to which persons are subjected to the infection varies according to the circumstances in which they live. The susceptibility to the disease varies in different persons, and in the same person according to the condition of his health. It would also seem to vary according to his age. Though disputed by some, there is little doubt that persons specially susceptible to the disease are, as a rule, of delicate constitution. The susceptibility to phthisis, as in all zymotic diseases, is largely hereditary." That meant that susceptibility was hereditary, that the disease was not transmitted from the parent to the child, but that the child started with a somewhat weakened constitution from the lower vitality of the parent, and that seemed to be the general theory at the present time. Dr. Lyon made those remarks before any of the gentlemen referred to in the present Paper, and he thought the authors might have made reference to a medical man who had put those bold statements before the Institute itself.

He was very pleased indeed to see that the subject had been again taken up. He did not say it was desirable that actuaries should attempt to divide their risks into numerous classes ; he thought it was their business to take all classes if they possibly could on the same level as they had hitherto done, and if they thought there was something in the personal or family history which put a man outside that class perhaps some little extra should be charged. He quite agreed with Mr. Besant that minute classification should, as far as possible, be avoided ; at the same time he thought it was desirable that such investigations as those made by the authors should be carried out. It was impossible to tell what such investigations might lead to ; they might in some cases lead to important results, and for that reason he welcomed the Paper. The members, as practical men, would know what value to put upon the results in their official work.

Dr. S. W. CARRUTHERS (a visitor) said that the valuable transactions of the Institute sometimes affrighted medical men by the *p*'s and *q*'s and *x*'s they saw there, but the present essay was clear, even to a medical man, and he was sure it

would repay closer and longer study than he had been able to give it so far. He hoped it would be the means of reviving an interest in the subject, which he thought had been perhaps somewhat neglected in England, especially in recent years, and that it would be a forerunner of very much more work on the same lines. He also wished to say, parenthetically, that he admired the discretion and caution with which the authors had dealt with medical questions. He was specially impressed with the sentence at the top of page 511 : " This at least raises a suspicion that the assured persons in this group were not all total abstainers." He hoped the members would pardon a personal reminiscence in dealing with that point. In the previous week he had had a liver submitted to him which was manifestly in the early stages of alcoholic cirrhosis —there was no question about it, but he was told subsequently that it was the liver of a calf ! He, therefore, greatly appreciated the caution shown in the sentence he had quoted. The Paper contained far too much information for him to discuss it in detail : if it were divided into sections and discussed on three or four evenings justice might be done to it. He wished, therefore, to confine himself to four points, two of which were personal and two of which dealt with general principles.

The first personal point was a very small one, which he mentioned only for the sake of accuracy. The authors had made a sympathetic reference to the Paper which he had had the honour to read before the Institute. But his classification was not based on Bertillon's system. Bertillon's system was a classification of causes of death, and it was of no use whatever for producing a systematic classification of impairments : the two things were different. He worked quite independently. The classification was not that of Melville-Dewey ; it was merely Melville-Dewey's system of notation of numbering that was used. The second personal matter was that he could not pass the Paper by without a brief word of tribute to his esteemed friend, the late Dr. Light. He was a man who did not say much at meetings ; he was retiring in his disposition ; but whenever one conversed with him on the medical aspects of insurance one found what an accurate thinker Dr. Light was, and that he was a man of very suggestive views. He deeply regretted that Dr. Light did not live to see the results of the work to which he gave so much thought and so much practical wisdom. That brought him to his first general point. He believed that had Dr. Light been present he would have led the authors to modify their position in regard to the causes of death. Actuaries always spoke with a genial tolerance of doctors' enthusiasm for causes of death, and he would like them to change their attitude. The authors attributed the enthusiasm of doctors to their humanity as professional men. Personally he thought the enthusiasm of doctors was due, at least in those who were really keen on insurance work, to an intense utilitarianism. He hoped the members would pardon him for making a personal reference when dealing with that point. Some years ago he investigated the

mortality in overweights. He not only found a high mortality, but he found that among the causes of death in cases where the mortality was most excessive were accidents at younger ages; Bright's disease and alcohol at all ages; apoplexy, diabetes, cancer, ulceration of the stomach and bowels at later ages. His enthusiasm was strictly utilitarian. As Mr. Besant had pointed out, it must be remembered that no class of impaired lives was absolutely homogeneous. It was impossible to make an absolutely homogeneous class, but the examiner with knowledge of the family history and by intelligent anticipation would be able to exclude the heavier risks. He thought he was right in saying that that had not been fully appreciated by the authors, because, for instance, in describing cases of tuberculous family history they had reported the deaths from tuberculosis and from other causes, which were interesting so far, but for practical utilitarian purposes it was necessary to know what those other causes were, and which of them bulked most largely. That was more important for the doctor's selection, which ultimately contributed to the stability of the company, than to know how many of them died of tuberculosis. If Dr. Light had been alive he would perhaps have dealt a good deal with that point.

The Paper was a most valuable and instructive one, but the authors, from first to last, lamented the paucity of their data. That, it seemed to him, proved that no single company could investigate impaired lives to the best advantage; it brought him back to an old dream of his own—a dream of a small permanent Bureau of Joint Investigation, a Bureau with both an actuary and a doctor in it, a Bureau which had card records of every impaired life—not every rated-up life—in every company that took part in the work of the Bureau. One of the chief difficulties was that the minor impairments had been excluded, because they dealt with rated-up lives. The committee of control of such a Bureau would decide from time to time what points it would pay to investigate, and even the least common impairments would in a short time afford sufficient data for investigation. They would not have to be spread over a time so long that, as Mr. Lutt pointed out, the experience would vary owing to improvements in medicine and sanitation. Such a Bureau need not be large nor expensive. Insurance companies had in their books material such as no one else could get for the study of heredity, and they could make it more accurate with a little care. They could utilize the methods of Professor Pearson, and other scientific men, in such a way that it might, for instance, appeal to an intelligent millionaire to give an endowment to such a Bureau. In any case he believed before very long the Bureau would justify its existence, if it were once started, by the immense value of collective investigation along standard lines of impaired lives.

Mr. W. PALIN ELDERTON, in closing the discussion, said that apart from the results in the Paper there were two points in regard to it that left a most pleasant impression on his mind. One was

that a large insurance company had arranged for what must have been an expensive investigation, and had then placed the results at the service of other companies which were to some extent its rivals. The other was that the authors had cut out all refinements and had dealt with the matter on broad lines. They had not troubled about duplicates and such things in their actuarial investigation, and they had not tried to graduate their data. Data of this kind were much better studied in the form of the expected and actual deaths than in any other way. He agreed with Mr. Lutt as to the warning that ought to be added to those given by the authors with regard to the possible difference between what might be called the "Prudential M Table" and the "Prudential E<sup>M</sup> Table" and the O<sup>(M)</sup> and O<sup>(E,M)</sup> Tables respectively. Without a comparison of the data in the Paper with the "Prudential" Experience it was, strictly speaking, impossible to tell what effect family history, obesity, etc., had had on the mortality. That warning was perhaps particularly necessary in view of the note on page 453 where the authors made the statement : "One thing seems obvious : that in the assessment of risks there should be a differentiation in practice as between whole-life and endowment assurances." Apart from such a caution it seemed to him quite likely that self-selection might be important. It might possibly be much more important than medical selection. Turning to the miscellaneous results, the high mortality among the rheumatic fever cases, especially in the early years, might be capable of explanation from the point of view that if the second attack did not follow the first very soon, possibly the life became normal after a certain number of years. Office practice there might come in. If any office accepted cases very soon after the first attack it might be getting all the heavy mortality, and so compare badly with offices which excluded all cases which had not been, say, for five years without an attack. In that connection he was tempted to ask the authors whether there was any special rule in the particular office as to the time that must elapse after the first attack of fever. The other miscellaneous group might be described as the Habits Group—the group that included obesity, past intemperance, and gout. To complete any such comparison it was necessary to obtain a definition of obesity, and to know how far the office would have gone in passing an overweight, &c. But the results so far as they went confirmed, he thought, their dread of the obese and mistrust of the reformed intemperate.

Turning to what he supposed was the most important part of the Paper, namely, the section dealing with tuberculous family history, the conclusions seemed to him to confirm the view of the majority of offices that the older the age at entry the less important was family history, and that the early years of insurance would show the greater excess of mortality. A newer point was the accentuation of the advisability of scrutinizing causes of death of collaterals. How seldom one noticed a brother or sister who was actually suffering from consumption when a proposal was made! A warning ought to be given in connection with the result that it did not necessarily follow, even if family history was unimportant when the age at

entry was over 35, that one could take such lives over that age without an extra premium regardless of their present condition, *i.e.*, if they were otherwise border-line cases, it might then be advisable to rate them up. With regard to the explanation of the anomaly of the comparative light mortality of cases with very bad family histories it seemed to him that the authors' explanation of specially-careful selection explained those results. It was well-known that, even apart from what medical officers might say, an office was particularly careful if there were several weak points about a case.

He desired in passing to make a remark on the question of unknown family history. The authors said they thought as a rule this indicated tuberculosis. He agreed, but he thought it might be wise to add "and insanity." He thought it would be found in unknown family history that one or other of those diseases covered nearly all the cases. There were three tables in the Paper which he wished had not been printed. The first two were the Correlation Tables on pages 482-483; he did not see how such tables as these, constructed from life office data, could prove anything, but perhaps he had not quite appreciated what they were intended to convey. The other table which he regretted was one that Dr. Carruthers had also regretted for quite different reasons: the table on page 477. For nearly all assurance statistical work causes of death were of very little interest to him. So long as he knew whether the mortality was heavier or lighter than he expected he did not care what the assured died of, although he was afraid his point of view would perhaps seem very wrong to doctors.

An interesting section of the Paper dealt with Tuberculosis and the Medical Profession. There were two points that seemed to him to require comment in that section. The first was the suggestion from two medical men, whose names were mercifully withheld, that life assurance companies should adopt a tuberculin test before they accepted lives. He was afraid that if it ever came to such an extreme thing as that it would be preferable to give up medical examinations altogether. The other point was one which he thought might possibly lead to misunderstanding as to what one of the doctors who was quoted meant; he referred to the quotation given from Dr. Otto May's Paper, on page 485. He felt when he read it that people might consider that Dr. Otto May advocated that a number of questions should be asked which in practice could not be asked. He felt quite sure that Dr. Otto May was much too practical to intend this. It was the practice of the majority of offices to pay attention to the question of surroundings. They were careful, for instance, that the type of case they considered was a good type of its class. They did not, as a rule, get business from what might be called insanitary surroundings. In that way, and by the selection of agents, companies did to a large extent guard themselves against the worst type of surroundings. This, he thought, covered part of the dangers Dr. May had in mind. Dr. Carruthers had made a remark as to the possibility of using the statistics of life offices for the study of heredity, and it was perhaps worth while to indicate a difficulty in interpreting results in this connection. If, in working

out the mortality of assured persons whose parents had died of consumption, it was found that they had a lighter mortality than those whose parents had not died of consumption, would they be right in concluding that it was better, from a mortality point of view, that the parents should die of that disease? He was putting an extreme case, in order to make it more obvious. Of course, they would not. The result would not prove that consumption was not inherited in some way or other. It might merely mean that, by medical examination or some other process of selection, it had been possible to pick out the particular cases that had escaped any inherited tendency to the disease.

He thought the Paper was a great step in the direction in which actuaries ought to go. He could quite understand that there were two distinct camps in the profession—the one that felt it was their business to pursue their investigations as far as possible, and the other which felt that they had got on successfully in the past, that the data would be small in quantity, and that it was not worth the trouble and expense of carrying out any further investigations. It might be thought by some of the members, owing to the authors having dealt with the subject on broad lines, that this latter view was accentuated. He did not think that was the case, as the authors themselves said they had merely dealt in a broad way with the subject and had left other people to fill in the details. If such subjects were to be investigated in the future, the problems they desired to solve should be carefully thought out, and the data should be obtained in such a form that those problems could be most usefully and simply attacked. It would not then be necessary to make the best they could of the data available; in other words, they would avoid to some extent the natural handicap of the ready-made statistics the authors had to use. It had, however, been shown that investigation was easily possible, and an indication had been given of the broad lines that such an investigation should follow. The next step, it seemed to him, was to fill in the details, and to replace their ignorance by the knowledge which could only be acquired by statistical investigation.

The PRESIDENT, in proposing a hearty vote of thanks to the authors for their excellent work, said he thought the Paper brought out very prominently two points, the first being the value of self-selection. It showed that a certain proportion of really damaged lives did manage to pass the doctor for the purpose of whole-life assurance. It also showed, on the other hand, that people who took out endowment assurance policies, and thus might be said to back themselves to live, were on the whole justified in their self-selection. He desired to refer to a point which Mr. Manly had already touched upon in regard to Mr. Besant's interesting remarks. Mr. Besant said he thought it would be extremely useful if the cases where there was a consumptive family history plus personal blemish or delicate constitution could be followed; but that was an impossibility, because such cases were declined altogether; the company did not know their subsequent history. The other point which had been brought out was that perhaps too much weight had been given to

family history in the past, and too little weight to personal blemishes which might perhaps have been considered of a trivial nature. Only those who had been concerned in an investigation such as that conducted by the authors could realize the enormous amount of thought and time that had to be put into the Paper. It literally had meant years of work for the authors—more particularly for Mr. Rusher—and he was quite certain the meeting would pass a most hearty vote of thanks to them for the excellent work they had done.

The resolution was put and carried with acclamation.

Mr. C. W. KENCHINGTON in reply, said there were a few points in the discussion on which he desired to comment. In the first place, Mr. Lutt said he assumed that the lives were rated-up lives. That was so ; all the lives were rated-up lives, the method of treatment being that of a decreasing contingent debt. Mr. Lutt also referred to the question of varying age constitution, and pointed out that there was scope for variation within the age groups -37, 38-52, and 53-62, for which the percentages were given. The authors referred to that very point at the top of page 453, where they made the statement : “ Owing to the variations in age distribution this warning against the use of percentages of totals is also required in a modified degree in making comparisons between the sub-groups for the same class of assurance.” The authors would not have attached so much importance to the phenomenally light rate of mortality where there was family history of consumption in lineals plus collaterals if it had not been that the results were so consistent. For instance, Mr. Lutt had referred to Table XX on page 460, and had confined his comparison to the percentage of 78·6 with the percentages of 80·8 and 84·5. If reference was made to the figures given in that table it would be seen that the percentages under Section  $\beta$  were in every instance lower than the corresponding percentages in either of the other Sections,  $\alpha$  or  $\gamma$ .

That led him to make an observation in regard to the tables referred to in the important remarks made by Mr. Besant. Comparing the figure which Mr. Besant quoted from column 10, of Table XI, with the corresponding figure in Table XXVII, in the section A + E, it would be seen that the percentage in the first case was 129, whereas in the latter it was 181·3. The classification was made entirely by Dr. Light. Those cases which came within the group  $A_0$  to  $A_{12}$  were classified as sub-normal for family history of consumption only. Those which combined tuberculous family history with some personal defect were included in the group A + E for which figures were given in Tables XXVII, XXVIII and XXIX, and the remark which Mr. Besant made was emphasized by the figures in those tables. Mr. Elderton had referred to the Correlation Tables given in the Paper. The idea in his own mind particularly was that in investigating family histories from the point of view of insurance risks, was it not universal experience that, perhaps unknowingly, attention was paid to the age at death of the member of the family concerned ? The tables, it seemed to him, showed that the age at death of the affected relative was of very little

consequence indeed. Apart from that, he did not think they were of any value.

Mr. E. A. RUSHER said that the President, in the kind words he spoke in proposing the vote of thanks, summed up very aptly the general results of the investigation. He thought the two outstanding features of the investigation were the enormous adverse force of self-selection, and the possibility of attaching too much weight to family history. With regard to the discussion generally, he thought the fact had been overlooked that there was a class of consumptive risks dealt with by the authors which took into account the personal condition as well as the family history—the class A + E ; and those were all the facts they were able to furnish on that point. Mr. Kenchington had already given an illustration showing how very much greater the mortality was under the combined group of consumptive family history and defective constitution as compared with family history alone. In reference to Mr. Lutt's remarks, he wished to point out that Mr. Hardy's suggested classification at the Vienna Congress was made too late for the authors to avail themselves of it, because it was not given until some years after the investigation had been put into some sort of shape. That led him to say that when they started the investigation they were absolutely in the dark ; they had no idea whatever as to how many facts they would get under each of the various classes of risk ; and it was not until about two years ago that they began to see where they were being led, what they would have to sacrifice, and what was likely to be useful to them. He desired to say one other thing generally about the Paper. In preference to going into any minor points the authors had endeavoured to give the broad facts, so that the members could judge for themselves and draw their own conclusions. The conclusions from the facts, he admitted, might be drawn differently by different people, and it was quite possible that in certain circumstances opposing opinions might legitimately be deduced from the facts before them. A question had been raised as to the experience of the general body of lives as compared with those dealt with in the Paper. He thought he could say that, so far as whole-life experience was concerned, the aggregate experience was about equal to the O<sup>M</sup> Aggregate Table—perhaps slightly better. Under endowment insurances there was every indication that the mortality was somewhat better than the O<sup>(EM)</sup>. He found it exceedingly useful in dealing with the various tables in the Paper whenever he came across endowment assurances to consider the normal as 75 instead of 100, and if the members adopted that principle in reading the Paper they would find it easier to draw conclusions, and would be sufficiently near the truth. Mr. Lutt referred to the "obesity" group and spoke about the necessity of a standard Table of Weight. He thought it had been made clear in the Paper that it was quite impossible to give a standard because the various circumstances surrounding each individual case were taken into account in accepting those lives. They were all lives which were rated up on account of excessive weight.

With reference to Mr. Manly's remarks, he was very glad that the results of the present Paper had testified to Mr. Manly's foresight and to the skill with which he drew from quite small statistics results which had been proved by the larger statistics now brought forward to be so closely in accordance with the facts. The reason Dr. Glover Lyon's Supplementary Paper was not quoted in dealing with the various medical opinions as to the heredity or otherwise of tuberculosis was that they were anxious to quote only opinions that had been quite recently expressed. That Paper was written over 20 years ago and during the intervening period medical opinion had very much altered on the subject, and the authors therefore thought it advisable to give quotations on both sides of the subject from quite recent writings. He was very glad to hear that Dr. Carruthers found the Paper clear, even to the medical mind, because he was afraid it was not quite clear even to his (Mr. Rusher's) mind : there were many points that he would like to see his way through. Dr. Carruthers had expressed a wish for a more detailed statement of the causes of death as being very useful to the medical profession, but the object they had in view in investigating the causes of death was to see if there was anything in the question of heredity, for which purpose the analysis given was sufficient and seemed to him to show that there was a distinct increase in the deaths from tuberculosis, where there was a family history of tuberculosis. Mr. Elderton, in referring to the question of rheumatic fever, pointed out that if the second attack did not soon follow the first the lives might possibly become normal and be treated as such. As emphasizing this he might say that in the previous week his attention had been called to the death of a lady at the age of 77 who had had six attacks of rheumatic fever, the first of which occurred before she was married. He might add that she was under 30 at marriage. The only other point to which he wished to refer was Mr. Elderton's remark as to his mistrust of the reformed intemperate. Personally he would put that in a little kinder way, and say "mistrust of the reformed intemperate who sought life assurance."

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*The Medico-Actuarial Investigation of the Mortality of American  
and Canadian Life Assurance Companies.*

[Communicated by MR. ARTHUR HUNTER, A.I.A., F.F.A., Chairman of the Central Bureau.]

**I**N Volume III.\* of their Report, the Joint Committee of the Medico-Actuarial Mortality Investigation deal with the effect of occupation on mortality.

\* Abstracts of the contents of the two preceding Volumes of the Report will be found in Vol. xlii of this *Journal*, pp. 384-394 and at pp. 414-421 of the current Volume.

When the plans were being prepared for the Investigation the Committee requested suggestions from the actuaries, medical directors, and other officers of the forty-three companies invited to contribute their statistics. There were fully 600 suggestions made as to the occupations which it would be desirable to investigate. From these the Committee selected 168, which were divided into two sets, one comprising the occupations suggested by a number of the companies, and the other, those for which there was little demand, or for which it was not believed that a sufficient amount of data existed at the time the investigation was commenced. The former were designated "Required," and the latter "Optional" occupations, for the reason that the companies were all required to give their experience in the former class, while it was optional in the latter class.

The occupation was taken as of the date of application for the insurance, and no subsequent changes were recorded. This was considered to be the practical course in view of the different customs of the companies. For example, one company charged extra premiums for hazardous occupations, and did not remove the extras on changes to favourable occupations until one year after the insured had left the former and were proved by medical examination to be first-class risks. Another company charged a lien against the policy in certain occupations, and removed it without evidence of insurability after the insured had been a sufficient length of time in a non-hazardous occupation to justify the expectation that he would not again enter his original, or any hazardous occupation.

A number of the occupations were only slightly hazardous, and, accordingly, the companies accepted applicants at the regular rate of premium, using more care in other respects than if the occupation were non-hazardous. In other cases an extra premium was charged, the policy placed in a special dividend class, limited to a high premium plan, or issued subject to a lien against the insurance. These cases were all included in the investigation, provided the reason for such treatment was the occupation alone. If the insured were overweight or underweight, or had a poor family history, or had a minor medical impairment, the case was included provided these factors were not in themselves of such moment as to make the insured a

sub-standard risk. The Committee states that there was no reason to assume that the occupation classes thus constituted included a larger proportion of risks with medical impairments than entered into the data upon which the standard table of mortality (Vol. I, p. 89) was based. In order to confirm this impression an investigation was made into the effect of including these minor impairments, and a synopsis of the results indicates that the effect is to raise the mortality to a slight extent only.

When there was a considerable departure from the normal proportion of overweights in any class, an investigation was made to determine the effect. As the effect of underweight is much less than that of overweight, it was deemed necessary to investigate only one class in which the proportion of underweights was larger than the average.

Attention is drawn to the fact that in many of the classes the medical selection was more severe than among policy-holders whose occupations were not hazardous. It is also pointed out that in certain of the wage-earning occupations the risks accepted by the companies were chiefly drawn from the more favourably placed workers. In some cases, therefore, the ratio of actual to expected mortality may be lower than 100 per-cent, although there may be a distinct hazard from occupation; or the ratio may be above 100 per-cent, and such excess not measure the full hazard from occupation.

For purposes of comparison, tables of the principal causes of death were prepared, based on policies issued to men at the regular rates of premium in January of the odd years and July of the even years 1885 to 1908 inclusive (Vol. II, p. 30). The corresponding tables on women were based on the entire experience of forty-one companies during the same years (Vol. II, p. 52). The comparisons in connection with the occupations are on the basis of the ratio of deaths from each cause to 10,000 exposed to risk. The tables of deaths in Vol. II will be referred to as the standard or normal.

A brief summary will now be given of some of the principal classes, the expected deaths being calculated by the M.A. Table, which represents the average mortality of the forty-three companies in the investigation by policy years and by grouped ages at entry.

## ARMY AND NAVY COMMISSIONED OFFICERS.

The following two classes cover the Commissioned Officers in the Army and Navy, excluding Chaplains, Surgeons, Physicians, and Paymasters :

	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Army ... ... ... ... ...	324	247.32	131%
Navy ... ... ... ... ...	149	97.79	152%

The division of the data according to years of issue 1885 to 1899, and 1900 to 1908, shows a marked improvement in the latter period over the former, the mortality in the earlier period being undoubtedly affected by the Spanish American War.

The death rate from accident and also from suicide was three times the normal among the Army Officers, and twice the normal among Navy Officers.

## SUBJECT TO HAZARD FROM ELECTRICITY.

	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Superintendents, Managers, and Chief Engineers of electric light, heat, and power systems ... ...	180	193.80	93%
Electrical Engineers who handle live wires ... ... ...	84	79.82	105%
Stationary Engineers and Firemen connected with electric light, heat, and power systems ...	78	84.76	92%
Linemen (pole climbers) and arc light trimmers ... ... ...	71	50.05	142%

In each of the first two mentioned classes the deaths from electricity were 3 per-cent of the total. Among the stationary engineers the deaths from accident were fully three times as high as the standard, while among the linemen and arc light trimmers, such deaths were five times the standard.

## FIRE DEPARTMENT (CITY).

	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Lieutenants, Captains, Chiefs, and Assistant Chiefs ... ... ...	51	42·00	121%
Firemen, Laddermen, Pipemen, and Hosemen ... ... ...	155	104·55	148%
Drivers, Engineers, Stokers, and Truckmen ... ... ...	54	43·40	124%

Among the Officers, and also among the Firemen, Laddermen, &c., the mortality from accident was five times the normal, while among the Drivers, Engineers, &c., it was twice the standard.

## GLASS WORKERS.

Two classes of glass workers were selected—namely, (a) glass blowers not using machinery, and (b) bevellers, grinders, engravers, and cutters of glass. The foremen and superintendents were excluded from both classes.

	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Glass Blowers ... ... ...	70	57·90	121%
Bevellers, Grinders, &c. ... ... ...	77	52·60	146%

In both these classes the mortality from tuberculosis of the lungs was twice the standard.

## LIQUOR TRADE.

		Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
<b>HOTELS WITH BAR :</b>				
Proprietors, Superintendents, and Managers :				
Not attending bar ... ...	529	391.66	135%	
Attending bar either occasionally or regularly... ...	519	291.94	178%	
<b>SALOONS, BILLIARD AND POOL ROOMS, BOWLING ALLEYS (where there is a bar) :</b>				
Proprietors and Managers :				
Not attending bar ... ...	222	122.23	182%	
Attending bar either occasionally or regularly... ...	830	478.75	173%	
<b>RESTAURANTS WITH BAR :</b>				
Proprietors, Superintendents, and Managers not attending bar ...	105	69.09	152%	
<b>GROCERIES WITH BAR :</b>				
Proprietors ... ... ...	39	23.76	164%	
<b>BREWERIES AND DISTILLERIES :</b>				
Proprietors, Managers, and Superintendents of Distilleries ...	119	140.17	85%	
Proprietors, Managers, and Superintendents of Breweries ...	483	358.99	135%	
Clerks in Breweries ... ...	112	86.35	130%	
Foremen, Maltsters, Beer-pump Repairers, Journeymen in Breweries ... ... ...	145	95.13	152%	
<b>WHOLESALE LIQUOR HOUSES :</b>				
Proprietors and Managers ...	992	810.67	122%	
Clerks ... ... ...	138	123.06	112%	
<b>TRAVELLING SALESMEN AND COLLECTORS :</b>				
For distilleries, breweries, and wholesale liquor houses ...	179	139.89	128%	
<b>WAITERS :</b>				
In hotels, restaurants, and clubs, where liquor is sold ... ...	83	46.83	177%	

The mortality is high among those who attend bar, whether connected with hotels, restaurants, groceries, or saloons. The Proprietors, Superintendents, and Managers of hotels, who do not attend bar are probably connected with larger and better hotels than the corresponding class who attend bar either occasionally or regularly. The slight difference in the mortality between Saloon-keepers who do not admit attending bar and those who do, is noteworthy.

The death rate of those connected with the sale of liquor is very high from cirrhosis of the liver, cerebral haemorrhage, apoplexy, diabetes, and Bright's disease.

## MARINE SERVICE.

There were four classes of Officers and Engineers on steam vessels, as follows (the Great Lakes being Lakes Michigan, Huron, Erie, Ontario, and Superior):—

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Officers and Engineers on Ocean Steamers ... ... ...	103	66·07	156%
Officers and Engineers on Steamers in Coast-wise trade, excluding those travelling to the Tropics ...	34	24·69	138%
Officers and Engineers on Steamers on Great Lakes ... ...	95	80·80	118%
Officers on Steamers on rivers, lakes, sounds, and harbours, excluding the Great Lakes ... ...	75	70·41	107%

The mortality among those engaged on rivers, lakes, sounds, and harbours is very slightly above the normal, although the death rate from accident was three times the standard. Those engaged on the Great Lakes had four times the normal, and those engaged in the coast-wise trade and on ocean steamers had five times the normal death rate from accident.

## MINING.

The conditions surrounding the mining industry are distinctly different in the United States from those in Britain, and, accordingly, a brief statement only of the important results is given. The relative mortality among working Miners in underground mines, exclusive of coal mines, was 226 per-cent, the death rate from accident being seven times the standard. Even among the Mining Engineers, Superintendents, and Managers who occasionally go underground, the mortality was 135 per-cent, the death rate from accident among these Supervisors being four times the standard. Among the working Miners of surface mines (placer, drift, hydraulic, &c.), the mortality was more than double the M.A. Table, and of the total deaths 70 per-cent were from accident, pneumonia, and tuberculosis.

## POLICE, WARDENS, JAILERS, &amp;c.

—	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
City Policemen ... ... ...	526	377·67	139%
Marshals, Sheriffs, and Constables ...	475	355·07	134%
Jailers, Wardens, and Prison Guards...	75	68·65	109%

(The duties of a constable in the United States are practically the same as those of a sheriff, in that he serves writs, executes warrants, and makes arrests on warrants.)

The death rate from accident in each of the foregoing classes was about twice the standard.

#### RAILROAD SERVICE.

The accident rate is much higher in the United States than in Great Britain in connection with railroad service, and the mortality in the various classes investigated reflects this condition :

	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Railway Locomotive Engineers ...	541	337.38	160%
Railway Locomotive Firemen ...	193	101.84	190%
Railway Passenger Trainmen (not conductors) ... ... ...	76	55.52	137%
Check Clerks, Freight Inspectors, Car Inspectors, Car Sealers, Yard Clerks, and Yard Masters ...	221	156.92	141%
Track Supervisors, Foremen, and Section Foremen ... ... ...	421	333.07	126%

Among the Engineers the death rate from accident was eight times, and among Firemen nine times the normal. Among the Check Clerks, Freight Inspectors, &c., whose work takes them more or less into the yard, the death rate from accident was four times the normal ; and among the Track Supervisors and Foremen, three times the normal.

#### THEATRICAL PROFESSION (MEN).

	Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Actors, including vaudeville performers, and excluding acrobats and circus performers ... ...	87	60.15	145%
Proprietors, managers, and treasurers of theatres, music halls, and vaudeville houses... ... ...	153	112.66	136%

The death rate from tuberculosis of the lungs, typhoid fever, pneumonia, cirrhosis of the liver, and diseases of the nervous system was much higher than the normal.

## ARTISANS, MECHANICS, FACTORY OPERATIVES, &amp;c.

There are a number of groups which come under the heading of "Skilled Workmen," such as :

		Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
<b>Barbers and Hairdressers—</b>				
Journeymen	...	130	119·11	109%
Bricklayers	...	82	76·13	108%
Cigar Makers	...	43	39·76	108%
Cotton Factory Operatives	...	46	42·50	108%
Firemen—Stationary	...	33	29·89	110%
Millers—Journeymen	...	134	126·61	106%
House Painters—Journeymen	...	132	119·42	111%
<b>Plumbers and Steam Fitters—</b>				
Journeymen	...	157	158·46	99%
<b>PRINTING—Compositors—</b>				
Journeymen	...	68	66·44	102%
Pressmen—Journeymen	...	70	59·74	117%
Retail Butchers—Journeymen	...	225	235·23	96%
Shoe Factory Operatives	...	91	90·29	101%
Tailors—Journeymen	...	193	195·48	99%
Woollen Mill Operatives	...	56	49·77	113%

In considering the mortality in the above occupations it should be borne in mind that the companies received applications only from the better paid workmen, the other grades generally taking industrial insurance.

## MISCELLANEOUS.

		Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Blacksmiths	...	250	309·76	81%
<b>BLAST FURNACES :</b>				
Foremen and Working Superintendents	...	37	33·72	110%
Forgemen, Foundrymen & Moulders	...	593	501·77	118%
Cooks and Chefs (Men) in hotels and restaurants	...	156	102·61	152%
Hatters — Journeymen, excluding makers of straw hats	...	84	62·74	134%
Life Saving Corps—Officers and Men	...	30	31·17	96%
Livery Stable Proprietors	...	274	216·84	126%
Stone Cutters—Journeymen	...	76	35·45	214%

The favourable mortality among the Blacksmiths was probably due to the robust type of men required for this occupation, as well as to the fact that they work in airy shops or out of doors, generally in rural communities.

The death rate among the Stone Cutters from tuberculosis was six times the standard.

### WOMEN.

			Actual Deaths	Expected Deaths	Ratio of Actual to Expected Deaths
Trained Nurses...	...	...	317	392·12	81%
Domestic Servants	...	...	188	148·18	127%

Among the spinsters insured by the forty-three companies the ratio of actual to expected deaths was 81 per-cent; among married women, 119 per-cent; and among all women, 104 per-cent (Vol. II, p. 36).

In some of the classes the results are not applicable to Great Britain, but in others they should be of value in the selection of risks.

### NATIONAL INSURANCE ACT, 1911.

*Life Tables, based upon the Estimated Population of England and Wales on 30 June 1909 and the Deaths registered in the three years 1908-1910, and adopted in computing Reserve Values for purposes of the Act.*

[Extracted from the Report for 1912-13, on the Administration of the National Insurance Act, Part I (Health Insurance) [Cd. 607], Appendices V and VI, pp. 577-601; \* by permission of the Controller of H.M. Stationery Office].

\* These Appendices also contain Rates of (1) Marriage and Widowhood, (2) Sickness and Disablement, and (3) Issue, with explanatory memoranda. In the two preceding Appendices will be found the Reports of the Actuarial Advisory Committee on Reserve Values, and on Reinsurance of Maternity Benefit.

TABLE I.

*Life Table. Males.*

Age	$l_x$	$d_x$	$p_x$	$q_x$	$\bar{e}_x$
16	99,708	300	.99699	.00301	47·10
17	99,408	309	.99689	.00311	46·24
18	99,099	319	.99678	.00322	45·38
19	98,780	330	.99666	.00334	44·53
20	98,450	341	.99654	.00346	43·68
21	98,109	351	.99642	.00358	42·82
22	97,758	363	.99629	.00371	41·98
23	97,395	375	.99615	.00385	41·13
24	97,020	388	.99600	.00400	40·29
25	96,632	401	.99585	.00415	39·45
26	96,231	416	.99568	.00432	38·61
27	95,815	431	.99550	.00450	37·78
28	95,384	447	.99531	.00469	36·94
29	94,937	465	.99510	.00490	36·12
30	94,472	483	.99489	.00511	35·29
31	93,989	502	.99466	.00534	34·47
32	93,487	523	.99441	.00559	33·65
33	92,964	545	.99414	.00586	32·84
34	92,419	567	.99386	.00614	32·03
35	91,852	592	.99356	.00644	31·22
36	91,260	618	.99323	.00677	30·42
37	90,642	646	.99287	.00713	29·63
38	89,996	676	.99249	.00751	28·84
39	89,320	707	.99208	.00792	28·05
40	88,613	742	.99163	.00837	27·27
41	87,871	778	.99115	.00885	26·50
42	87,093	816	.99063	.00937	25·73
43	86,277	857	.99007	.00993	24·97
44	85,420	900	.98946	.01054	24·21
45	84,520	947	.98880	.01120	23·47
46	83,573	995	.98809	.01191	22·73
47	82,578	1,047	.98732	.01268	21·99
48	81,531	1,102	.98648	.01352	21·27
49	80,429	1,161	.98556	.01444	20·56
50	79,268	1,223	.98457	.01543	19·85
51	78,045	1,289	.98349	.01651	19·15
52	76,756	1,358	.98231	.01769	18·46
53	75,398	1,430	.98103	.01897	17·79
54	73,968	1,506	.97964	.02036	17·12
55	72,462	1,585	.97813	.02187	16·47
56	70,877	1,666	.97649	.02351	15·82
57	69,211	1,751	.97470	.02530	15·19
58	67,460	1,838	.97276	.02724	14·58
59	65,622	1,927	.97064	.02936	13·97

TABLE I—*continued.*  
*Life Table. Males.*

Age	$l_x$	$d_x$	$p_x$	$q_x$	$\hat{e}_x$
60	63,695	2,017	.96834	.03166	13.38
61	61,678	2,107	.96584	.03416	12.80
62	59,571	2,196	.96313	.03687	12.23
63	57,375	2,285	.96018	.03982	11.68
64	55,090	2,369	.95699	.04301	11.15
65	52,721	2,450	.95352	.04648	10.62
66	50,271	2,526	.94976	.05024	10.12
67	47,745	2,593	.94569	.05431	9.63
68	45,152	2,651	.94129	.05871	9.15
69	42,501	2,698	.93653	.06347	8.69
70	39,803	2,731	.93138	.06862	8.25
71	37,072	2,750	.92582	.07418	7.82
72	34,322	2,752	.91982	.08018	7.40
73	31,570	2,736	.91335	.08665	7.00
74	28,834	2,700	.90637	.09363	6.62
75	26,134	2,643	.89886	.10114	6.25
76	23,491	2,566	.89078	.10922	5.90
77	20,925	2,467	.88210	.11790	5.56
78	18,458	2,349	.87276	.12724	5.24
79	16,109	2,211	.86273	.13727	4.93
80	13,898	2,057	.85197	.14803	4.64
81	11,841	1,889.5	.84043	.15957	4.36
82	9,951.5	1,711.1	.82806	.17194	4.09
83	8,240.4	1,526.1	.81480	.18520	3.83
84	6,714.3	1,338.8	.80060	.19940	3.59
85	5,375.5	1,153.5	.78541	.21459	3.36
86	4,222.0	974.6	.76916	.23084	3.14
87	3,247.4	806.1	.75178	.24822	2.93
88	2,441.3	651.3	.73320	.26680	2.74
89	1,790.0	513.1	.71336	.28664	2.55
90	1,276.9	393.04	.69219	.30781	2.38
91	883.86	292.02	.66961	.33039	2.21
92	591.84	209.78	.64554	.35446	2.05
93	382.06	145.22	.61989	.38011	1.90
94	236.84	96.49	.59258	.40742	1.76
95	140.35	61.259	.56353	.43647	1.63
96	79.091	36.962	.53266	.46734	1.51
97	42.129	21.069	.49989	.50011	1.39
98	21.060	11.264	.46515	.53485	1.28
99	9.796	5.600	.42837	.57163	1.18
100	4.196	2.562	.38950	.61050	1.08
101	1.634	1.065	.34850	.65150	.99
102	.569	.395	.30534	.69466	.91
103	.174	.129	.26000	.74000	.83
104	.045	.035	.21246	.78754	.77
105	.010	.008	.16270	.83730	.70
106	.002	.002	.11071	.88929	.50

TABLE I—*continued.**Life Table. Females.*

Age	$l_x$	$d_x$	$p_x$	$q_x$	$\hat{e}_x$
16	99,745	261	.99738	.00262	49.83
17	99,484	269	.99730	.00270	48.96
18	99,215	276	.99722	.00278	48.09
19	98,939	284	.99713	.00287	47.23
20	98,655	293	.99703	.00297	46.36
21	98,362	302	.99693	.00307	45.50
22	98,060	312	.99682	.00318	44.64
23	97,748	323	.99670	.00330	43.78
24	97,425	334	.99657	.00343	42.92
25	97,091	346	.99644	.00356	42.07
26	96,745	358	.99630	.00370	41.22
27	96,387	371	.99615	.00385	40.37
28	96,016	386	.99598	.00402	39.52
29	95,630	401	.99581	.00419	38.68
30	95,229	416	.99563	.00437	37.84
31	94,813	432	.99544	.00456	37.00
32	94,381	450	.99523	.00477	36.17
33	93,931	469	.99501	.00499	35.34
34	93,462	488	.99478	.00522	34.52
35	92,974	508	.99454	.00546	33.69
36	92,466	529	.99428	.00572	32.88
37	91,937	551	.99401	.00599	32.06
38	91,386	574	.99372	.00628	31.25
39	90,812	598	.99342	.00658	30.45
40	90,214	622	.99310	.00690	29.65
41	89,592	648	.99277	.00723	28.85
42	88,944	675	.99241	.00759	28.06
43	88,269	704	.99203	.00797	27.26
44	87,565	734	.99162	.00838	26.48
45	86,831	766	.99118	.00882	25.70
46	86,065	800	.99071	.00929	24.92
47	85,265	836	.99019	.00981	24.15
48	84,429	876	.98963	.01037	23.39
49	83,553	918	.98901	.01099	22.63
50	82,635	964	.98833	.01167	21.87
51	81,671	1,015	.98757	.01243	21.13
52	80,656	1,070	.98673	.01327	20.39
53	79,586	1,131	.98579	.01421	19.65
54	78,455	1,197	.98474	.01526	18.93
55	77,258	1,269	.98357	.01643	18.22
56	75,989	1,347	.98227	.01773	17.51
57	74,642	1,431	.98083	.01917	16.82
58	73,211	1,521	.97923	.02077	16.14
59	71,690	1,617	.97745	.02255	15.47

TABLE I—*continued.*  
*Life Table. Females.*

Age	$l_x$	$d_x$	$p_x$	$q_x$	$\bar{r}_x$
60	70,073	1,717	.97549	.02451	14·81
61	68,356	1,822	.97334	.02666	14·17
62	66,534	1,930	.97099	.02901	13·55
63	64,604	2,041	.96841	.03159	12·94
64	62,563	2,152	.96560	.03440	12·34
65	60,411	2,263	.96254	.03746	11·76
66	58,148	2,371	.95922	.04078	11·20
67	55,777	2,476	.95560	.04440	10·66
68	53,301	2,576	.95167	.04833	10·13
69	50,725	2,667	.94742	.05258	9·62
70	48,058	2,748	.94282	.05718	9·12
71	45,310	2,817	.93783	.06217	8·65
72	42,493	2,871	.93243	.06757	8·19
73	39,622	2,908	.92660	.07340	7·74
74	36,714	2,926	.92030	.07970	7·32
75	33,788	2,923	.91349	.08651	6·91
76	30,865	2,897	.90614	.09386	6·52
77	27,968	2,847	.89822	.10178	6·14
78	25,121	2,771	.88968	.11032	5·78
79	22,350	2,671	.88047	.11953	5·43
80	19,679	2,548	.87054	.12946	5·10
81	17,131	2,401	.85983	.14017	4·79
82	14,730	2,235	.84829	.15171	4·49
83	12,495	2,051	.83585	.16415	4·20
84	10,444	1,854·3	.82245	.17755	3·93
85	8,589·7	1,649·2	.80800	.19200	3·67
86	6,940·5	1,440·8	.79241	.20759	3·42
87	5,499·7	1,234·2	.77559	.22441	3·18
88	4,265·5	1,034·6	.75744	.24256	2·96
89	3,230·9	846·9	.73786	.26214	2·75
90	2,384·0	675·3	.71672	.28328	2·55
91	1,708·7	523·1	.69388	.30612	2·36
92	1,185·6	392·20	.66920	.33080	2·18
93	793·40	283·62	.64253	.35747	2·01
94	509·78	196·92	.61371	.38629	1·85
95	312·86	130·60	.58257	.41743	1·70
96	182·26	82·21	.54894	.45106	1·55
97	100·05	48·761	.51263	.48737	1·42
98	51·289	27·006	.47345	.52655	1·29
99	24·283	13·812	.43122	.56878	1·17
100	10·471	6·431	.38579	.61421	1·06
101	4·040	2·678	.33703	.66297	.96
102	1·362	.974	.28491	.71509	.86
103	.388	.299	.22960	.77040	.77
104	.089	.074	.17164	.82836	.69
105	.015	.013	.11222	.88778	.63
106	.002	.002	.05363	.94637	.50

MEMORANDUM ON THE BASIS AND GRADUATION OF THE MORTALITY TABLES.

BASIS.

It was decided to employ the data for England and Wales rather than those for the United Kingdom, as the former appeared on the whole to be more complete and trustworthy.

*Deaths.*

The latest year for which details of the number of registered deaths were obtainable was 1910, but as it appeared inadvisable to make use of the figures of one year only, it was decided to base the mortality tables upon the deaths registered in the three years 1908–1910, in decennial groups of ages, of which the numbers are given in the tables appended.

*Population.*

In order to obtain an estimate of the population living in the various age groups corresponding to the deaths for the three years above mentioned, the population as at 31 March 1911 (the census figures not being available) was first computed by the method described in §§ 10–14 of Messrs. Hardy and Wyatt's report (Cd. 5,681). The numbers living on 30 June 1909, the middle point of the three years 1908–1910, were then obtained by an arithmetical interpolation between the census figures for 1901 and the estimated numbers for 31 March 1911. Thus for each decennial age group the population on 30 June 1909, was taken as equal to

$$\text{Population } 1901 + \cdot825 \quad (\text{Population } 1911 - \text{Population, 1901}).$$

The resulting numbers are shown in the appended Tables.

GRADUATION.

From the estimated population and the registered deaths in the decennial age groups 15–25, 25–35, &c., and in the group 85 and over, the numbers living and the deaths were obtained for each age by a process of graduation.

The functions operated upon were  $\Sigma L_x$  and  $\Sigma l_x$ , respectively, being the total Population and the total Deaths at and above age  $x$ , the numbers being graduated by the formulas described later.

The population figures having first been graduated, the next step, normally, would have been to introduce a correction into the ungraduated number of deaths in each group proportionate to the changes introduced by the process of graduation into the number representing the population in the corresponding group, thus preserving the original central death rate in the various age groups. In the Female Table this was done, but in the Male Table, as the deviations were small, this process was dispensed with.

The Deaths were graduated by formulas similar to those used

for the Populations, with certain modifications in the case of the Female Table.

Finally the values of  $q_x$  were calculated by the formula

$$d_x \div (\bar{L}_x + \frac{1}{2}d_x)$$

In the following explanations a detailed description is given of the processes employed in obtaining the Male Population Table. It will be noticed that the whole of the data was used in the application of the formula which was employed.

### Males—Population.

After some experiments as to the most suitable formula to employ for the purpose, the function  $\Sigma L_x$  was finally graduated by a curve of the form

$$\log \Sigma L_x = a + br + c \frac{x \cdot x - 1}{2} + mr^x = A + Bx + Cx^2 + mr^x$$

In the actual work as the data were arranged in decennial age groups the equations employed to determine the constants were as follows :—

$$\log \Sigma L_{15} = u_0 = a + m$$

$$\log \Sigma L_{25} = u_1 = a + b + mr$$

$$\log \Sigma L_{35} = u_2 = a + 2b + c + mr^2$$

$$\begin{matrix} \vdots & \vdots \\ \vdots & \vdots \end{matrix}$$

$$\log \Sigma L_{85} = u_7 = a + 7b + 21c + mr^7$$

Differencing alternate terms throughout

1st

2nd

3rd

$$2b + c + m(r^2 - 1)$$

$$2b + 3c + mr(r^2 - 1) \quad 4c + m(r^2 - 1)^2$$

$$2b + 5c + mr^2(r^2 - 1) \quad 4c + mr(r^2 - 1)^2 \quad m(r^2 - 1)^3$$

$$2b + 7c + mr^3(r^2 - 1) \quad 4c + mr^2(r^2 - 1)^2 \quad mr(r^2 - 1)^3$$

$$2b + 9c + mr^4(r^2 - 1) \quad 4c + mr^3(r^2 - 1)^2$$

$$2b + 11c + mr^5(r^2 - 1)$$

$r \left( = \frac{mr(r^2 - 1)^3}{m(r^2 - 1)^3} \right)$  was thus obtained from the 3rd differences

The value of  $r$  found in this manner was  $\log^{-1} \cdot 3748$ , but that adopted was  $\log^{-1} \cdot 375$ , and this value was retained for the Male Deaths and subsequently for the Female Population and Deaths.

There remained the constants  $a$ ,  $b$ ,  $c$  and  $m$  which were obtained as follows :—

Combining the original functions in pairs,

$$u_0 + u_1 = 2a + b + m(r+1)$$

$$u_2 + u_3 = 2a + 5b + 4c + mr^2(r+1)$$

$$u_4 + u_5 = 2a + 9b + 16c + mr^4(r+1)$$

$$u_6 + u_7 = 2a + 13b + 36c + mr^6(r+1)$$

Differencing

$\Delta$

$\Delta^2$

$\Delta^3$

$$\begin{array}{lll} 4b + 4c + m(r+1)(r^2-1) & 8c + m(r+1)(r^2-1)^2 & m(r+1)(r^2-1)^3 \\ 4b + 12c + mr^2(r+1)(r^2-1) & 8c + mr^2(r+1)(r^2-1)^2 & \\ 4b + 20c + mr^4(r+1)(r^2-1) & & \end{array}$$

Utilizing the constants previously found there were obtained successively  $m$  from  $\Delta^3$ ,  $c$  from  $\Delta^2$ ,  $b$  from  $\Delta$  and finally  $a$  from one of the pairs of the original functions.

The values of the constants thus found for the Male Population in decennial groups were as under :—

$$a = 7 \cdot 06926$$

$$b = - \cdot 13169$$

$$c = - \cdot 03303$$

$$m = - \cdot 0027279$$

Log  $r$  adopted as  $\cdot 375$

The constants having been found in this manner the functions  $u_0$ ,  $u_1$ , &c., were reconstructed. The deviations from the un-graduated figures are shown in the tables appended.

The next process was to interpolate the values for the individual ages on the basis of the formula.

This might have been done by giving to the variable the successive values of  $0$ ,  $\cdot 1$ ,  $\cdot 2$ , . . .  $\cdot 9$ ,  $1 \cdot 0$ ,  $1 \cdot 1$ , &c., but the following method was obviously superior and more easily applied.

Splitting the function into two portions, namely,

$$a + bx + c \frac{x \cdot x-1}{2} \text{ and } mr^x,$$

the first part is seen to be of the second order, the first and second differences being  $b$  and  $c$  respectively for decennial intervals.

The subdivided differences for single age intervals,  $\delta$  and  $\delta^2$ , are readily ascertained.

The second portion of the function  $mr^x$  was modified by taking  $R (= r^{1/10} = \log^{-1} \cdot 0375)$  as the ratio for single age intervals. The

constants calculated on this basis and actually used in the construction were—

$$a = 7.06926$$

$$\delta = - .0116826$$

$$\delta^2 = - .0003303$$

$$m = - .0027279$$

$$\text{Log R} = .0375$$

The final table was formed by combining the two portions, the first having been built up by continuous summation, and the second by taking the antilogarithm of the continuous addition of  $\log R$  to  $\log mR^x$ .

#### *Males—Deaths.*

The deaths were graduated by a formula of the same character as that taken to represent the Population. The constants are shown in the accompanying tables.

#### *Females—Population.*

The formula used was similar to that of the Male Table, the value of  $\log r$  being retained at .375 since this value, with suitable values for the remaining constants, appeared to give very satisfactory results.

#### *Females—Deaths.*

The deaths were first corrected to make allowance for the differences in the population arising from graduation. A modification was made in the method employed in the Men's table as it was not found to be practicable to represent the whole range of the table by a single formula.

In the first place a base curve was constructed of the form

$$\log \Sigma d_x = a + bx + c \frac{x(x-1)}{2} + mr^x$$

the constant  $r$  being taken as  $\log^{-1} .375$  as in the Population Table and the remaining constants being found from the values of the function at ages 55, 65, 75, and 85 where the mortality would probably not be affected by any special female risk. These are shown in the table in respect of the base curve.

The deviations in  $\log \Sigma d_x$  produced by calculating the constants from the values at these ages were, as expected, considerable, showing a maximum at age 15 and a gradual decrease to zero at age 55.

A supplementary curve was therefore formed so that

$$\log \Sigma l \text{ (base)} + \log \Sigma l \text{ (supplementary)} = \log \Sigma l \text{ (final table)}.$$

The form of this curve was made similar to that of the underlying series, but it was found convenient to apply it in the following manner :—

$$\log \{\log \Sigma l \text{ (supplementary)}\} = a + bx + mr^x$$

The value of  $\log r$  was taken as .4 (decennial groupings) so that the effect of this supplementary series should become negligible as soon after age 55 as possible.

The remaining constants were determined from the values of the function for ages 15, 25, 35, and 45, special care being taken to ensure the approximate reproduction of the value at age 15, i.e., the total number of deaths.

The constants for decennial intervals were found to be

$$a = -2.726$$

$$b = -0.079$$

$$m = -0.037$$

$$\log r = .4$$

The values for annual intervals are shown in the following table:

*Table of Constants used in the various Graduations.  
Annual Intervals.*

Table	$a$	$\delta$	$\delta^2$	$m$	$\log R$
<b>MALES.</b>					
Population ... ...	7.06926	-0.0116826	-0.00033030	-0.0027279	.0375
Deaths ... ...	5.23019	-0.0021390	-0.00007374	-0.0026750	.0375
<b>FEMALES.</b>					
Population ... ...	7.11753	-0.0125020	-0.00025245	-0.0025703	.0375
Deaths (base) ... ...	5.17988	-0.0009950	-0.00003910	-0.0024534	.0375
Do. (supplementary)	2.726	-0.0079		-0.037	.04

#### *Tables showing the original data and the Graduated Results.*

MALES. Population as at 30 June 1909.

Age Group	UNGRADUATED			GRADUATED	
	$L \div 10$	$\Sigma L \div 10$	$\log \Sigma L$	$\log \Sigma L$	Deviations $\times (10)^5$
15-25 ... ...	318,183	1,168,974	7.06780	7.06653	+ 127
25-35 ... ...	278,745	850,791	6.92982	6.93110	128
35-45 ... ...	226,281	572,046	6.75743	6.75751	8
45-55 ... ...	165,154	345,765	6.53878	6.53872	6
55-65 ... ...	105,237	180,611	6.25674	6.25806	132
65-75 ... ...	55,602	75,374	5.87722	5.87595	127
75-85 ... ...	17,750	19,772	5.29605	5.29857	252
85-end ... ...	2,022.3	2,022.3	4.30585	4.30345	240
	...	...	...	...	520 500

## Deaths. One-third of total for 1908, 1909 and 1910.

Age Group	UNGRADUATED			GRADUATED	
	d	$\Sigma d$	$\log \Sigma d$	$\log \Sigma d$	Deviations $\times (10)^5$
15-25	10,614	168,822	5.22742	5.22752	+ 10
25-35	14,060	158,208	5.19923	5.19914	- 9
35-45	18,887	144,148	5.15881	5.15836	- 45
45-55	24,888	125,261	5.09781	5.09827	- 46
55-65	32,262	100,373	5.00160	5.00252	- 92
65-75	37,245	68,111	4.83322	4.83231	- 91
75-85	24,836	30,866	4.48948	4.49564	- 616
85-end	6,030	6,030	3.78032	3.77434	- 598
...	...	...	...	...	... 764 743

## FEMALES. Population as at 30 June 1909.

Age Group	UNGRADUATED			GRADUATED	
	$L \div 10$	$\Sigma L \div 10$	$\log \Sigma L$	$\log \Sigma L$	Deviations $\times (10)^5$
15-25	339,432	1,291,800	7.11120	7.11496	- 376
25-35	311,813	952,368	6.97880	6.97505	- 375
35-45	245,300	640,555	6.80656	6.80507	- 149
45-55	178,081	395,255	6.59688	6.59838	- 150
55-65	118,801	217,174	6.33681	6.33926	- 245
65-75	69,619	98,373	5.99288	5.99043	- 245
75-85	25,176	28,754	5.45870	5.46350	- 480
85-end	3,578.0	3,578.0	4.55364	4.54882	- 482
...	...	...	...	...	... 1251 1251

## Deaths. One-third of total for 1908, 1909 and 1910.

Age Group	UNGRADUATED			GRADUATED	
	d un-corrected	$d^1$ corrected i.e. $\times$ grad. pop. ungrad. pop.	$\Sigma d^1$	$\log \Sigma d^1$	$\log \Sigma d^1$
15-25	9,993	10,567	168,403	5.22635	5.22629
25-35	13,273	13,020	157,836	5.19821	5.19817
35-45	16,527	16,287	144,816	5.16082	5.16036
45-55	20,867	20,883	128,529	5.10900	5.10829
55-65	28,110	28,530	107,646	5.03200	5.03286
65-75	38,413	37,933	79,116	4.89826	4.89825
75-85	31,165	31,610	41,183	4.61472	4.61468
85-end	9,682	9,573	9,573	3.98105	3.98120
...	...	...	...	...	... 101 132

The following comparisons between the graduated and ungraduated death rates for decennial age groups show the modifications made by the graduations.

*Ratio of Deaths to Population.*

Age Group	MALES		FEMALES	
	Ungraduated	Graduated	Ungraduated	Graduated
15-25...	.0033	.0034	.0029	.0029
25-35...	.0050	.0050	.0043	.0043
35-45...	.0083	.0082	.0067	.0068
45-55...	.0151	.0151	.0117	.0115
55-65...	.0307	.0308	.0237	.0238
65-75...	.0670	.0663	.0552	.0552
75-85...	.1399	.1419	.1238	.1238
85-end	.2982	.2959	.2706	.2706

*Comparison of the New Rates of Mortality with Standard Tables.*

Age	MALE			FEMALE	
	New Table	Manchester Unity (Whole Society)	English Life No. 6	New Table	English Life No. 6
15	.00292	...	.00305	.00255	.00306
20	.00346	.00314	.00457	.00297	.00414
25	.00415	.00428	.00568	.00356	.00502
30	.00511	.00507	.00671	.00437	.00618
35	.00644	.00630	.00897	.00546	.00782
40	.00837	.00835	.01190	.00690	.00995
45	.01120	.01078	.01481	.00882	.01170
50	.01543	.01455	.01936	.01167	.01495
55	.02187	.02106	.02568	.01643	.02023
60	.03166	.03039	.03596	.02451	.02928
65	.04648	.04672	.04969	.03746	.04187
70	.06862	.06898	.07212	.05718	.06243
75	.10114	.10481	.10540	.08651	.09311
80	.14803	.15726	.15200	.12946	.13626
85	.21459	.21412	.21398	.19200	.19345
90	.30781	.27116	.29186	.28328	.26378
95	.43647	.33266	.38281	.41743	.34194
100	.61050	...	.47908	.61421	.41695
105	.83730	...	.56806	.88778	.47320

## LEGAL NOTES.

*By ARTHUR RHYS BARRAND, F.I.A., Barrister-at-Law.*

Income tax in respect of double endowment assurance premiums. THE case of *Gould v. Curtis* [1913] 3 K.B. 84, was reported in these Notes when it came before the High Court (*J.I.A.* vol. xlvi, p. 191), and the decision of the Court of Appeal was referred to briefly in the last instalment of Legal Notes (*J.I.A.* vol. xlvii, p. 426). In view, however, of the importance of the matter it is thought that an extended report of the case will be useful, especially in view of certain passages in the judgment relating to previous decisions on the subject.

The case is concerned with the contention of the Crown that in the case of a double endowment assurance, only that portion of the premium which represents the temporary assurance premium is entitled to the benefit of the rebate of income tax given by section 54 of the Income Tax Act, 1853, and that the portion of the premium due to the endowment portion is not so entitled.

When the case came before Hamilton, J. ([1912] 1 K.B. 635), he decided against the contention of the Crown, and this decision was affirmed by the Court of Appeal in April last, when Cozens-Hardy, M.R., Buckley, L.J., and Kennedy, L.J., held unanimously that the assured was entitled to the statutory rebate in respect of the whole of such a premium. In delivering judgment to this effect Cozens-Hardy, M.R., said : "This appeal raises an important question upon the true construction of section 54 of the Income Tax Act of 1853. The question is really whether the right to an abatement in respect of premiums payable by a man on a policy on his life includes payments made by him in respect of a policy, which is often called an endowment policy, under which £100 is payable in the event of his death within fifteen years; or whether the benefit conferred by section 54 is limited to the case in which a sum of money is insured payable only to his executors, administrators, or assigns on his death. Hamilton, J., has held that an endowment policy of the nature I have described is within the section, and I agree with his decision."

" . . . It is important to observe the enactment in the Act of 1853 in two points. First of all, it is limited to contracts of insurance with an insurance company, whereas the old Act

" had no such limitation. In the next place there are words " here added to the former words : ' Who shall have made " insurance on his life or on the life of his wife, or shall have " contracted for any deferred annuity on his own life or on the " life of his wife, in or with any insurance company.' It was " contended on behalf of the Crown that in construing these " words we must suppose that that language in 1853 had pre- " cisely the same meaning as it had in 1799 or in 1806. I do " not assent to that. I think in 1853 Parliament was dealing " with a perfectly well known, well established, and very large " class of business, namely, the business of insurance companies " which entered into contracts of insurance on people's lives " to an enormous amount ; and in order to consider the meaning " of the words used in 1853, limited as they are to contracts " with insurance companies, I decline to refer for that purpose " to the meaning of those words in the Acts of 1799 or 1806, " which had no special reference whatever to insurance " companies.

" Now what was the meaning of those words ' contract of " insurance on his life or on the life of his wife ' as understood " commonly in the business world, by insurance companies, " and by other people, in the year 1853 and onwards. I think " the shortest way in which I can deal with this proposition is " to repeat what I said in *Joseph v. Law Integrity Insurance* " *Company* ([1912]. 2 Ch. 581, *J.I.A.* vol. xlvi, p. 300). I " thought it was legitimate to look at works of authority on the " branch of law dealing with this matter, and I referred to Mr. " Bunyon's book as a book of authority on this subject, the " first edition of which was written in 1853 and published in " 1854. It was contemporary with this Act. It was not dealing " with a subject which was then first started, but it was dealing " with what was well known in the insurance world. Mr. " Bunyon gives this as a definition of life insurance : ' The " contract of life insurance may be further defined to be that " in which one party agrees to pay a given sum upon the " happening of a particular event contingent upon the duration " of human life in consideration of the immediate payment " of a smaller sum or certain equivalent periodical payments " by another.' . . . The subject matter with which we are " now dealing was one with which in the year 1853 insurance " companies were dealing largely. It is a topic which must have " been familiar to any writer of authority who was considering

“ what the meaning of ‘insurance on his life’ was at that date.  
“ I think, therefore, that there is no reason to doubt, and so far  
“ as I am concerned I feel no doubt, that under the language of  
“ that section the policy in question is a ‘contract of insurance  
“ ‘on his life’, and none the less because in the event of his  
“ surviving fifteen years, which is a contingency dependent  
“ upon the continuance of his life, of course, he will receive £200,  
“ although in the other event, in the event of his death before  
“ the expiration of fifteen years, his executors will only receive  
“ £100.

“ It has been suggested that this cannot be a contract of  
“ insurance because the continuance of life is not an event  
“ which can be fairly said to be adverse to the interests of the  
“ insurer. This is based really upon a passage in the judgment  
“ of Channell, J., in the Prudential Company’s case ([1904]  
“ 2 K.B. 658, *J.I.A.* vol. xli, p. 114). He was there dealing  
“ with an argument which had been addressed to him by counsel.  
“ Dealing with the case of an insurance by a man who was of  
“ the artisan class, with small weekly payments, he pointed out  
“ that a payment made to a man in that class when he attains  
“ sixty-five, or whatever the age might be, is one which may be  
“ fairly said to contemplate an adverse event to the man who  
“ probably might not then be able so well to maintain himself  
“ and his wife. If, however, the learned judge intended to  
“ lay down that there cannot be a contract of insurance unless  
“ the event is of a character more or less adverse to the interests  
“ of the person effecting the assurance, with the greatest possible  
“ respect to the learned judge, I do not think that is accurate.  
“ I do not think it is true to say that there can be no insurance  
“ except the event covered by the contract is one which is, in its  
“ nature adverse to the assured. . . .”

Buckley, L.J., said: “ What . . . is the meaning of the  
“ words ‘insurance on his life’? There would, to my mind, be  
“ a significant difference if the preposition were ‘of’ and not  
“ ‘on.’ I can agree that the phrase ‘insurance of the life’ may  
“ as a matter of English mean a guarantee of a sum to be paid  
“ if the life drops. Insurance ‘on’ it is, to my mind, a different  
“ thing. It means the insurance of a sum dependent upon it.  
“ The life is mentioned as a contingency upon which the insur-  
“ ance is to be paid. The contingency is death or no death—  
“ death or life. Insurance ‘on’ life is an insurance of a sum  
“ payable or not payable according as the contingency of life or

"death is answered one way or the other. Regarded thus, it is plain that an insurance 'on' life includes as much an obligation to pay a sum of money if life continues at a date as an obligation to pay a sum of money if life ceases. An insurance 'on' life expresses an obligation to pay a sum of money on an event dependent upon the contingency of human life. If that be sound, it follows that the whole of this premium is deductible, because this is altogether an insurance 'on' life."

The recent case of *Hewson v. Shelley* [1913], 29 T.L.R. 699, is concerned with the position of a purchaser from an administrator when the letters of administration are subsequently revoked. The material facts of the case are as follows: One George Hewson died on 30 January 1899, without issue, leaving a widow, to whom letters of administration were granted on 13 June 1899, no will having been found. The estate included certain freehold property known as Ovington Glebe, which was sold by the administratrix on 13 October 1902, to Sir John Shelley for £3,500, and conveyed by her to him. Of the proceeds of the sale one-third was invested as dower for the widow and the remainder was divided among three co-heiresses of the deceased.

Administration revoked on subsequent discovery of will. Position of purchaser from administrator.

In September 1911 the widow died, and her executor, on looking through her papers, discovered in an envelope at the back of an old writing desk the will of her husband, George Hewson. The document was between the back and shelves of some pigeon holes, where it appeared to have been placed by the testator. This will was dated 24 April 1894, and by it the testator left his property to his widow for life, and after her death to his cousin, one of the plaintiffs, whom he exhorted "to hold Ovington Glebe as an heirloom and on no account to sell it, but should such occur", then the proceeds were to be equally divided among certain persons named. The testator then appointed the plaintiffs his executors. On 9 February 1912 the letters of administration granted to the widow were revoked, and probate of the will granted to the plaintiffs. The executors then brought the present action to set aside the sale of Ovington Glebe to Sir John Shelley who, in reply, contended that the sale was *bonâ fide*, and took place in the due course of administration, and that therefore it could not now be set aside.

The case came before Astbury, J., on 17 July last, when he

decided in favour of the executors, holding that they were entitled to recover possession of the estate, they undertaking to hand over the dower fund in exchange for the title deeds. He considered himself bound by the old decision in *Graysbrook v. Fox* (1560, 1 Plowd. 275) where it was held that if A makes his will and appoints an executor, and the ordinary commits administration after his death to another, who sells the goods of the deceased, and the executor afterwards proves the will, and brings detinue of the goods against the purchaser, the probate supersedes the administration *ab initio*, and the sale made under it. He also quoted from the judgment of Warrington J., in *Ellis v. Ellis* ([1905] 1 Ch. 613) who, in dealing with a case where a grant of administration had been made and revoked in circumstances similar to the present case, said : "I think "it is clear law that the grant of letters of administration is "wholly void, and that speaking generally, dispositions of the "assets by the supposed administrator are void also, the ground "of this being that the assets are vested in the executor from the "death, and the supposed administrator has no property in "them, and no power of dealing with them."

The decision in this case has excited much interest and has given rise to considerable comment, in view of its effect on existing titles ; and it seems probable that in the event of its being upheld on appeal, efforts will be made, by means of fresh legislation, to place the law on the point on a more satisfactory basis.

*Liability of  
insurance  
company to  
income tax in  
respect of interest  
received and  
re-invested abroad,  
when the company  
is taxed on profits*

The case of *The Liverpool and London and Globe Insurance Company v. Bennett* (Surveyor of Taxes)

1913, 29 T.L.R. 757, which has already been referred to in these Notes (*J.I.A.* vol. xlvi, p. 387) came before the House of Lords on 31 July last on appeal. The facts relating to the case have been given previously, and it will be sufficient to say that the question at issue was as to whether certain interest and dividends received abroad and re-invested there were to be treated as profits or gains of the company for income tax purposes, the company being assessed for income tax on the basis of profits, and not on the basis of interest, as in the case of most life assurance companies. The House of Lords dismissed the insurance company's appeal and affirmed the decision of the Court of Appeal, holding that such interest and dividends were taxable as profits or gains arising from the company's business.

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EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE  
(PART I).*First Paper.*

1. A company has issued whole life assurances and endowment assurances. The square root of the number of whole life assurances added to one-half of the square root of the number of endowment assurances is equal to the square root of the total number of assurances. The geometric mean between the respective numbers of whole life and endowment assurances is ten times the number which expresses the average amount of all the assurances in £'s. If the amount of the whole life assurances were halved and that of the endowment assurances doubled the business would be increased 10 per-cent. If the average amounts of the whole life and endowment assurances were interchanged and a further £40,000 of business issued the business would be increased 30 per-cent. Find the number and amount of whole life and endowment assurances respectively.

2.  $a, b, c \dots k$  being  $n$  positive quantities not all equal, find which is the greatest of the three expressions :

$$\left( \frac{a^2 + b^2 + c^2 + \dots + k^2}{a+b+c+\dots+k} \right)^{a+b+c+\dots+k};$$

$$\left( \frac{a+b+c+\dots+k}{n} \right)^{a+b+c+\dots+k};$$

and  $a^a b^b c^c \dots k^k$ .

3. If  $A = \log(1 - \cdot 1)$

$$B = \log(1 - \cdot 04)$$

$$C = \log(1 + \cdot 0125)$$

show how to express  $\log 2$  in terms of A, B, and C.

4. Sum to infinity the series

$$x^3 - x^4 - \frac{x^6}{2} + \frac{x^8}{2} + \frac{x^{10}}{3} - \frac{7x^{12}}{12} + \frac{x^{15}}{5} + \frac{x^{16}}{4} \dots$$

5. If  $u_x$  be any function of  $x$ , show how to express the  $n$ th difference of  $u_x$  in terms of  $u_x$  and its successive values.

Find the sum to  $n$  terms of the series

$$3 + 4 + 7 + 15 + 35 + 83 + \dots$$

6. Given  $u_0 = 11$ ,  $u_3 = 18$ ,  $u_6 = 74$  and  $u_9 = 522$ , find the intermediate terms.

7. Out of a large number of invitations replies are received to 90 per-cent. Assuming that one letter in 100 is lost in the post, find the probability that the invitation reached a given person from whom no reply was received.

8. Four coins are tossed together and A is to receive £2 if exactly 2 heads turn up, and to pay £1 in any other event. Find to three decimal places the probability that after four trials A is £1 out of pocket.

9. An urn contains counters marked with the digits 6, 7, 8, and 9; and the number of times each digit occurs is equal to the value of the digit. If counters are drawn one at a time each counter being replaced when drawn, what is the probability

(1) that the digit 6 is drawn before the digit 9;

(2) that the sum of the first three digits drawn is exactly 20?

### *Second Paper.*

1. Prove that if  $a$  and  $b$  are any quantities whatever, and  $n$  is a positive integer,

$$(a+b)_n = a_n + n a_{n-1} b_1 + \frac{n(n-1)}{2} a_{n-2} b_2 + \dots$$

$$+ \frac{n}{r(n-r)} a_{n-r} b_r + \dots + b_n$$

where  $a_n = a(a-1)(a-2) \dots (a-n+1)$ .

2. Prove that if  $a_n$  diminishes as  $n$  increases and converges to the limit zero, the sum to infinity of the series  $a_1 - a_2 + a_3 \dots$  is the same as the sum to infinity of the series  $\frac{1}{2}a_1 - \frac{1}{4}\Delta a_1 + \frac{1}{8}\Delta^2 a_1 - , \text{ &c.}$

Find the sum to infinity of the series  $\frac{1}{10} - \frac{1}{11} + \frac{1}{12} \dots$  true to 4 decimal places.

3. Find from first principles the differential coefficient with regard to  $x$  of  $f(v)$ , where  $v = \phi(x)$ ,  $f$  and  $\phi$  being single-valued functions.

Differentiate  $a^x \cdot b^{c^x}$ , where  $a$ ,  $b$ , and  $c$  are constant.

4. Prove that

$$\int f(x)dx = xf(x) - \int xf'(x)dx$$

Show that if  $m$  is an integer

$$\int_0^1 (1-x^2)^m dx = \frac{(2^m \cdot m!)^2}{(2m+1)!}$$

5. Explain the meaning of  $\int_a^b \phi(x)dx$  where  $\phi(x)$  is a function of  $x$  which is finite and continuous between the limits  $a$  and  $b$ .

Evaluate

$$\int \frac{dx}{\sqrt{x^2 + a^2}}, \int \log x dx, \int \frac{dx}{(a + cx^2)^2}.$$

6. If  $u_x = a + bx + cx^2$ , find expressions for  $u_{\frac{1}{2}}$ ,  $\frac{du_0}{dx}$ , and  $\int_0^1 u_x dx$  in terms of  $u_0$ ,  $u_1$ , and  $u_2$ .

7. Explain how the slope of a curve is measured when its equation is known, and how the points of greatest slope can be determined.

Find the values of  $x$  at the points where the graph of the function  $(1+x^2)e^{1-x^2}$  has its greatest slope.

8. If  $f(x)$  either increases continually or decreases continually as  $x$  increases, show that  $\int_1^n f(x)dx$  differs from  $\sum_1^{n-1} f(x)$  by less than the difference between  $f(1)$  and  $f(n)$ .

Prove that the difference between  $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-1}$  and  $\log_e n$  is less than unity however great  $n$  may be.

9. In a table of common logarithms to 10 decimal places of all numbers to 100,000 find within what range of numbers the differences of the logarithms lie between .00000999 and .00001001. Given  $M = .4342945$ .

### *Third Paper.*

1. A series of payments is to be made as follows: the first after a period  $n_1$ , the second after a further period  $n_2$ , the third after a further period  $n_3$ , and so on. The amount of the payment for any period bears to unity the ratio which interest for that period bears to interest for one year. Prove that if  $n_1 + n_2 + n_3 + \dots = n$ , then the present value of the series of payments  $= \frac{1 - r^n}{i}$ .

If a debt is to be liquidated by 3 payments, the relative values of which are to be calculated by the above method, and the last of which is to be made at the end of two years, at what periods should the debtor make the intermediate payments so as to make his total disbursements a minimum?

\*2. Find the annuity which will liquidate a debt of £10,000 in 10 years, on the assumption that the lender is to receive 4 per-cent on his capital during that period, and to be enabled to replace it at the end by a sinking fund invested at  $2\frac{1}{2}$  per-cent. Construct a schedule showing the division of the yearly payments into principal and interest.

\*3. A foreign government borrows £100,000 on 14 June 1900, repayable by a half-yearly annuity, including principal and interest at 5 per-cent, to run for 20 years.

On 13 May 1905, it borrows a further £50,000, repayable by a similar annuity for 18 years.

In 1912, an arrangement is come to whereby the two loans are consolidated as from 31 December of that year, while repayment of the amount then outstanding is to be made by a half-yearly annuity, including principal and interest at 5 per-cent, to run for 25 years. What should be the amount of this annuity?

\*4. A government is issuing a loan of £100,000, bearing interest at 5 per-cent, and repayable by drawings of £20,000 at the end of every 5 years. A syndicate takes up the entire issue at a price of £104,500. What approximate rate of interest does it realize on the investment?

\*5. A corporation on taking over a water undertaking offers to the shareholders in respect of each £100 stock the following alternatives :

- (1) £125 debenture stock bearing interest at the rate of 5 per-cent per annum payable half-yearly, and redeemable at par at the end of 50 years.
- (2) An increasing annuity of amount £1 in the first half-year, rising by half-yearly increments of 2s. to a maximum of £5 per half-year, and ceasing after 50 years.

Find at 4 per-cent which of these alternative offers is the more advantageous to the shareholder.

6. Prove that the yield on a debenture redeemable in  $n$  years and purchased at a premium is approximately equal to the rate realized on the average invested capital if  $\frac{1}{n}$ th part of the premium be annually written off out of dividend.

Find by this means the approximate yield on a debenture bearing interest at 5 per-cent payable half-yearly, redeemable in 10 years at 125, and purchased at 137.

7. Prove that  $a_n^- = r(1 + a_{n-1}^-)$ .

Show how by the application of this formula to construct a table of values of  $a_n^-$  by a continuous process.

\* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

## EXAMINATION FOR ADMISSION TO THE CLASS OF ASSOCIATE (PART II).

### *First Paper.*

1. Find expressions for the probabilities that

(1) ( $x$ ) will die in the same year as ( $z$ ) ;

(2) ( $z$ ) will die in the  $t$ th year after ( $y$ )'s death, and in the  $2t$ th year after ( $x$ )'s death.

2. If the value of  $l_{x+t}$  for all values of  $t$  from  $-\frac{1}{2}$  to  $+\frac{1}{2}$  can be expressed in the form  $A - \beta t$ , where  $A$  and  $\beta$  are independent of  $t$ , prove algebraically that  $\mu_x = m_{x-\frac{1}{2}}$ .

3. Having given the values of  $\log p_{20}$ ,  $\log p_{30}$ ,  $\log p_{40}$ , where the table of mortality strictly follows Makeham's Law, show clearly how you would calculate the values of  $l_x$  and  $\mu_x$  for all ages.

4. If the death curve has one minimum point (age 20), one maximum point (age 75), and two points of inflection (ages 45 and 85), find in what sections of the life table the expression  $2d_x/(l_x + l_{x+1})$  is greater than the expression  $d_x/(l_{x+\frac{1}{2}})$  and in what sections it is less.

5. The staff of an office consists of 10 clerks aged 40, 10 aged 30, 20 aged 21, 20 aged 20, 20 aged 19, 20 aged 18, and 20 aged 17. Clerks are pensioned at 60. The chance of ceasing to be in the employ of the office from any cause is .01 at every age under 40, and after age 40 the only decrement in the staff is due to death or the attainment of the pension age. Give expressions for (1) the number of the staff in 40 years' time, assuming that two new clerks age 16 are employed at the beginning of each year; and (2) the number of pensioners in 40 years' time. You may assume that all the ages are exact ages.

6. The staff of a company at 1 January is represented by the  $D_x$  column from age 20 to age 64. Pensions are given on attaining age 65, and there are no withdrawals except by death and superannuation.

Find an expression for the present value at 1 January of all future salaries payable to the then existing staff in terms of the commutation columns  $N$ ,  $S$ ,  $W$ , where  $W_x = S_x + S_{x+1} + \&c.$ , the salary for the year of age  $x$  to  $x+1$  being  $A + Bx$ . Assume that all birthdays fall on 1 January, that salaries are payable at the end of the year, and that at the end of the year of death a half-year's salary only is paid.

7. A loan of  $a_{\bar{n}}$  to a life ( $x$ ) is to be repaid by equal annual amounts of 1 each, including principal and interest. Assuming the same rate of interest throughout, prove that the single premium for an assurance to secure the amount outstanding should ( $x$ ) die during the  $n$  years is  $a_{\bar{n}} - a_{x\bar{n}}$ .

\*8. What approximate methods can you suggest for finding a value for a compound survivorship annuity?

Find by approximate summation a value for  $a_{\substack{30:40|50}}$  by  $H^M$  3 per-cent, and work out one of the suggested approximations to it by the same table and by  $O^{NM}$  3 per-cent.

\* A Short Collection of Actuarial Tables will be supplied for use in answering this question.

## Second Paper.

\*1. A man aged 35 wishes to be allowed to pay a premium of  $P_{[30]}$  for a whole-life assurance, and is prepared to have his sum assured reduced by  $30X$  in the first year,  $29X$  in the second year, and so on, the deduction from the sum assured decreasing by  $X$  per annum. Find an approximate value for  $X$  by  $O^{[NM]} 3$  per-cent.

\*2. An endowment assurance policy for £500 to mature at 65 was effected at 25 at an annual premium of £13.

It has been 18 years in force and has reversionary bonus additions of £180. It is desired to apply the bonus to make the assurance payable at an earlier age than 65.

Assuming  $O^{[NM]} 3$  per-cent, find the maturity age.

3. Derive the formula

$$\overset{\circ}{a}_x = a_x + \bar{A}_x \frac{i - \delta}{i\delta}$$

and thence show that

$$\overset{\circ}{a}_x^{(m)} = a_x^{(m)} + \bar{A}_x \frac{j(m) - \delta}{j(m)\delta}$$

4. Having given  $P_{xr}^1$  and  $a_{xr}$  find  $a_{xr}^{(m)}$  and  $P_{xr}^m$ .

5. Prove that

$${}_nV_{x+n+t}^1 + (P_{x+n+t} - P_{x+n}) (s_{n+1} - 1) < {}_nV_{x+n+t}$$

and find the future annual premium payable if a term assurance for  $n+t$  years effected  $n$  years ago be converted into an endowment assurance payable  $t$  years hence or at previous death.

6. Derive the following formula by general reasoning

$${}_nV_x = \frac{A_{r+n} - A_r}{1 - \bar{A}_x}$$

If  $1 - A_{x+2n} = A_{x+2n} - A_{x+n} = A_{x+n} - A_x$ , express  ${}_nV_{x+n}$  in terms of  ${}_nV_x$  and give the numerical values of  ${}_nV_{x+n}$  and  ${}_nV_x$ .

7. Find the value of an annuity, deferred until the failure of the joint lives ( $x$ ) and ( $y$ ), to be payable thereafter for a term of 20 years, or until 10 years after the death of the survivor, whichever be the longer term.

\*8. Find the office annual premium by H<sup>M</sup> 3 per-cent for a whole life policy of £100 on a life aged 40, the policy providing that the assured shall receive at the end of each year interest at the rate of  $2\frac{1}{2}$  per-cent on the total premiums previously paid.

Assume that the office premium is derived from the net premium by adding 15 per-cent.

\*A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

### *Third Paper.*

1. Prove that

$$(IB)_x = -(1+i) \frac{dB_x}{di}$$

and hence show how to find the value at  $3\frac{1}{2}$  per-cent interest of a guaranteed bonus of 2 per-cent per annum payable at death or maturity under a 25-year endowment assurance on a life aged 35.

\*2. A (aged 50) is in possession during his life of a perpetuity of £100 per annum for 20 years from the present time, to be increased to £200 per annum thereafter; and B (aged 35) and C (aged 25) are entitled, during their respective lives, to the perpetuity, in succession to one another, after A's death.

What is the value of C's interest by H<sup>M</sup> 3 per-cent?

3. If  $w$  is the purchase money for a continuous annuity on a life  $(x)$  with the proviso that if death occur before the annuity payments amount to the purchase money the difference shall be refunded, prove that  $w = \bar{a}_x + \int_c^w \bar{A}_{xt}^1 dt$ .

4. Give an accurate formula for an annuity, payable by half-yearly instalments, and apportionable, during the joint lives and the life of the survivor of  $x$  and  $y$ , with the proviso that in any event the annuity is to be payable for ten years certain.

5. Two partners  $(x)$  and  $(y)$  effected  $t$  years ago at an annual premium an assurance for £1,000 (in which they have equal interests) payable should the joint lives fail during  $n$  years or at the end of  $n$  years should both be then alive. It is now desired to convert the policy into two separate assurances for £500 each, one being payable at the end of  $n-t$  years or the previous death of  $(x)$ , and the other being payable at the end of  $n-t$  years or the

previous death of ( $y$ ). What annual premiums should be paid by ( $x$ ) and ( $y$ ) respectively?

6. A sum of money is, on the death of ( $x$ ), to be divided equally among the survivors of ( $w$ ), ( $y$ ) and ( $z$ ). Find the present value of ( $w$ )'s share. Give a general expression for the present value of ( $w$ )'s share if the division has to be made equally among the survivors of  $m$  lives ( $w$ ), ( $x$ ), ( $y$ ) . . .

7. Describe fully how you would construct, and verify, a table of policy values for whole-life assurances by limited payments.

8. You are given tables of values of  $\log D$ ,  $D$ ,  $N$ ;  $\log C$ ,  $C$ ,  $M$ ;  $a$  and  $A$  for all ages and at various rates of interest.

Show clearly how you would, without checking any individual values, test the general accuracy of the tables.

\* A Short Collection of Actuarial Tables will be supplied for use in answering this question.

#### *Fourth Paper.*

1. What entries would you make in the books of a company in respect of claims under the following policies: (a) at the time the claim is intimated; (b) when payments are made?

(1) Where payment of the sum assured is deferred 10 years, interest at 6 per-cent per annum being payable on the sum in the meantime.

(2) Where the sum assured is payable in 20 equal yearly instalments.

2. (1) A company takes credit for interest at 4 per-cent per annum on the book values of its reversions.

(2) A reversion belonging to the above company, which stands in the books at £5,000, falls in, and the proceeds, namely £7,000, are received.

What entries in the books would you make in respect of (1) and (2)?

3. A foreign policyholder of a mutual life office surrenders his policy. The agent hands the policyholder £28. 10s. in full settlement, and brings this sum into account. There was a loan of £150 on

the policy with interest at 5 per-cent for 3 months owing, and a premium of £15. 7s. 6d. and a fine of 7s. 6d. were also due and deducted from the surrender value. Trace the details of the transaction through the accounts and show what items will eventually appear in the Revenue Account.

4. Draft the headings for a set of registers recording
  - (1) The carrying out of purchases and sales of Stock Exchange securities.
  - (2) The market prices from time to time of such securities.
  - (3) The valuation of such securities at the close of the books.

State also how such registers may be conveniently subdivided.

5. What are Floater? Give a short list of such securities.

6. Explain the following terms:—

Pari Passu.

Inscribed Stock.

“ Bull.”

“ Stag.”

Special Settlement.

Scrip.

7. Distinguish between the Funded and Unfunded Debts of the United Kingdom. Of what securities does each group now consist?

8. Give the principal items comprising the remunerative and non-remunerative Local Debt of this country. Under what headings may the 26,000 authorities, classed as local bodies, be grouped?

#### EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW (PART III).

##### *First Paper.*

1. Describe how Messrs. Hardy and Wyatt ascertained the average rates of sickness applicable to the male population in connection with their estimates of the cost of the National Health Insurance Scheme.

2 Give a general comparison of the rates of mortality, at different ages, between the following sets of tables :

- (a) English Life No. 3 with English Life No. 6.
- (b) Government Annuitants 1883 with Government Annuitants 1912. (Male Lives.)

3. Describe what you consider the best method of determining and adjusting the constants for Makeham's formula in the graduation of select mortality tables. What criticisms of the method you suggest can be urged?

4. The following particulars were given in the report of a company which gives special terms to abstainers.

Period	GENERAL SECTION			TEMPERANCE SECTION		
	Expected Deaths (H <sup>m</sup> )	Actual Deaths	Per-cent	Expected Deaths (H <sup>m</sup> )	Actual Deaths	Per-cent
1884-1888	233	184	79	100	55	55
1889-1893	282	233	83	160	95	59
1894-1898	314	250	79	250	148	59
1899-1903	356	274	77	400	216	54
1904-1908	350	280	80	600	360	60

From these figures it was stated by someone unconnected with the company that the abstainers assured by the particular company had a lighter mortality than the non-abstainers. Prove that this statement might be incorrect and mention briefly the limits within which it is legitimate to draw conclusions from such figures.

5. Find from the following census particulars the rate of mortality at age 50 :

Age last Birthday	Population on 1 May 1900	Population on 1 May 1908	Deaths for the calendar years 1900-1908, both inclusive
50	40,100	45,150	6,500

If you were also informed that the population at the same age had been 37,600 on 1st May 1896, would you modify your method?

6. The following statistics are submitted for the purpose of deducing the rates of mortality from the experience of a life

assurancee society during the quinquennium ending 31 December, 1912 :—

Age $x$	No. of members 31.12.1907	En- trants $n_x$	Deaths $\theta_x$	With- drawals $w_x$	No. of members 31.12.1912	$s_x + n_x$	$\theta_x + w_x$ + $s'_{x+1}$	(7) - (8)	Exposed to Risk $\Sigma(9)$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
25	1	...	...	...	...	1	1	...	...
26	49	7	1	2	1	47	7	49	40
27	60	12	1	4	4	72	15	57	97
28	60	20	2	6	10	80	23	57	154
29	90	30	4	8	15	120	32	88	242
30	60	10	6	10	20	70	56	14	256
31	40	1	6	10	40	41	71	-30	226
32	4	...	6	5	55	4	71	-67	159
33	...	...	4	5	60	...	64	-64	95
34	...	...	2	5	55	...	57	-57	38
35	...	...	2	...	50	.	33	-33	5
36	...	...	1	...	31	...	5	-5	...
37	...	...	...	...	4	...	...	...	...
	355	80	35	55	345	435	435	{ + 256 } - 256 }	1,312

In tabulating the numbers at the various ages shown in the first column the ages were arrived at in the first instance as follows :

Col. (2) . . . . . 1907 minus year of birth.

Col. (6) . . . . . 1912 , , , , ,

Cols. (3), (4), and (5)—Year in which entry, death or withdrawal took place minus year of birth.

Do you approve of the manner in which the exposed to risk has been arrived at? If not, explain where the method is defective and calculate the correct values of the exposed to risk.

### Second Paper.

1. Set out (1) the objections urged against the net premium method of valuation with a full aggregate table; (2) any answer you can urge to the objections; (3) the steps you would take to overcome the objections.

[Note.—This question must be answered concisely.]

2. What bases should be employed to produce fair scales of premium for whole of life and endowment assurances for an

industrial office? Discuss the question of bringing in the element of lapse.

How would you value these policies?

3. In making up your valuation balance sheet, after calculating the ordinary reserves against the various classes of policies what extra reserves (if any) would you keep? Explain how you would arrive at them.

4. What are the considerations that would guide you if you were asked to advise an office, that had not hitherto granted immediate life annuities, as to the advantages or otherwise of doing so? If it were decided to grant such annuities, upon what bases do you consider the rates should be formed? Set out your answer in the form of notes, dealing, *inter alia*, with the rates you would charge for last survivor annuities on lives under 40.

5. It has been decided to revise scales of single and limited payment premiums for with and without profit policies. Explain in detail the bases you would employ and your reasons for their use.

6. What information would you call for if you were asked to value the liabilities of an employers' liability insurance company?

7. The following are abridged particulars of an ultimate (after 5 years) mortality table for annuitants:

Age Group	Exposed to Risk	Actual Number of Deaths	Expected Number of Deaths found from Graduated Rate of Mortality
under 24	100	...	.6
25-29	250	3	1.6
30-34	700	2	4.9
35-39	1,000	4	7.5
40-44	2,500	15	20.0
45-49	4,000	45	40.0
50-59	9,000	146	135.0
60-69	10,000	248	250.0
70-79	8,000	434	400.0
80-89	2,500	238	250.0
90 and over	700	150	175.0
	38,750	1,285	1284.6

Do you consider the graduation a good one so far as you can tell from the facts given, and what criticisms, if any, would you offer?

*Third Paper.*

\*1. An office charges an annual premium at age 1 of 16*s. 5d.* per-cent and at age 8 of £1. 2*s.* per-cent for a child's deferred assurance commencing at age 21: premiums to be returned in event of death before age 21: a cash option to be given at age 21 of £21·6 per-cent in case of an entrant at age 1 and £16 per-cent in the case of an entrant at age 8, and in event of the policy being surrendered before age 21 a guaranteed surrender value of 90 per-cent of the premiums paid excluding the first year's. Assuming commission to be paid at the rate of 10*s.* per £100 assured in respect of the first premium and 2½ per-cent on renewals, on what terms at the outset would you commute all the premiums payable up to age 21 in the cases of entrants at ages 1 and 8 and what commission would you allow?

\*2. An office that calculates its premiums for 5 year term policies by  $O^{(NMI)} 3$  per-cent with a percentage loading of 12 per-cent and a constant of 4*s.* for each £100 assured and pays 5 per-cent commission, states in its prospectus that if the assurance is effected for £1,000 or over it may be enlarged at any time during its term into a whole life or endowment assurance at the current rate for the age of the assured at date of alteration. Up to what limits is this justifiable?

3. With what restrictions and under what conditions would you accept risks in the following general classes:—

(1) Persons with history of syphilis.

(2) Persons proceeding to Central America (Panama, etc.)?

4. An office takes a life aged 22, proceeding to Brazil, at an extra premium of £2 per-cent limited to 7 yearly payments.

The proposer explains that he does not require insurance at the moment, but may want it five years hence, and asks for a policy similar to a child's deferred assurance. What extra would you charge and what return would you make in the event of death within the 5 years?

5. What, if any, modification should be made in the usual extras charged on whole life policies in the following cases: joint life policies (one or both lives exposed to extra risk), last survivor policies (one or both lives exposed to extra risk), short term assurances, endowment assurances and whole life single premium policies? Distinguish between extras imposed for family history, occupation and residence.

6. State concisely the various ways in which the profit from mortality in a life assurance society can be ascertained. Which method do you prefer, and why?

7. A life office granting bonuses on the tontine principle—the first bonus vesting on the assured surviving the ordinary expectation of life—starts a series under which ordinary simple reversionary bonuses are to be allotted. State the points which have to be taken into consideration in apportioning the surplus available for distribution between the two series. What additional reserves should in your opinion be made for tontine policies which have not been in force long enough to rank for vested bonuses?

8. A life office valuing by  $O^M$  and  $O^{M(5)}$  earning  $4\frac{1}{4}$  per-cent interest and experiencing 70 per-cent of the  $O^{[M]}$  rates of mortality (taking into account the duration of the various policies), grants quinquennial bonuses in the form either of a simple reversionary bonus or of reduction of premiums, as the assured may select at the first distribution. The reversionary bonus corresponding to the cash surplus is ascertained by dividing the cash allotment by the  $O^M$  3 per-cent reversion, and when bonuses are applied in reduction of premiums the reduction is found by multiplying the reversionary bonus so ascertained by the  $O^M$  3 per-cent net premiums. Do you consider this practice equitable? State reasons for your answer.

\* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

#### *Fourth Paper.*

Table I gives the total business on the books of an office as at 1 January 1912; Table II gives the new business, and Table III the cancellments for the year 1912; Table IV gives the items for the revenue account for the year 1912, which cannot be obtained from Tables I, II, and III. The net premiums are calculated by  $H^M$  3 per-cent for the age next birthday at entry and policies are valued at the nearest age on 31 December.

Make a valuation of the business as at 31 December 1912, by  $H^M$  3 per-cent; ascertain from the completed revenue account the surplus; trace this surplus to its various sources and state what rate of simple reversionary bonus could be declared. In completing the revenue account you can assume that there was no new business or cancellation during 1911. The whole of the business consists of whole-life policies.

*Note.*—The work should be done throughout in tabular form and all working shown.

TABLE I.

Nearest Age on 31 Dec. 1911	Sum Assured	Reversionary Bonus	Annual Office Premium	Net Premium H <sup>m</sup> 3	Annual Extra Premium
40	3,000	100	60	50	...
41	4,000	120	85	70	...
42	5,000	150	125	100	10
43	4,000	140	105	85	...
44	3,000	120	80	65	5
Total	19,000	630	455	370	15

TABLE II.

Date of Birth	Date of Entry	Sum Assured	Annual Office Premium	Renewal dates
20 Aug. 1870	29 Feb. 1912	300	10·0	28 Feb.
15 Feb. 1871	10 Mch. 1912	400	13·5	10 Mch. and 10 Sept.
29 Nov. 1869	15 July 1912	300	10·5	15 July
20 July 1869	16 Dec. 1912	1,000	360	16 Dec. and 16 June
Total ...	...	2,000	70·0	...

NOTE.—Office Premiums are charged according to age next birthday.

TABLE III.

Cause of Cancellation	Date of Cancellation	Nearest Age on 31 Dec. 1911	Sum Assured	Reversionary Bonus	Annual Office Premium	Renewal Dates	Net Premium H <sup>m</sup>
Death	29 Feb. 1912	40	100	5	2·0	14 May	1·0
Death	15 Aug. 1912	42	100	...	4·0	15 Mch. & 15 Sept.	2·0
Surrendered for £12	14 June 1912	42	300	...	9·0	30 Aug.	7·0
Lapse	1 Feb. 1912	44	250	...	8·5	1 Feb.	7·0
Bonus surrendered for £5	2 Dec. 1912	42	...	15	...	...	...
Total ...	...	...	750	20	23·5	..	18·0

TABLE IV.

Fund at beginning of year	3,400	Claims	...	...	...	...
Premiums ...	...	Surrenders, including surrenders of bonus	...	...	...	...
Interest (less tax)...	138	Commission	...	...	32	
		Expenses	...	...	30	
		Fund at end of year	...	...	...	

A Short Collection of Actuarial Tables will be supplied for use in answering this question.

### EXAMINATION FOR ADMISSION TO THE CLASS OF FELLOW (PART IV).

#### *First Paper.*

1. An insurance company transacting fire, life and accident (including employers' liability) business has entered into an agreement for amalgamation with a proprietary life office. What legal formalities must be carried out by both sides, and what returns are necessary?
2. Describe the circumstances in which money paid under a mistake can or cannot be recovered back. What is the position where the money in question has been paid under compulsion of legal process? Illustrate your answer by examples.
3. What is a chose in action? Explain the difference between a legal and an equitable chose in action, and give examples of both. State the provisions of Section 25, Sub-section 6, of the Judicature Act, 1873, relating to the assignment of a chose in action.
4. What is the position of a *bonâ fide* purchaser for value without notice, whose title depends on (*a*) a fraudulent exercise of a power of appointment; (*b*) a transaction declared void under the Moneylender's Act, 1900; (*c*) a voluntary settlement?
5. Describe in detail the courses open to a mortgagee who desires to enforce his security.
6. A man effects a policy of life assurance on his own life. He mortgages the policy, and being unable to pay the premiums, he

requests his wife to pay them, which she does. He subsequently becomes bankrupt, but his wife continues to pay the premiums after the bankruptcy, until the death of the assured. Discuss the question of the interest of the mortgagee, the trustee in bankruptcy and the widow of the assured respectively in the policy-moneys.

7. A policy of life assurance is issued by an English life assurance company to a person domiciled in Scotland. Under what system of law will such a contract be interpreted? What courts will have jurisdiction to deal with the matter?

8. Give an account of the nature and effect of the power of nomination conferred by Section 56 of the Friendly Societies Act, 1896, and refer to any legal decisions on the subject with which you are acquainted.

*Second Paper.*

1. State the requirements of the Assurance Companies Act, 1909, in regard to underwriters transacting employers' liability insurance business, and compare these with the corresponding provisions of that Act relating to employers' liability insurance companies.

2. What arguments have been advanced for and against the following bases of scales of payment in pension funds? Which method do you prefer, and why?

- (1) The "average salary" system;
- (2) The "last seven years average salary" system;
- (3) The "money value" or "single premium" system.

3. In making the periodical valuation of a pension fund, in which the pensions at retirement are a percentage of the total salary received whilst in active service, it is proposed to modify the assumed rates of retirement for ages over 60 which were previously used, but to adhere to the bases adopted in the previous valuation for the retirement rates for ages up to 60, the rates of mortality and withdrawal, and the salary scale. What effect will this change have on the value of the liability for prospective pensions to present contributors?

4. A British railway company is desirous of establishing, on a contributory basis, a widows' and orphans' annuity fund for its employees (excluding the salaried staff at the head office). You are

asked what weekly contribution would be required for each 1,000 employees to provide

- (a) Annuities of 7s. a week to widows of employees dying while in active service, the annuity to cease on re-marriage;
- (b) Annuities of 2s. a week for children under 14 of such employees dying while in active service, the annuities to cease at age 14, and in any event to be limited to 6s. a week in respect of any one employee.

What statistical information would you require, and how would you proceed to estimate the cost?

5. A proprietary life office declaring uniform compound reversionary bonuses, valuing by the O<sup>M</sup> 3 per-cent Table and earning  $4\frac{1}{4}$  per-cent (gross) on its funds, is considering whether it is desirable to take steps to secure a rapid expansion in its new business. Draft a report on the subject for submission to a Board of Directors.

6. You are asked to consider whether it is advisable to revise the non-profit rates of your office, and to make the conditions of the policies with regard to suicide, and non-forfeiture of surrender values more generous to the assured. Draft a report on the subject.

### *Third Paper.*

1. Discuss whether the following classes of securities are suitable for the investment of life assurance funds at the present time :

- (1) South American Government Securities ;
- (2) Indian Railway Annuities, class B ;
- (3) Shares in British Insurance Companies (a) transacting only life assurance, (b) transacting fire and accident business ;
- (4) Preference Shares in high-class British Industrial Companies ;
- (5) American Street Railway Bonds.

2. A ground rent which bears a high proportion to the rack rent is purchased, and after some years of bad management and low rentals, the ground rent falls into arrear. In the meantime the

leaseholder has mortgaged his interest. What courses are open to the freeholder?

3. Discuss the question of the influencees that control the course of the chief European exchanges.

4. Describe briefly the principal changes which have occurred in the Money Market since 1 October last.

5. Discuss the changes that have taken place during the past 20 years in the practice of Life Assurance Companies with regard to the proportions of their funds invested in the following classes of securities :

- . Mortgages on Property within the United Kingdom ;
- Mortgages on Property without the United Kingdom ;
- British Government Securities ;
- Railway and other Debentures and Debenture Stocks ;
- Foreign Government Securities.

6. Recently there have been public issues of the following bonds and stocks. Mention any points suggested by the titles of the investments which in your opinion should be considered when deciding as to their suitability for investments of a life insurance company :

- New York Telephone Company 1st Mortgage 4½ per-cent Gold Bonds ;
- Anglo-Argentine Tramways Company 5 per-cent Debenture Stock ;
- Rio de Janeiro State 5 per-cent Bonds ;
- Caledonian Trust Company 4 per-cent Debenture Stock ;
- Cuba Railroad Company Improvement and Equipment 5 per-cent Fifty Year Gold Bonds.

7. Explain the following terms :

- Amortisation ;
- Form of Renunciation :
- Refunding Bonds :
- Adjustment Bonds ;
- Underlying Bonds ;
- Unifying Bonds.

8. To what extent do you consider that an assurance office is justified in taking interest accrued but not due into its annual accounts? Mention the treatment of (a) Mortgages, (b) Indian

Railway "A" Annuities, (c) Preferred Ordinary Railway Stock, and explain how the method you adopt would affect the prices at which the various securities would be valued.

*Fourth Paper.*

1. How would you calculate surrender values for children's deferred assurances after the expiration of the period of deferment?

\*2. A is entitled to the reversion to the balance of the following funds after payment, out of whichever fund falls into possession first, of the sum of £5,000. What is the value of A's interest?

Fund (a)—(subject to the life interest of a lady aged 70): £5,000 Consols.

Fund (b)—(subject to the life interest of a man aged 65): £5,000 invested on mortgage at 4 per-cent and a freehold farm containing 150 acres of land and producing a rental of £200 a year.

3. Under a settlement made on the marriage of A and B, now aged respectively 50 and 46, a life interest in settled funds is reserved to A. After his death the income is payable to B for the remainder of her life and after the death of both of them the capital goes to the issue of the marriage. There has been one child only of the marriage who has just attained the age of 21 years. It is now proposed to realize the settled funds and divide the balance between the interested parties. The settled funds consist of:—

New Zealand 4 per-cent Stock redeemable 1929.

London Brighton and South Coast Railway 4½ per-cent Debenture Stock.

Leasehold Ground Rents having 30 years to run.

Indicate the methods you would follow to ascertain the amount you consider each should receive.

4. A Board of Directors is considering the question of extending its field of investment to include the purchase of reversions and policies of assurance. Draw up a report to the Board as to the advisability of the course and as to the principles on which such purchases, if decided on, should be made.

\*5. A, aged 50, is entitled to the undermentioned fund, subject to the payment thereout of an annuity of £200 to a lady aged 65. He is also entitled to a life interest in another fund

consisting of a sum of £3,000 invested on mortgage of freehold estates at 5 per cent. A wishes to borrow as much as possible on his interests; what are the enquiries you would make and what sum would you be prepared to advise a lender to advance?

Fund: £105 per annum East India Railway Annuity  
Class "A."

£1,300 Bank of England Stock.

£3,000 India 3½ per-cent Stock.

\*6. A, aged 25, is entitled, expectant on the death of his mother, aged 63, to the life interest in estates having a net rental of upwards of £5,000. He wishes to obtain a sum of £10,000 in consideration of a charge on his reversionary life interest, but desires to have the right to repurchase at the end of 5 years by repayment of the advances with interest at 4½ per-cent compounded half-yearly. To keep down the figures during the first five years it is agreed that the whole-of-life policy effected at the outset should be for the amount of the repurchase money at the end of the first five years only.

Using 4½ per-cent interest throughout calculate the amount of the necessary charge on the reversionary life interest and the amount of the redemption money payable (1) at the end of the first five years; (2) on the death of the life tenant.

\* A Short Collection of Actuarial Tables will be supplied for use in answering these questions.

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PROCEEDINGS OF THE INSTITUTE.—SESSION 1912-1913.

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*First Ordinary Meeting, 25 November 1912.*

The President (Mr. FREDERICK SCHOOLING) in the Chair.

The President delivered an Inaugural Address.

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*Second Ordinary Meeting, 16 December 1912.*

The President (Mr. FREDERICK SCHOOLING) in the Chair.

A paper entitled "Some observations on Currency and Credit and their influence on Trade and Exchange", was read in abstract by the Author, Mr. Walter T. May.

The following gentlemen took part in the discussion:—Messrs. G. M. Reeve, H. E. Raynes, Owen Kentish, and G. E. May; and Messrs. Hartley Withers, and P. W. Matthews (visitors).

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*Third Ordinary Meeting, 27 January 1913.*

The President (Mr. FREDERICK SCHOOLING) in the Chair.

A paper entitled "Some aspects of the National Insurance Act, 1911

(Part I—National Health Insurance)", was read in abstract by the Author, Mr. Reginald C. Simmonds.

The following gentlemen took part in the discussion:—Messrs. R. C. Fippard, F. J. Cameron, J. Bacon, A. W. Tarn, R. G. Maudling, V. A. Burrows, and J. Burn.

*Fourth Ordinary Meeting, 24 February 1913.*

The President (Mr. FREDERICK SCHOOLING) in the Chair.

A paper entitled "‘House Purchase’ Companies: The ‘Bond Investment’ sections of the 1909 Act, and some actuarial features of the business returned thereunder", was read in abstract by the Author, Mr. C. H. Maltby.

The following gentlemen took part in the discussion:—Messrs. L. E. Clinton, T. G. Ackland, D. C. Fraser, and G. King.

*Fifth Ordinary Meeting, 31 March 1913.*

The President (Mr. FREDERICK SCHOOLING) in the Chair.

Mr. William Gilbert Craig, F.F.A., was duly elected an Associate of the Institute.

A paper entitled "On the estimated Age-Distribution of the Indian Population, as recorded at the Census of 1911, and the estimated Rates of Mortality, deduced from a comparison of the Census Returns for 1901 and 1911", was read in abstract by the Author, Mr. Thomas G. Ackland.

The following gentlemen took part in the discussion:—Messrs. A. Henry, G. King, and A. T. Winter.

*Sixth Ordinary Meeting, 28 April 1913.*

The President (Mr. FREDERICK SCHOOLING) in the Chair.

A paper entitled "An investigation into the effects of Family and Personal History upon the Rates of Mortality experienced in various classes of Life Assurance risks, with special reference to Tuberculosis", by Messrs. E. A. Rusher and C. W. Kenchington, was read in abstract by Mr. Kenchington.

The following gentlemen took part in the discussion:—Messrs. H. E. W. Lutt, A. D. Besant, H. W. Manly, and W. Palin Elderton; and Dr. S. W. Carruthers (a visitor).

*The Sixty-Sixth Annual General Meeting, 2 June 1913.*

The President (Mr. FREDERICK SCHOOLING) in the Chair.

The proceedings at the Annual General Meeting will be found on page 595.

**REPORT, 1912-1913.**

The Council have the pleasure to report to the Members upon the progress of the Institute during the Session of 1912-1913, the sixty-fifth year of its existence.

There has been a *decrease* of 12 in the total number of members, as compared with the previous year. At the end of the official year in which the Institute was incorporated by Royal Charter the number

[Continued on page 590.]

Dr.			Revenue Account for the	
1912.			£	s. d.
£	s.	d.	Amount of Funds at the beginning of the year—	
9,861	9	7	General Fund (including Stock of Publications, other than <i>Journal</i> )	10,025 0 4
424	11	2	Messenger Legacy Fund	374 5 11
312	4	5	Brown Prize Fund	321 11 9
<u>10,598</u>	<u>5</u>	<u>2</u>		<u>10.720</u> 18
Subscriptions—				
819	0	0	Fellows	838 19 0
708	15	0	Associates	714 0 0
286	13	0	Students	276 3 0
.97	2	6	Probationers	107 12 6
<u>1,911</u>	<u>10</u>	<u>6</u>		<u>1,936</u> 14 6
10	10	0	One Annual Subscription compounded for	31 10 0
3	3	0	Fines on Reinstatement	8 8 0
<u>1,925</u>	<u>3</u>	<u>6</u>		<u>1,976</u> 12
Application Fees—				
4	4	0	Associates	2 2 0
14	3	6	Students	16 5 6
34	13	0	Probationers	41 9 6
53	0	6		59 17
139	11	6	Profit on Sales of Publications	147 18
Dividends and Interest—				
303	2	10	General Fund	324 10 7
12	14	9	Messenger Legacy Fund	11 4 7
9	7	4	Brown Prize Fund	9 12 11
<u>325</u>	<u>4</u>	<u>11</u>		<u>345</u> 8

£13,041 5 7

£13,250 14 ;

*Publications Account for*

<i>£</i>	<i>s.</i>	<i>d.</i>		<i>£</i>	<i>s.</i>
472	12	9	Stock (excluding <i>Journal</i> ) at the beginning of the year . . . . .	371	19
23	16	6	Expenditure on Account of A. W. Watson's Lectures on Friendly Societies	140	11
42	6	7	Cost of New Edition of "Short Collection of Actuarial Tables" . . . . .	...	...
...			Cost of Reprints of Text-Book, Part I . . . . .	43	8
16	6	3	Binding and Advertising . . . . .	6	14
139	11	6	Profit on Sales . . . . .	147	18
694	13	7		£710	13

Balance She.

£	s.	d.	LIABILITIES.	£	s.	d.	£	s.	d.	£	s.
10,025	0	4	General Fund . . . . .				9,523	7	9		
233	9	2	Messenger Legacy Fund . . . . .				233	9	2		
140	16	9	Accumulated Dividends . . . . .				152	1	4		
374	5	11								385	10
200	0	0	Brown Prize Fund . . . . .				200	0	0		
121	11	9	Accumulated Dividends . . . . .				131	4	8		
321	11	9					331	4	8		
81	18	0	Examination Fees for year 1913 . . . . .							80	17
86	4	0	Sundry unpaid Accounts . . . . .							50	10
£10,889	0	0								£10,371	10

Year ending 31 March 1913.

£. s. d.

1912.			1913.		
£	s.	d.	£	s.	d.
771	13	9	Printing of Nos. 249, 250, 251, 252 . . . . .	523	11 10
86	5	0	Clerical assistance . . . . .	86	5 0
857	18	9		609	16 10
280	3	1	Less Sales during the year . . . . .	260	12 8
577	15	8			349 4 2
76	8	0	Library—Binding, Purchases, &c. . . . .		62 14 8
64	10	4	Meetings . . . . .		66 11 9
303	15	6	Examination charges . . . . .	357	10 1
225	15	0	Less Fees received from Candidates (1912) . . . . .	208	19 0
78	0	6			148 11 1
296	2	0	Tutors for classes in Parts I and II . . . . .	329	14 0
212	2	0	Less Fees received from Students . . . . .	245	14 0
84	0	0			84 0 0
66	16	0	Legal Charges . . . . .		...
600	0	0	Office Expenditure—Rent . . . . .	600	0 0
352	6	6	Salaries . . . . .	404	2 1
73	13	5	House expenses . . . . .	72	10 2
15	5	11	Corporation Duty . . . . .	16	0 6
29	8	11	Fire and other Insurance . . . . .	30	8 2
118	12	9	Stationery and Printing . . . . .	133	4 5
15	18	3	Furniture and Fittings . . . . .		...
43	18	6	Postage and Telegrams . . . . .	45	12 3
8	2	10	Sundries . . . . .	3	8 1
257	7	1			1,305 5 8
52	10	0	Donation to Sir Francis Galton Laboratory Fund . . . . .		...
63	0	0	Honorarium for Lectures on Friendly Societies, by A. W. Watson (charged to Messenger Legacy Fund) . . . . .		...
...			Amount written off in respect of decrease in value of Stock Exchange Securities . . . . .	994	4 3
720	18	0	Amount of Funds at the end of the year as per Balance Sheet . . . . .	10,240	2 11
			Examined and found correct, 1 May 1913.		
41	5	7	H. J. PEARCE, } H. LUCEY, } W. G. TITMUSS, } Auditors.	£13,250	14 6

Year ending 31 March 1913.

£	s.	d.	£	s.	d.
302	13	11	Sales (excluding Journal) . . . . .	367	4 4
20	0	0	Received in Settlement of Claim on Fire Insurance Policy for Depreciation . . . . .		
71	19	5	Stock (excluding Journal) at the end of the year . . . . .	343	8 8

Examined and found correct, 1 May 1913.

H. J. PEARCE, }  
H. LUCEY, }  
W. G. TITMUSS, }

£710 13 0

3 March 1913.

£	s.	d.	ASSETS.	£	s.	d.
60	0	0	£3,000 Natal 3 per-cent Inscribed Stock . . . . .	2,370	0	0
128	0	0	£1,200 Metropolitan Railway 3½ per-cent Debenture Stock . . . . .	1,008	0	0
279	11	3	£2,000 Great Eastern Railway 4 per-cent Debenture Stock . . . . .	1,980	0	0
95	0	0	£1,000 Great Northern Railway Preferred Ordinary Stock . . . . .	835	0	0
130	10	0	£1,350 Great Western Railway 4½ per-cent Debenture Stock . . . . .	1,431	0	0
78	3	6	£500 Dominion of Canada 3½ per-cent Registered 1930-50 Stock . . . . .	465	0	0
11	18	6	£1,000 New South Wales 3½ per-cent Inscribed 1930-50 Stock . . . . .	890	0	0
71	19	5	Stock of Publications (excluding Journal) in hand . . . . .	343	8	8
00	0	0	Cash on Deposit Account . . . . .	350	0	0
33	17	1	Cash on Current Account and in hand . . . . .	699	1	7

Examined and found correct, 1 May 1913.

H. J. PEARCE, }  
H. LUCEY, }  
W. G. TITMUSS, }

£10,371 10 3

of Members was 434, while twenty years later, at 31 March 1905, it was 881. Since that time the numbers have been as follows:

On 31 March	Fellows	Associates	Students	Honorary and Corresponding Members	Total
1906	232	301	367	22	922
1907	248	303	383	22	956
1908	253	313	421	22	1,009
1909	254	325	400	19	998
1910	259	335	348	21	963
1911	267	339	308	20	934
1912	278	354	268	20	920
1913	282	355	252	19	908

The following schedule shows the additions to, and the changes and losses in the membership which have occurred during the year ending 31 March last :

*Schedule of Membership, 31 March 1913.*

	Fellows	Associates	Students	Corre-sponding Members	Total
i. Number of Members in each class on 31 March 1912 .	278	354	268	20	920
ii. Withdrawals by (1) Death . . .	7	5	1	1	52
(2) Resignation or otherwise . . .	1	9	28	...	
	270	340	239	19	868
iii. Additions to Membership (1) By Election . . .	...	2	...	...	40
(2) By Order of Council . . .	...	...	22	...	
(3) By Re-instatement . . .	...	4	12	...	
	270	346	273	19	908
iv. Transfers (1) By Examination: <i>from Associates to Fellows</i> . . .	...	9	...	...	...
9	...	...	...	...	...
	279	337	273	19	908
(2) By Examination: <i>from Students to Fellows</i> . . .	...	...	3	...	...
3	...	...	...	...	...
	282	337	270	19	908
(3) By Examination: <i>from Students to Associates</i> . . .	...	...	18	...	...
...	18	...	...	...	...
	282	355	252	19	908
v. Number of Members in each class on 31 March 1913 .	282	355	252	19	908

There are also 197 candidates admitted as Probationers, and 55 as Students conditionally on their passing Part I of the Examination. These are not included in the above Schedule of Membership. The numbers in these two classes since 31 March 1908 have been as follows:

On 31 March	Probationers	Conditional Students
1909	128	39
1910	141	42
1911	160	58
1912	181	59
1913	197	55

The Council have, with great regret, to report the loss by death, since the last Annual Meeting, of seven Fellows, Messrs. A. H. Bailey, T. H. Cooke, R. C. Fisher, J. B. Gillison, N. B. Gunn, R. P. Hardy, and R. Wilson, senior; five Associates, Messrs. R. G. Hann, R. A. Hunt, E. W. Scott, H. S. Smith, and T. S. Wornum; one Student, Mr. F. Bedford; and one Corresponding Member, M. Léon Marie.

By the deaths of Mr. Arthur Hutcheson Bailey and Mr. Ralph Price Hardy, the Institute loses two of its most prominent and highly esteemed members.

Mr. Bailey's connection with the Institute dates back to 29 January 1849. He was one of the candidates who sat for the first examination held by the Institute in 1850, when he was awarded the certificate of competency and elected a Fellow. He was President of the Institute for the four years 1878-82. He died in his eighty-ninth year. His genial and courteous manner endeared him to all with whom he came in contact.

Mr. Hardy's career was no less distinguished. As Vice President of the Institute and especially as occupant during the twelve years of its existence of the arduous position of Chairman of the joint Institute and Faculty Committee which conducted the last Mortality Investigation, he rendered to his profession service of the highest value.

The Annual Subscriptions, together with admission and other fees, amounted to £2,491. 2s. 6d., as compared with £2,416. 1s. 0d., received in the previous year. The net Income and Expenditure for the year were £2,529. 16s. 6d., and £2,016. 7s. 4d. respectively.

Six years ago the Stock Exchange securities held by the Institute were written down to the value of the day. The Council have considered it advisable to write off the sum of £994 4s. 3d., having regard to the further depreciation which has taken place. This amount accordingly appears in the Revenue Account as an additional charge.

The stock in hand of the Institute publications on 31 March was as follows:

No. of Copies	Description of Work
22,727 . . . .	Parts of <i>Journal</i> .
751 . . . .	Index to Vols. 1 to 40.
100 . . . .	<i>Text-Book</i> , Part I (New Edition).
365 . . . .	" Part II (Second Edition).
645 . . . .	Government Joint-Life Annuity Tables.
743 . . . .	Select Life Tables.
603 . . . .	A Short Collection of Actuarial Tables (New Edition).

No. of Copies.	Description of Work.
1,281 . . . .	Frequency-Curves and Correlation (W. P. Elderton).
36 . . . .	Messenger Prize Essay (Friendly Societies).
42 <i>in cloth</i> } . . . .	(Lectures on Finance and Law (Clare and Wood Hill).
2,372 <i>in paper</i> } . . . .	
1,542 . . . .	Lectures on the Companies Acts (A. C. Clauson).
1,256 . . . .	Lectures on the Law of Mortgage (W. G. Hayter).
732 . . . .	Lectures on the Measurement of Groups and Series (A. L. Bowley).
1,519 . . . .	Lectures on the Construction of Tables of Mortality, &c. (G. F. Hardy).
1,052 . . . .	Lectures on Stock Exchange Investments (J. Burn).
1,606 . . . .	Lectures on Friendly Society Finance (A. W. Watson).
332 . . . .	South African War Mortality (F. Schooling and E. A. Rusher).
299 . . . .	Barrand's Paper on Life Assurance Law.
710 . . . .	British Offices' Valuation Tables.
650 . . . .	Transactions of the Second International Congress of Actuaries.
1,500 . . . .	Examination Questions, 1909-12.

The following papers were submitted at the sessional meetings of the Institute, namely :

- 25 November 1912.—Inaugural Address by the President, Mr. Frederick Schooling.
- 16 December 1912.—“Some Observations on Currency and Credit and their Influence on Trade and Exchange.”—Mr. W. T. May.
- 27 January 1913.—“Some aspects of the National Insurance Act, 1911 (Part I.—National Health Insurance).”—Mr. Reginald C. Simmonds.
- 24 February 1913.—“‘House Purchase’ Companies: The ‘Bond Investment’ sections of the 1909 Act, and some actuarial features of the business returned thereunder.”—Mr. C. H. Maltby.
- 31 March 1913.—“On the estimated Age-Distribution of the Indian Population, as recorded at the Census of 1911, and the estimated Rates of Mortality, deduced from a comparison of the Census Returns for 1901 and 1911.”—Mr. Thomas G. Ackland.
- 28 April 1913.—“An investigation into the effects of Family and Personal History upon the Rates of Mortality experienced in various classes of Life Assurance risks, with special reference to Tuberculosis.”—Messrs. E. A. Rusher and C. W. Kenchington.

For the Examinations held in the United Kingdom and the Colonies on 21, 22, 23, 24, 25 and 26 April 1913, 228 entries were received, namely :

98	for Part	I.
17	“	I, (§) 3.
45	“	II.
45	“	III.
23	“	IV.

The results will be duly announced. The Council warmly acknowledge the valuable services of the Board of Examiners, and also those of the Honorary Supervisors at centres other than London.

The Council have to record their thanks to Mr. R. Todhunter and Mr. J. Spencer for their services as Joint Editors of the Journal during the past year. The Council are pleased to be able to report that the Editorship of the Journal will continue for the present in the efficient hands of these gentlemen.

The Seventh International Congress of Actuaries was held in Amsterdam last September under the patronage of His Royal Highness Prince Henry of the Netherlands, Duke of Mecklenburg. The Council appointed as official delegates on behalf of the Institute the following gentlemen, namely, the President of the Institute (Mr. Frederick Schooling), Messrs. H. W. Andras and W. P. Phelps (Vice-Presidents), Mr. R. R. Tilt, one of the Honorary Secretaries, and Messrs. T. G. Ackland, A. R. Barrand, J. Burn, V. Marr, J. D. Watson and E. Woods. Mr. Woods again performed the onerous duties of Honorary Correspondent to the Congress for England. The subjects discussed at the Congress were of considerable professional interest, and several papers were contributed to the Proceedings by members of the Institute.

## THE INSTITUTE OF ACTUARIES.

### EXAMINATIONS, 1913.

Examinations were held on the 21st, 22nd, 23rd, 24th, 25th and 26th of April 1913, in the United Kingdom, the Colonies, and India, at London, Liverpool, Edinburgh, Dublin, Melbourne, Sydney, Perth, Wellington, Montreal, Toronto, Ottawa, Winnipeg, Calcutta, with the following results.

The successful candidates are placed in two classes only, the names being printed in alphabetical order in each class.

#### PART I.

Ninety-eight candidates sent in their names, of whom eighty-nine presented themselves (seventy-nine in the United Kingdom, and ten in the Colonies), and thirty-one passed, namely:—

#### *Class I:*

Andras, J. B.	Haalmeyer, B. P.
Fox, G. A.	McConaghy, C.A.

#### *Class II:*

Barkworth, A. G.	Menzler, F. A. A.
Barrett, C. C. C.	Moore, W. R.
Berry, J. R.	Nicholson, W. R. I.
Blake, W. T. C.	Rackham, G.
Brown, E. A. L.	Rowell, A. H.
Carse, J. W.	Sen-Gupta, S.
Chase, P. C.	Snell, R. A.
Chatham, E. F.	Waller, F. W.
Clarke, J. H.	Watson, D. J.
Coleman, H. D.	Weyer, D.
Cundall, H. A.	Williamson, E. R.
Johnson, A. N.	Woodrow, G.
Kemp, E. C.	Young, H. M.
Kerr, A. P. T.	

## PART II.

Forty-five candidates sent in their names, of whom forty-three presented themselves (thirty-five in the United Kingdom, and eight in the Colonies), and seventeen passed, namely:—

*Class I:*

None.

*Class II:*

Bazell, H.	Richardson, G. R.
Carpmael, C.	Ruddle, F.
Denmark, R. J.	Shepherdson, H. J.
Hawkins, L. F.	Smith, V. R.
Hurd, H. G.	Spurgeon, C. B.
Lithgow, J. H. F.	Tambe, M. R.
Martin, W. A.	Wallis, H.
Morton, Francis.	Williamson, W. W.
O'Brien, H.	

## PART III.

Forty-five candidates sent in their names, of whom forty-four presented themselves (thirty-seven in the United Kingdom, and seven in the Colonies), and twenty-one passed, namely:—

*Class I:*

None.

*Class II:*

Alison, S. H.	Jones, E. S.
Anderson, R. D.	Linton, M. A.
Baker, S. H.	Mann, F. C.
Bennett, S.	Monkhouse, C. C.
Blake, F. S.	Owen, D. J.
Bradshaw, F. L.	Spiegel, E. W. R.
Burrows, G. E.	Sturgeon, R. W.
Clarke, H. T.	Tyler, V. W.
Craig, W. G.	Wisdom, S. H.
Doyle, J. P.	Wolfenden, H. H.
Gawler, O.	

## PART IV.

Twenty-three candidates sent in their names, all of whom presented themselves (twenty-two in the United Kingdom, and one in the Colonies), and sixteen passed, namely:—

*Class I:*

None.

*Class II:*

+Blyth, R. O.	+Lever, E. H.
+Bullwinkle, L. A.	+Lewty, F. A.
+Duffell, J. H.	+McKechnie, J. B.
+Epps, G. S. W.	+Nathan, E. B.
+Fulford, W. J.	+Osborne, W. A.
+Harvey, P. N.	+Peters, C. F.
+Holness, A. S.	+Phillips, E. W.
+Hutton, G. C.	+Rhodes, F.

Those marked (+) have now completed the Examination for the Class of Fellow.

PART I, § 3.  
(COMPOUND INTEREST AND ANNUITIES.)

Seventeen candidates, who had already passed, or been exempted from, Part I of a Syllabus prior to 1908, entered for this section alone and presented themselves (fifteen in the United Kingdom, and two in the Colonies), of whom nine passed, namely:—

PART I.—THIRD PAPER ONLY.

Aldridge, W. H.	Smith, W.
Grant, F. J.	Thompson, J. W.
Hobbins, C. B.	Wellisch, F.
Kearns, W. N.	Williamson, W. W.
Purry, W. B.	

By Order of the Council,

A. LEVINE,  
*Chairman of Board of Examiners.*

R. R. TILT,  
R. TODHUNTER,  
*Joint Honorary Secretaries.*

11th June 1913.

PROCEEDINGS AT THE ANNUAL GENERAL MEETING.

The Sixty-sixth Annual General Meeting of the Institute was held at Staple Inn Hall on Monday, 2 June 1913, the President (Mr. Frederick Schooling) being in the Chair.

The Report of the Council having been taken as read,

The PRESIDENT, in moving the adoption of the Report and Accounts, said it would be noticed that during the year the Institute had lost many eminent members of the profession; he had on other occasions referred to the services rendered to actuarial science by Mr. Bailey, Mr. R. P. Hardy, Mr. N. B. Gunn, and M. Leon Marie, and to the appreciation in which they were held by their comrades and professional brethren. It would also be

noticed that the number of Fellows was greater at the present time than in any previous year, although the number of members was one hundred less than it was five years ago. It must be borne in mind, however, that a class of Probationers now existed; that class, together with the conditional students—who might be termed probationers residing abroad—showed an increase of 85; so that the total membership exhibited a decrease of 15 only.

The year, from an actuarial point of view, had been a busy one. The work connected with the National Insurance Act had been heavy, and had given direct employment to no fewer than 20 members of the Institute, who had received appointments under the National Insurance Commissioners. A vast amount of labour had also been thrown upon actuaries generally in connection with the Approved Societies. Another feature of the year had been the continuous success of the Students' Society. It was a matter of great congratulation that that was so, and the Council rejoiced in the great interest and ability shown in the proceedings of that society. At the same time the importance of the Institute's Sessional Meetings, and the opportunity thus given to all members, both seniors and juniors, must be borne in mind. The Honorary Secretaries were anxious to fill the dates for the next session as soon as possible, so that they might be able to make certain of having good, able, and interesting papers.

During the whole of the late session the Institute, in conjunction with the Faculty of Actuaries, had been endeavouring to lay the foundations for a permanent Research Bureau. At one time it was thought the Institute would be able to obtain from the life offices details of business which had been running during the last ten years, but the replies received to the circular letter proved that that would not be practicable. It was, perhaps, well that that was so, because there would have been very grave doubt as to the uniformity of the classifications, and, although it might mean waiting for results, accuracy was of more importance than speed. It was hoped that a start would be made on a common basis, whereby not only under-average lives could be dealt with, but, what he thought was of more importance, the mortality of all assured lives and of all annuitants could be followed and tabulated. The Institute had received a communication from their Scottish friends, saying that they would be glad to fall in with their views, and he thought the time had now arrived when a Provisional Joint Committee should be appointed for the purpose of settling the very many preliminary questions involved. With those remarks he had very great pleasure in moving that the Report and Accounts be adopted.

Mr. GEOFFREY MARKS (Vice-President), in seconding the motion, said he had been very pleased—as he was sure everybody who studied the proceedings of the Institute would be—to see the very practical nature of the questions which were dealt with in the papers read before the Institute during the past session. He thought the choice of subjects and the manner in which they had been dealt with were very much to the credit of those who undertook to read papers. The highly technical essays, which at one time formed the subject of the papers read before the Institute, were not so suitable for that purpose as for actuarial notes in the *Journal*.

The motion for the adoption of the Report and Accounts was then put and carried unanimously.

#### ELECTION OF OFFICERS.

Messrs. E. F. SPURGEON and W. A. SIM were appointed Scrutineers of the ballot for the election of officers; and the President subsequently announced that the following gentlemen had been unanimously elected:

*President.*

FREDERICK SCHOOLING.

*Vice-Presidents.*

GEOFFREY MARKS.  
GEORGE JAMES LIDSTONE.

WILLIAM PEYTON PHELPS, M.A.  
LEWIS FREDERICK HOVIL.

*Council.*

THOMAS GANS ACKLAND.  
\*HENRY JAMES BAKER.  
ARTHUR RHYS BARRAND.  
\*ARTHUR DIGBY BESANT, B.A.  
JOSEPH BURN.  
\*FREDERICK TIMOTHY MASON BYERS.  
ROBERT CROSS.  
GEORGE FRANCIS HARDY.  
JAMES ROBERT HART.  
LEWIS FREDERICK HOVIL.  
\*GEORGE KING.  
ABRAHAM LEVINE, M.A.  
GEORGE JAMES LIDSTONE.  
GEOFFREY MARKS.  
VVYVAN MARR.

ALFRED MOORHOUSE.  
\*HARRY ETHELSTON NIGHTINGALE.  
WILLIAM PEYTON PHELPS, M.A.  
SIR GERALD HEMMINGTON RYAN.  
FREDERICK SCHOOLING.  
WILLIAM RICHARD STRONG.  
HERBERT CECIL THISELTON.  
ROBERT RUTHVEN TILT.  
GEORGE TODD, M.A.  
RALPH TODHUNTER, M.A.  
HAROLD MOLTKE TROUPER, M.A.  
SAMUEL GEORGE WARNER.  
ALFRED WILLIAM WATSON.  
ERNEST WOODS.  
FRANK BERTRAND WYATT.

*Treasurer.*

SAMUEL GEORGE WARNER.

*Honorary Secretaries.*

ROBERT RUTHVEN TILT.

| RALPH TODHUNTER, M.A.

\* New Members of the Council.

MR. STANLEY HAZELL, in moving "that Messrs. H. Lucey, W. G. Titmuss, and A. Taylor, be elected Auditors for the ensuing year", said that the names of the Auditors were appended to the Report, and, in view of the excellent way in which they carried out their work it would, perhaps, be the simplest plan if the Members were to ask them to make their office a permanent one and to act as Auditors until they became Fellows, or for the term of their natural lives. But there were two drawbacks to that plan, the first being that it would debar other Associates from taking their share in the official work of the Institute, a privilege they prized very highly and would be loth to give up, and the second that it would not be in accordance with the regulations of the Institute, which provided that two of the old Auditors should continue their work and that another Associate should be elected to join them. He had great pleasure in moving the resolution.

MR. E. B. NATHAN seconded the motion, which was carried unanimously.

MR. H. E. NIGHTINGALE, in moving a vote of thanks to the President, Vice-Presidents, Council, Officers, Examiners, and Honorary Supervisors at centres other than London, for their excellent services during the past Session, said that although the vote had become almost customary at the annual meetings he thought real importance should be attached to it because the work of the Council was of a very responsible nature. In the present days of competition it was no easy task to preserve the membership and influence of the Institute in the life assurance world.

MR. OWEN KENTISH, in seconding the motion, said that they were fortunate in having had in the Chair of the Institute during the past year a President who had rendered many services to the profession in connection

with financial questions. They were indebted also to the Vice-Presidents and other Officers for the work they performed.

Mr. RICHARD TEECE (Sydney, Australia) said it went without saying that a resolution of such a character required no advocacy from any Fellow present, especially when they read the very satisfactory report which had been presented. He desired, however, to express the pleasure he felt in being present for the first time at a meeting of the Institute, and to convey a message of thankfulness from the Actuaries of Australia to the home body in this country. He especially wished to emphasize the value of the concessions which had been recently made with regard to examinations conducted in Australia. Australia was a country in process of formation. Its citizens were largely occupied with utilitarian matters, and there was a strong disposition to avoid that culture which was so essential to the building up of the mental calibre of a young nation. There was a fear that, under the existing regulations, the supply of candidates for the Degree of the Institute would have been materially minimized, if it had not entirely ceased. But, thanks to the concessions which had been made, and to the consideration with which the Council had treated Australia, it would now be able to send in increasing numbers young men of the best talent who would seek advancement in the actuarial profession in Australia, and for that concession he desired in the name of Australia to accord their very best thanks to the Council. In conclusion, he expressed the wish that the Institute would continue in the future, as it had done in the past, to be the home of all that was best and most advanced in actuarial science, and that it would continue to afford those examples of high conduct in business by which it had been distinguished in the past, and which Australians, as far as their opportunities had offered, had endeavoured to imitate.

The resolution was then put to the meeting by Mr. Nightingale and carried with acclamation.

The PRESIDENT thanked the mover and seconder of the resolution, and the whole of those present, most heartily for the kind vote which had been passed. He was quite certain that all his colleagues had done their best throughout the session to carry on the work of the Institute. Perhaps the Examiners had had the most difficult rôle to play, because it was very hard work to find new and suitable questions for the students each year, and he thought they had succeeded wonderfully well. He thanked Mr. Teece very much indeed for his remarks, and was certain that the Institute was proud of being a pioneer body which had its members all over the world, and that it would always do its utmost to maintain those members in a high state of efficiency.

Mr. A. W. TARN, in proposing a vote of thanks to the Auditors—Messrs. Pearce, Lucey and Titmuss—for their services during the past year, said that he had the curiosity that afternoon to look through the accounts of the first meeting he had the honour to attend, in 1879. He found that the funds of the Institute then amounted to £2,800, whereas they now reached the total of £10,000, notwithstanding the fact that £1,000 had been written off their value. That showed that the duties of the Auditors were considerably more arduous than they were in 1879, particularly when the increase of membership that had also taken place was borne in mind. As the only acknowledgment the Auditors received was the appreciation of their fellow members, he was sure they would accord them a very hearty vote of thanks.

Mr. H. E. MELVILLE, in seconding the motion, said the services of the Auditors, though not ostentatious, were very valuable, and the members recognized them as such.

The resolution was put and carried unanimously.

The PRESIDENT then adjourned the meeting to Monday, the 24th November, at 5 o'clock.

*Additions to the Library.*

The following works have been added to the Library since the publication of the *Journal* for October 1912:

*By whom presented  
(when not purchased).*

Accountants and Auditors, Society of Incorporated		
List of Members, &c., 1912-13.		<i>The Society.</i>
Accountants, Institute of Chartered, in England and Wales.		
List of Members, 1913.		<i>The Institute.</i>
Actuarial Society of America.		
Transactions, 1912-13.		<i>The Society.</i>
Containing <i>inter alia</i> —		
“The effect on Select Tables of a variation in the Rates of Mortality to which the lives involved are subject”, by P. C. H. Papps.		
“The Mortality Experience of the Mutual Benefit Life Insurance Company on paid-up Policies issued in lieu of surrendered Policies”, by E. E. Rhodes.		
“The basis for Employers’ contributions toward Service Pensions”, by A. H. Mowbray.		
“Select and Ultimate gain on single premium Policies”, by E. W. Marshall.		
“Modern Surrender Values”, by J. F. Little.		
“A New Annuity Experience”, by J. S. Thompson.		
“A Theory of Sub-standard lives”, by A. W. Whitney.		
“Mortality surplus and its distribution”, by H. Moir.		
“Concerning the American Experience Table of Mortality”, by S. A. Joffe.		
“Effect on mortality of selection by the insured: mortality after the deferred dividend period”, by A. Hunter.		
“Provisions for expenses in Savings Contracts”, by M. M. Dawson.		
“Total disability benefits; with special reference to Pension Fund methods of calculation”, by J. F. Little.		
Actuaries, Faculty of		
Transactions, 1912-13.		<i>The Faculty.</i>
Containing <i>inter alia</i> —		
“Errors in Age”, by J. A. Rankin.		
“Peerage Males. Statistics of Mortality, First Marriage and Issue”, by R. M. Hunter.		
American Mathematical Society.		
Transactions, 1912-13.		<i>The Society.</i>
American Statistical Association.		
Transactions, 1912-13.		<i>The Association.</i>
Assecuranz-Almanach.		
Handbuch für Versicherungs-Recht und Technik. Herausgegeben von Dr. A. F. Elsner's Erben. Svo. Berlin. 1913.	}	<i>The Publishers.</i>
Assecuranz Jahrbuch.		
Herausgegeben von A. Ehrenswieg. Vol. 34. 8vo. Wien. 1913.	}	<i>Purchased.</i>

Association des Actuaires Belges.	<i>By whom presented (when not purchased).</i>
Bulletin, 1912-13.	<i>The Association.</i>
Associazione Italiana Attuari.	<i>The Association.</i>
Bollettino, 1912-13.	
Australian Mutual Provident Society.	<i>The Society.</i>
Sixty-fourth annual report, 1913.	
Austria-Hungary.	<i>The Society.</i>
Bericht der Arbeiter-Unfall-Versicherungs-Anstalt für das Königreich Böhmen, 1911.	
Die Privaten Versicherungsunternehmungen in den im Reichsrathe vertretenen Königreichen und Ländern, 1910.	<i>The Austrian Government.</i>
Versicherungswissenschaftliche Mitteilungen der Mathematisch - Statistische Vereinigung des Österreichisch-ungarischen Verbandes der Privat-Versicherungs-Anstalten, 1912-13.	
Zeitschrift für öffentliche und private Versicherung, 1912-13.	<i>The Editor.</i>
The Mortality Experience of Hungarian Assured Lives, 1876-1900. 4 vols. 8vo. Budapest. N.D.	<i>The Executive Committee.</i>
Bachelier (L.).	
Caleul des Probabilités. Les Probabilités semi-uniformes. Paris. 1913.	
Les Probabilités cinématiques et dynamiques. Paris. 1913.	<i>The Author.</i>
Barker (D. A.).	<i>Purchased.</i>
The Theory of Money. Sm. 4to. 1913.	
Barrand (A. R.).	
Legal Notes. Reprinted from the Journal of the Institute of Actuaries, Vols. XLI-XLV. In one volume. 8vo. 1907-1911.	<i>Official.</i>
Belgium.	
Bulletin du Comité central du travail industriel, 1912-13.	<i>Le Comité.</i>
Bulletin du Syndicat des Compagnies d'Assurances-vie populaires opérent en Belgique, 1912-13.	<i>Le Syndicat.</i>
Compte Rendu des Opérations et de la Situation de la Caisse Générale d'Epargne et de Retraite. 1912.	<i>The Belgian Government.</i>
"Biometrika."	<i>Purchased.</i>
Volume IX, Parts I and II. Containing <i>inter alia</i> —	
"On the probable error of a coefficient of Correlation as found from a fourfold table", by Prof. Karl Pearson.	
"The intensity of natural selection in man", by E. C. Snow.	
"On errors of random sampling in certain cases not suitable for the application of a 'normal' curve of frequency", by M. Greenwood, Jr.	
"On the probable error of the Correlation coefficient to a second approximation", by H. E. Soper.	
Blaschke (Dr. E.).	
Die Sterblichkeit der Versicherten zu verschiedener Zeit. Wien. 1913.	<i>The Author.</i>
Cameron (J. C.).	
Reinsurance of excess risks. Texas. 1913.	<i>The Author.</i>

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**Canadian Annual Financial Review.**

Compiled by W. R. Honston. With Appendix. Ob. 8vo. }  
Toronto. 1912, 1913. }  
Purchased.

**Charlier (C. V. L.).**

Contributions to the Mathematical Theory of Statistics :  
1. Theorems of Poisson and Lexis. 2. Homograde and heterograde statistics. 3. Some fundamental problems in the Theory of Probability. 4. The statistical series in homograde statistics. Stockholm. 1912.  
} Purchased.

**Clare (G.).**

The A B C of the Foreign Exchanges. 5th Edit. 8vo. }  
1911. }  
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**Compagnie d'Assurances Générales.**

Notes pour servir à une Bibliographie française de l'assurance sur la vie. 8vo. Paris. 1878. }  
} Purchased.

**Dawson (M. M.).**

Elements of Life Insurance. 3rd edit. 8vo. New York, 1911. }  
} Purchased.

**Denmark.**

Beretning fra Forsikringsraadet for aaret, 1911. }  
Another Copy. }  
Statsanstalten for Livsforsikring. Den 8de femaarige Risiko-Opgorelse. Copenhagen. 1912. }  
} The Danish Government.  
II. W. Andras. The Danish Government.

**Deutscher Verein für Versicherungs-Wissenschaft.**

Veröffentlichungen, 1912-13. Zeitschrift für die gesamte Versicherungs-Wissenschaft, 1912-13. }  
} The Society.

**Dowell (S.).**

The Acts relating to the Income Tax. 7th edit., by J. E. Piper. 8vo. 1913. }  
} Purchased.

**Drapers' Company Research Memoirs.**

Studies in National Deterioration. 4to.  
VIII. A fourth study of the statistics of Pulmonary Tuberculosis. The mortality of the Tuberculous. Sanatorium and Tubereulin treatment. By W. Palin Elderton and S. J. Perry. 1913.  
Biometric Series. 4to.

VIII. A Monograph on Albinism in Man. Parts II and IV. Text and Atlas. By IX. Prof. Karl Pearson, E. Nettleship, and C. H. Usher.

**Dryden (J. F.). 1839-1911.**

In Memoriam volume. 4to. New York. 1912. }  
} Prudential Insur. Co. of America.

**Duboisdenghien (L.).**

L'oeuvre de Comptabilité. Répartition et Capitalisation. Institutions de Prévoyance, etc. 4to. Brussels. N.D. }  
} L'Association des Actuaires Belges.

**Dumas (S.).**

Le taux de l'intérêt dans l'assurance sur la vie en Suisse. Bern. 1913. }  
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Journal of the, 1912-13. }  
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**Eugenics Laboratory.**

Memoirs. 4to.

XVIII. On the correlation of Fertility with social value. A co-operative study. By Ethel M. Elderton, Prof. Karl Pearson, and others. 1913.

Lecture Series. 8vo.

IX. Darwinism, Medical progress, and Eugenics. By Prof. Karl Pearson. 1912.

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Versicherungs-Statistik über die Reichsaufsicht stehenden Unternehmungen. 1911.

*K. A. für  
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ung.*

**Gibbon (I. G.).**

Medical benefit. A study of the experience of Germany and Denmark. 8vo. 1912.

*Purchased.*

**Gotha.**

Rechenschaftsbericht der Gothaer Lebensversicherungsbank auf Gegenseitigkeit für das Jahr 1912.

*The Gotha  
Life Office.*

**Harman (J. E.).**

The Finance Act, 1894, and subsequent legislation as to the Death Duties, and particularly Estate Duty and Settlement Estate Duty, and the Finance (1909-10) Act, 1910, as to Increment Value Duty, especially as to its payment on the occasion of Death. 3rd edit. 8vo. 1910.

*Purchased.*

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Insurance Law in Australasia. (Life, Fire, Marine, and other kinds of insurance). 8vo. Sydney. 1912.

*The A.M.P.  
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Archief voor de Verzekerings-Wetenschap. 1912-13.

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*The Society.*

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*The Institute.*

Catalogue de la Bibliothèque. 8vo. Paris. 1913.

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*The Institute.*

List of Members, 1913.

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**Insurance and Actuarial Society of Glasgow.**

Transactions 1905-1910. Sixth Series.

*The Society.*

Containing *inter alia*—

“Fluctuations in Security Values”, by W. S. Nicol.

“Progress and Position of Friendly Societies”, by R. Gordon-Smith.

“Some observations on the comparative death rates of Abstainers and Non-abstainers in Life Assurance Companies”, by E. Duncan.

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(when not purchased).*

**Insurance Institute of America.**

Proceedings of the Fourth Conference, 1912. Svo. }  
Boston. 1912. } ) The Institute.

**Insurance Institute of Birmingham.**

Transactions, 1911-12. }  
Containing *inter alia*— } The Insurance  
Institute.

- “Some notes on present-day competition”, by W. A. Robertson.
- “Early British Assurance Offices (Provincial)”, by G. W. Bain.

**Insurance Institute of Great Britain and Ireland.**

Journal of the. Volume XV. 1912. }  
Containing *inter alia*— } The Insurance  
Institute.

- “Insurance against Unemployment”, by D. C. Fraser.
- “Life Office Reserves”, by P. L. Newman.
- “The Medical Report: Its place in the selection of lives”, by Dr. G. Williamson.
- “The National Insurance Act”, by W. Worthington.

**Insurance Institute of London.**

Journal of the. 1907-8 to 1911-12. }  
Containing *inter alia*— } The Insurance  
Institute.

- “The early history of Accident Insurance”, by A. Foot.
- “The investments of Life Assurance Companies”, by J. Burn.
- “Some notes on the Workmen’s Compensation Act, 1906”, by H. T. Sneezum.
- “Special Policies”, by W. Penman.
- “A review of the working of the Workmen’s Compensation Act”, by H. Collett.
- “Returns of Life Assurance Companies to the Board of Trade”, by H. W. Andras.
- “Income Tax as affecting Life Assurance Companies”, by C. H. Maltby.
- “Side-lights on medical technicalities from an insurance point of view”, by Dr. R. J. Collie.
- “Some points arising in Life Assurance practice”, by H. E. Melville.
- “The National Insurance Bill in its relation to the Insurance Companies”, by A. Hair.
- “Fraud and its detection in Accident Insurance claims”, by Sir J. Collie.

**Insurance Institute of Toronto.**

Proceedings, 1911-12. }  
Containing *inter alia*— } The Insurance  
Institute.

- “The British National Insurance Bill, 1911”, by H. R. Stephenson.
- “Some points on Life Insurance from an actuary’s standpoint”, by A. F. Hall.
- “The reinstatement of Life Policies”, by W. A. Hines.
- “The adjustment of Liability claims”, by W. V. Blissett.
- “Pension Funds for staffs of Insurance Companies”, by F. Sanderson.

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**Insurance Institute of Yorkshire.**

Proceedings, 1911-12.

}      *The Insurance  
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C. & E. Layton.*

**Insurance Register, 1913.**

**International Congresses of Actuaries.**

Bulletin of the Permanent Committee, 1913.

}      *The Permanent  
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Proceedings of the Seventh Congress held in Amsterdam, September, 1912. 2 vols. Svo. 1912-13.

}      *Purchased.*

**Jacobs (B.).**

A short treatise on the law of Bills of Exchange, Cheques, Promissory Notes, and Negotiable Instruments generally. Svo. 1913.

}      *The Publishers.*

**Josephson (A. G. S.).**

A list of books on the History of Science. 8vo. Chicago. 1911.

}      *The John Crerar  
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Vergleichende Zusammenstellung der Versicherungsbedingungen und Prämiensätze der im Deutschen Reiche arbeitenden Lebensversicherungs-Gesellschaften. Berlin. 1913.

}      *The Author.*

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1. International nosological classification, etc. 2. Secular progress of Pulmonary Tuberculosis, Cancer, etc. 3. The improvement in Infantile Mortality. 4. Secular and annual fluctuations of Deaths from several diseases. Sydney. 1913.

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**Lalaguna (M. P.).**

Memoria acerca de las Reservas Matemáticas con aplicaciones numéricas á una compañía de seguros. Madrid. 1912.

}      *The Author.*

**Laycock (F. U.).**

Political Economy in a nutshell. Svo. 1907.

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**Lefrancq (E.).**

Sur la comptabilisation des Emprunts par obligations. Brussels. Svo. 1912.

}      *The Author.*

**Life Offices' Association.**

Memoranda as to the Meetings of the Standing Committee, 1912-13.

}      *The Association.*

**London Mathematical Society.**

Proceedings, 1912-13.

*The Society.*

**McAdam (L.).**

Total disability benefits. Based on Mead's adjusted American Maccabees' Experience Tables at  $3\frac{1}{2}$  per cent. interest. Svo. Chicago. 1913.

}      *The Author.*

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## Macaulay (T. B.).

The supposed inferiority of First and Second born members of Families. Statistical fallacies. 8vo. }  
1912. } Purchased.

## Manes (Dr. A.).

Versicherungs - Lexikon. Ergänzungsband für die Jahre 1908 bis 1912. 4to. Tübingen. 1913. }  
Versicherungswesen. 2nd Edit. 8vo. Leipzig. 1913. }  
The Author.

## Medico-Actuarial Mortality Investigation.

Influence of build on mortality among Men. Causes of death among Men. Mortality among Women. }  
Influence of build on Mortality among Women. }  
Causes of death among Women. Mortality among North American Indians, Negroes, Chinese, and Japanese in North America. Vol. II. 4to. New York. 1913. }  
A. Hunter.

## Minutilli (G.).

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## Moir (H.).

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1912. }  
Purchased.

Moody's Manual of Railroad and Corporation Securities, 1913. }  
Purchased.

## Murai (H.).

Tables d'intérêts composés de Dépôts, de Rentes et d'Amortissements, calculées à dix décimales. }  
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## National Health Insurance Commission.

Statutory Rules and Orders, Regulations, Forms, Circulars, and other papers issued by the Commissioners relative to the National Insurance Act, 1911. }  
The Insurance Commissioners.

## Orr (L.P.).

Insurance Research and Eugenics. Edinburgh, 1912. The Author.

## Parliamentary Papers.

Acts of Parliament.  
Consolidated Fund, 1912.  
Elementary School Teachers (Superannuation), 1912.  
Finance, 1912.  
Public Works Loan, 1912. }  
Purchased.

Assurance Companies. Returns to the Board of Trade. 1912. }  
The Board of Trade.

Censuses.  
1911. England and Wales. Vols. II to VI, and Vol. VII containing Report by G. King on Graduation of Ages. Vols. VIII and XII. Fol. 1912-13.  
1911. Ireland. General Report, with Tables and Appendix. Fol. 1913.  
1911. Scotland. Vol. I, Parts 7 to 37. Fol. 1912-13.  
1911. Islands in the British Seas. Isle of Man, Jersey, Guernsey and adjacent Islands. Fol. 1913. }  
Purchased.

*By whom presented  
(when not purchased).*

**Parliamentary Papers—(continued).**

**Censuses—(continued).**

1911. New Zealand. Results of a Census of the Dominion. Fol. 1912.  
Report of the Government Statistician. 4to. } 1913. } *The Government of New Zealand.*

**Colonies.**

- Australia, Commonwealth of.  
Official Statistics, 1912–13. }  
Official Year Book, 1913. } *The Commonwealth Govt.*

**Canada.**

- Report of the Superintendent of Insurance for } the year 1911. } *The Government Insurance Dept.*

**New South Wales.**

- Friendly Societies. Report of the Registrar for 1911. }  
Government (including Municipal and Shire) Services Superannuation Fund. Report of the Committee and the Actuarial Sub-Committee on the establishment of. Fol. Sydney. 1912. } *The Government of N.S.W.*

- Official Year Book, 1911–12.

- Statistical Register for 1911 and previous years.

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